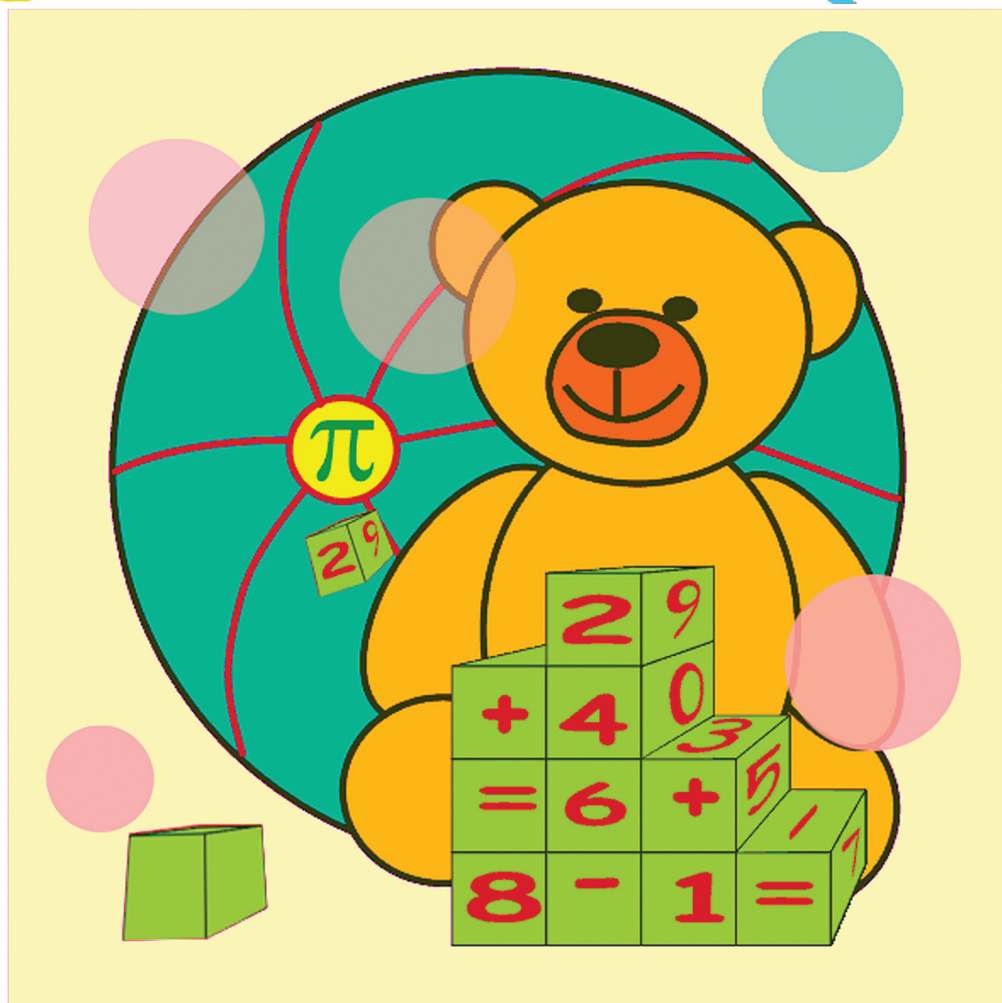




The 3rd International Scientific Colloquium **MATHEMATICS AND CHILDREN** (The Math Teacher)

Osijek, March 18, 2011

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**The Third International Scientific Colloquium
MATHEMATICS AND CHILDREN
(The Math Teacher)**

**Treći međunarodni znanstveni skup
MATEMATIKA I DIJETE
(Učitelj matematike)**

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Osijek, March 18, 2011

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Editor's Note

The Organising Committee of the *Third International Scientific Colloquium Mathematics and Children* persists in its attempts to encourage scientists at international level to conduct research in the area of teaching mathematics. In Croatia, much like in Europe and around the world¹, when drafting the proposal for the National framework curricula, special attention is given to the field of mathematics². The mathematics classroom in Croatia, however, faces slow changes regarding strategic and organisational goals, as well as social and technical conditions of education. Principally well-set tasks (introducing unsplit schedule into primary schools, broader inclusion of children with special needs – both the gifted ones and those with difficulties into regular public instruction, assigning assistants to pupils with special needs, organising extended daycare for all interested pupils, healthy meals in schools, higher mobility of pupils and teachers) are not successfully implemented due to the poor economic situation. Negative atmosphere of hesitation is mirrored in the results of much research in mathematics teaching.

For example, the focus of PISA research³, OECD's international programme for the assessment of knowledge and skills of fifteen-year-olds conducted in 2009 in 65 countries around the world, 32 of which are members of OECD, were students' competences in reading, mathematics and science proficiency. Reading competences of the participants were assessed on six levels. The results, made public in December 2010, bring up many questions which in Croatia must be answered in a quick and ingenious fashion. Within this research the result that causes most concern is that 27% of participants from Croatia never read for pleasure, and more than 57% claim to read only in order to find specific information [8, 15–19]. What is positive is that PISA research includes countries of different educational strategies and social systems. In this way it is possible to critically learn from comparative analyses of student achievements, in view of the turning points in education in countries which have between two instances of research achieved statistically significant improvement.

Yet, do we learn fast enough? Despite all the changes attempted in higher education⁴ and science⁵, the society in Croatia is still at the crossroads in search

¹ *Key Competences for Lifelong Learning-European reference framework* http://ec.europa.eu/dgs/education_culture/publ/pdf/II-learning/keycomp_en.pdf

² *Prijedlog Okvirnog matematičkog kurikula*, radna skupina Hrvatskoga matematičkoga društva, Poučak X(38) 2009., 4–20.

³ PISA (Programme for International Student Assessment) program-međunarodno procjenjivanje znanja i vještina petnaestogodišnjih učenika

⁴ *Londonška ministarska konferencija o Bolonjskom procesu* održana 17. i 18. 05. 2007., dostupno 4. 02. 2011., <http://public.mzos.hr/fgs.axd?id=12926>.

⁵ *Pravilnik o znanstvenim i umjetničkim područjima, poljima i granama* (NN 78/08) od 22. 09. 2009.

of the right path. The defining of educational areas and, in connection to that, the enactment of Framework national curricula with clearly defined learning outcomes for each field and each term, including lifelong learning, is of strategic interest for every country. However, this is only a part of work still to be done within the educational system in the Republic of Croatia. The changes in education should be aligned with strategic guidelines in economy (which, unfortunately, does not have a well-defined direction) and the uniqueness of our natural and social resources. The school needs to become and remain the fruitful soil for motivated personalities with interest to, by means of acquired knowledge and skills, take charge of the situation in production, economy, culture, art or politics, and then, in accord with their own ability, better the society as a whole. Economy and science, institutes and faculties, should synergise to create a type of instruction implemented by competent teachers in order for children and students to change the ways of society as soon as possible – for the better [5].

New technologies in the classroom also require changes in methodology of (mathematics) education, of both children and students, especially those involved in teacher studies [1]. The development of intelligent systems in the educational domain naturally implies the presence of a skilled teacher equipped for their implementation in practice. Faculties of teacher education need to, without delay and by means of flexible programmes and resources, lead the way to modern education of students – mastering of tutor systems which can be used for learning support and acquisition of new knowledge and skills, integrating multimedia in the classroom and personalising the approach to pupil (student) [6]. Long distance learning, the usage of expert systems for recognising gifted pupils with the interest of encouraging their development, as well as other forms of artificial intelligence, are imperatives of this age [4]. Simultaneously, the market is overwhelmed with programme languages, tools, and educational computer games. Are we capable of successfully classifying them according to their function? Are we sure that the computer games currently on the market have been scientifically and professionally verified for possible application in the (mathematics) classroom? Do the available games intended for fun really have no bad influence on a child's health and their inner world? Such and similar questions are raised by teachers and parents who are in need of scientific answers.

Numerous legal sources of the European Union contribute to the easier co-ordination of the national legislation in the area of special interest to people with disabilities. Teachers are expected to more formally be familiar with some, for example the Council Resolution put forth on May 5 2003 regarding equal education opportunities for pupils and students with disabilities 320030604(01)⁶. In our society there are approximately 10 – 15% of children with special needs, who, along with their families, expect some support from the community when their education is concerned [2]. Besides the regulation of legal and, subsequently, material issues for people with disabilities, it is necessary to equip students of teacher studies with knowledge and skills required to recognise, educate and bring up children with dyscalculia, dyslexia, dysgraphia or simply attention deficit disorder [3]. Are the faculties of teacher education ready for educating students in this field? Students

⁶ *Nacionalna strategija izjednačavanja mogućnosti za osobe s invaliditetom od 2007. do 2015.*, Vlada RH, Zagreb, 2007.

in this respect ask for more! What are experiences of school teachers? Can any of the omissions be compensated for within lifelong education of teachers, and in which manner, is yet another question that requires a speedy answer.

It is necessary to systematically review approaches to educating (mathematics) teachers, however, even more important is the readiness of the programme creators to implement inventive change [7, 21–34].

Nowadays the society and academic community expect a great deal from teachers. On the other hand, this goes hand in hand with the essence of the profession. In children's play little leaders still often and gladly identify with the role of a teacher if they want to control the group. For children of younger school age in particular, the teacher becomes a role model in every aspect. Yet, the tasks that are imposed on the teacher by the society, and for which they are during their studies neither theoretically, nor practically equipped, become a real burden in practice. It is in the interest of general well-being of our children, and the progress of society as a whole, that faculties of teacher education become aware of their responsibility and educate the teacher of tomorrow.

Likewise, faculties of teacher education are expected to create doctoral study programmes directed towards the scientific promotion of teachers. The academic community is required to give every teacher the opportunity for scientific advancement. This task became more realistic in 2009 when the National Science Council expanded its Statute on areas, fields and branches of science and arts. This statute includes in 1. Areas of natural science the field of 1.07. *Interdisciplinary natural science* accompanied by the branch 1.07.01 *Teaching methodology in natural science*. In addition to that, there is a new 8. Interdisciplinary area of science (science; arts) with a field 8.05. *Educational science*, within which are, among others, listed disciplines of pedagogy. However, according to the Statute on changes and additions to the Statute on conditions for entering scientific vocations (NN 71/10) within 8. Interdisciplinary area of science it is also possible to enter scientific vocations by combining criteria for two (or more) fields in different scientific areas (for example, for mathematics teaching methodology the appropriate combination might be 1.01 mathematics and 5.07 pedagogy).

Currently there is a list being compiled with appropriate scientific journals which accept contributions on teaching mathematics within the aforementioned fields. However, this still does not solve all the problems for teachers of mathematics who want to do scientific research in teaching (mathematics), and faculties of teacher education which are expected to create such programmes. Because the declarative confession supposes that research in mathematics embraces multidisciplinary and interdisciplinary approaches, it is important to create doctoral studies which will allow participants to take part in such programmes, though we still support the attitude that development of mathematics teaching methodology should remain within the boundaries of mathematical teaching practice.

While reading the works which deal with the issue of being a *teacher*, we find that the society has always focused more on what is expected from a teacher, rather than ensuring them a high quality of education, better living conditions, or a more agreeable status in society. This subject is often brought up by, for example, Croat-

ian novelists⁷, as well as those from around the world⁸, celebrated film actors⁹, and occasionally we also find romantic sketches on the subject of *teacher* in poems.¹⁰

Within the colloquia *Mathematics and Children* we continue to honour teachers and scientists in the field of mathematics who have sprung from our midst, given a great contribution to teaching and popularising mathematics, and who are no longer present among us. At the first conference we talked about the life and work of prof. dr. sc. Boris Pavković (1931. – 2006.). During the second one we marked the hundred-year anniversary of academician Stanko Bilinski (1909. – 1998.) by providing comments and analysis of his article which is of topical interest even today, 'Shall we study mathematics?', published in 1959 in the Mathematics and Physics Paper with the intention to encourage pupils to enroll mathematics studies. Unfortunately, during the previous years Croatia has lost a number of distinguished mathematicians who have contributed greatly to teaching mathematics. These contributions can be roughly divided into two crucially different, yet inseparable wholes. Two leading figures in mathematics – prof. dr. Svetozar Kurepa (1929. – 2010.) and prof. dr. Krešimir Horvatić (1930. – 2008.) have in this area (Osijek) initiated, organised and coordinated the implementation and realisation of supplementary studies in teaching mathematics during the 1970's and 1980's at the Department of Mathematics of the Faculty of Science in Zagreb. The supplementary studies gave the opportunity for a hundred participants to receive the diploma for teaching mathematics, which resulted in a far-reaching incentive for economic and scientific growth in Slavonia, Baranja and, partially, Bosnia and Herzegovina. The second group of mathematicians will be mostly remembered for the brilliant books they wrote, their journals and magazines, and activities done in order to popularise mathematics, as well as their scientific contribution to teaching mathematics. Those are the aforementioned professor Boris Pavković (1931. – 2006.) and academician Bilinski (1909. – 1998.), as well as Dominik Palman (1924. – 2006.), Vladimir Devide (1925. – 2010.) and Zdravko Kurnik (1937. – 2010.), all professors at the Department of Mathematics of the Faculty of Science in Zagreb. Worldwide, there is also news that a great opus has been completed by the famous populariser of mathematics Martin Gardner (1914. – 2010.).

Accordingly, popularising mathematics plays an important part in motivating pupils and students to do mathematics. This is particularly an important and vital question when considering the children of early, pre-school and younger school age. We are, therefore, proud that one of the initiatives of the participant of the first conference *Mathematics and Children* has reached its closing. Our nominee for the state award for science awarded by the Ministry of Science, Education and Sports in the year 2007, dr. sc. Mirko Polonijo, full professor at the Department of Mathematics of the Faculty of Science, University of Zagreb, is the recipient of this award! We are happy to know that the award for the year 2007 was given to professor Mirko Polonijo for popularising and promotion of mathematics among the young.

⁷ Šenoa, August., *Branka*, Alfa, Zagreb, 2008.

⁸ Frank Mc Courta, *Učitelj (Teacher Man)*, Algoritam, Zagreb, 2008.

⁹ Sidney Poitier u ulozi učitelja Marka Thackeraya u engleskoj filmskoj drami, *To Sir, with Love* (1967).

¹⁰ Dragutin Tadijanović, *Da sam ja učiteljica*/ Pjesme i proza/Dragutin Tadijanović. Zagreb: Matica Hrvatska; Zora, 1969. str. 134.

The Organising Committee of the *Third Scientific Colloquium Mathematics and Children* has chosen as the central topic of the conference *The Math Teacher*, which includes current issues from the professional, scientific and everyday surroundings of a teacher of mathematics. It is hereby important to know that the term teacher denotes a person who teaches, and the context of each work will reveal whether this implies a teacher in a pre-school institution, school or a university instructor.

We believe that the presentations, discussion and exchange of experience of host and foreign presenters at the *Third Scientific Colloquium Mathematics and Children* will encourage and strengthen scientific cooperation in the field of education, work and position of the teacher of mathematics within the boundaries of professional, cultural, academic – shortly, diverse community.

On behalf of the Organising Committee and myself, I would like to thank all the present persons for the time dedicated to the exchange of opinions and ideas. Also, we would like to thank the officials and hard-working entrepreneurs of the local community, as well as the Ministry of Science, Education and Sports of the Republic of Croatia for providing financial support vital for the organisation of this conference.

Osijek, March 18, 2011

Margita Pavleković

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Riječ urednice

Organizacijski odbor *Trećega međunarodnoga znanstvenoga kolokvija Matematika i dijete* ustrajava u poticanju znanstvenika na međunarodnoj razini u istraživanjima iz područja nastave matematike. U Hrvatskoj se, po ugledu na Europu i svijet¹, pri izradi prijedloga Nacionalnoga okvira kurikula posebna pozornost pridaje području matematike². U nastavi se matematike u Hrvatskoj još uvijek sporo mijenjaju strateški i organizacijski ciljevi te socijalni i tehnički uvjeti odgoja i obrazovanja. Deklarativno dobro postavljene zadaće (uvođenje jednosmjenske nastave u osnovne škole, sveobuhvatnija inkluzija djece posebnih potreba – darovitih i onih s teškoćama u redovite odjele, dodjeljivanje asistenata učenicima posebnih potreba, organizacija produženoga boravka za sve zainteresirane učenike, ponuda zdrave prehrane u školama, veća pokretljivost učenika i učitelja) zbog loših gospodarskih prilika ne zaživljavaju. Negativne konotacije oklijevanja zrcale se u rezultatima mnogih istraživanja iz nastave matematike.

Primjerice, u težištu PISA istraživanja³, OECD-ova međunarodnoga programa za procjenu znanja i vještina petnaestgodišnjaka koje je provedeno 2009. godine u 65 zemalja svijeta od kojih su 32 članice OECD-a, bile su učeničke kompetencije u čitalačkoj, matematičkoj i prirodoslovnoj pismenosti. Čitalačke kompetencije ispitanika procjenjivale su se na šest razina. Rezultati objavljeni u prosincu 2010. zahtijevaju otvaranje niza pitanja na koja u Hrvatskoj moramo brzo i domišljeno odgovoriti. Iz toga istraživanja najviše zabrinjava podatak da 27% ispitanika iz Hrvatske nikada ne čita iz zadovoljstva, a više od 57% ih izjavljuje da čita samo kako bi pronašli određenu informaciju [8, 15–19]. Dobro je što u PISA istraživanju sudjeluju zemlje različitih obrazovnih strategija i društvenih uređenja. Tako je moguće kritički učiti iz usporednih analiza učeničkih postignuća s jedne strane, prema učinjenim zaokretima u obrazovanju u onim zemljama koje su između dvaju razdoblja ispitivanja postigle statistički značajnija poboljšanja.

No, učimo li dovoljno brzo? Unatoč poduzetim promjenama u visokom obrazovanju⁴ i zanosti⁵ društvo u Hrvatskoj još uvijek tapka na raskrižju izbora pravoga puta. Definiranje obrazovnih područja te s tim u vezi donošenje Okvira

¹ *Key Competences for Lifelong Learning-European reference framework* http://ec.europa.eu/dgs/education_culture/publ/pdf/II-learning/keycomp_en.pdf

² *Prijedlog Okvirnog matematičkog kurikula*, radna skupina Hrvatskoga matematičkoga društva, Poučak X(38) 2009., 4–20.

³ PISA (Programme for International Student Assessment) program-međunarodno procjenjivanje znanja i vještina petnaestogodišnjih učenika

⁴ *Londonska ministarska konferencija o Bolonjskom procesu* održana 17. i 18. 05. 2007., dostupno 4. 02. 2011., <http://public.mzos.hr/fgs.axd?id=12926>.

⁵ *Pravilnik o znanstvenim i umjetničkim područjima, poljima i granama* (NN 78/08) od 22. 09. 2009.

nacionalnoga kurikula s prepoznatim ishodima učenja za svako područje i svako obrazovno razdoblje, uključujući i cjeloživotnu izobrazbu strateškoga je interesa svake zemlje. No, to je tek dio posla unutar sustava obrazovanja u RH koji još uvijek stoji pred nama. Promjene u obrazovanju trebalo bi uskladiti sa strateškim smjernicama u gospodarstvu (koje na žalost nema definiran pravac razvoja) i posebnostima naših prirodnih i društvenih resursa. Škola treba postati i ostati rasadnik motiviranih ličnosti, zainteresiranih da znanjem i vještinama ovladaju prilikama u proizvodnji, gospodarstvu, kulturi, umjetnosti, politici pa potom, u skladu sa svojim mogućnostima unaprjeđuju društvo u cjelini. Privreda i znanost, instituti i fakulteti trebali bi skladnom suradnjom kreirati nastavu vođenu od kompetentnih učitelja kako bi djeca i studenti što ranije mijenjali zbivanja u društvu – nabolje [5].

Nove tehnologije u nastavi također iziskuju promjene u metodologiji (matematičkoga) obrazovanja, kako djece tako i studenata, osobito učiteljskih studija [1]. Razvojem inteligentnih sustava u domeni obrazovanja prirodno je očekivati osposobljenog učitelja za njihovu primjenu u praksi. Nastavnički fakulteti bez odlaganja trebaju fleksibilnim programima i resursima predvoditi suvremenu izobrazbu studenata – ovladavanje tutorskim sustavima koji mogu biti potpora učenju i svladavanju novih znanja i vještina, uključivanje multimedije u nastavu te personalizirani pristup učeniku (studentu) [6]. Učenje na daljinu, korištenje ekspertnih sustava za prepoznavanje darovitih učenika s ciljem poticanja njihova razvoja, ali i drugih oblika umjetne inteligencije imperativi su vremena [4]. Istodobno, tržište je preplavljeno programskim jezicima, alatima, edukativnim računalnim igrama. Jesmo li ih u stanju dobro klasificirati s obzirom na primjenu? Jesmo li sigurni da su računalne igrice koje kolaju tržištem prošle znanstvenu i stručnu provjeru za moguću primjenu u nastavi (matematike)? Jesu li raspoložive igrice namijenjene zabavi doista bez lošeg utjecaja na djetetovo zdravlje i njegov unutarnji svijet? Na ova i slična pitanja učitelji iz prakse, ali i roditelji traže znanstvene odgovore.

Brojni pravni izvori Europske unije pridonose lakšem usklađivanju nacionalnoga zakonodavstva na području od posebnoga interesa za osobe s invaliditetom. Od učitelja se očekuje temeljitije poznavanje primjerice Rezolucije Vijeća od 5. svibnja 2003. o jednakim mogućnostima obrazovanja i izobrazbe za učenike i studente s invaliditetom 320030604(01)⁶. U društvu ima oko 10 – 15% djece s teškoćama, koji zajedno sa svojim obiteljima očekuju od društvene zajednice podršku u obrazovanju [2]. Osim uređivanja pravnih pa onda i materijalnih pitanja za osobe s invaliditetom, nužno je studente učiteljskih studija osposobiti za prepoznavanje, obrazovanje i odgoj djece s diskalkulijom, disleksijom, disgrafijom ili samo poremećajem pažnje [3]. Jesu li nastavnički fakulteti dorasli edukaciji studenata u ovom području? Studenti/ce traže u ovom segmentu više! Što o tome govore učitelji iz prakse? Može li se što od propuštenoga popraviti, i na koji način u okviru cjeloživotne izobrazbe učitelja, još je jedno pitanje na koje treba što prije odgovoriti.

Neophodno je sustavno preispitivanje pristupa obrazovanju učitelja (matematike), ali je još važnija spremnost kreatora programa na provođenje domišljenih promjena [7, 21–34].

⁶ Nacionalna strategija izjednačavanja mogućnosti za osobe s invaliditetom od 2007. do 2015., Vlada RH, Zagreb, 2007.

Danas društvo i akademska zajednica iznimno mnogo očekuju od učitelja. S druge strane, moglo bi se reći da je to konstanta kroz povijest u bitku toga zvanja. U dječjim igrama mali predvodnici još se uvijek često i rado poistovjećuju s ulogom učitelja/ice kada nastoje ovladati skupinom. Osobito za mlađu školsku dob učitelj/ica postaje uzorom u svakom pogledu. No, zadatci koje društvo stavlja pred učitelja, a za koje tijekom studija nije ni teorijski, niti praktično osposobljen, postaju ozbiljan teret u praksi. U interesu sveopćeg boljitka naše djece, ali i napretka društva u cjelini iznimno je važno da učiteljski fakulteti osvjeste svoju odgovornost i obrazuju učitelja sutrašnjice.

Također se od učiteljskih fakulteta očekuje da kreiraju programe doktorskih studija usmjerenih znanstvenom napredovanju učitelja. Akademska zajednica dužna je dati svakom učitelju realnu priliku da znanstveno napreduje. Taj zadatak postao je izglednijim 2009. godine kada je Nacionalno vijeće za znanost proširilo Pravilnik o znanstvenim i umjetničkim područjima, poljima i granama. Tim pravilnikom u okviru 1. Područja prirodnih znanosti javlja se polje 1.07. *Interdisciplinarne prirodne znanosti s granom 1.07.01 Metodika nastavnih predmeta prirodnih znanosti*. Osim toga, usustavljeno je novo 8. Interdisciplinarno područje znanosti (znanost; umjetnost) s poljem 8.05. *Obrazovne znanosti*, gdje se, između ostaloga nabrajaju pedagoške discipline. No, prema Pravilniku o izmjenama i dopunama Pravilnika o uvjetima za izbor u znanstvena zvanja (NN 71/10) unutar 8. Interdisciplinarnoga područja znanosti moguće je također napredovati u znanstvenim zvanjima kombinirajući kriterije za dva ili više izbornih polja iz različitih znanstvenih područja (primjerice, za metodiku matematike prikladna kombinacija mogla bi biti 1.01 matematika i 5.07 pedagogija).

Ovih dana radi se na izradi liste s popisom odgovarajućih znanstvenih časopisa koji prihvaćaju znanstvene radove iz metodike matematike u okvirima spomenutih polja. No, time još uvijek nisu riješena sva pitanja za učitelje matematike koji žele znanstveno napredovati u području nastave (matematike) i učiteljskih/nastavničkih fakulteta od kojih se očekuje da kreiraju takve programe. S deklarativnoga priznanja da istraživanja u nastavi matematike pretpostavljaju multidisciplinarnost i interdisciplinarnost važno je u kreiranju doktorskih studija zainteresiranima osigurati takve programe, iako i dalje ustrajavamo na tomu da razvoj metodike matematike treba njegovati u okvirima matematičke struke.

Čitajući djela s problematikom *učitelj*, nalazimo da se društvo oduvijek više bavilo očekivanjima od učitelja, a manje osiguranjem kvalitete njegove izobrazbe, uvjetima života, statusom u društvu. O tome nam govore primjerice romanopisci iz Hrvatske⁷ i svijeta⁸, proslavljeni glumci iz filmova⁹ a samo ponekad nailazimo i na romantičarske crtice o temi *učitelj* u poeziji.¹⁰

Na kolokvijima *Matematika i dijete* nastavljamo sjećanjima i zahvalom učiteljima i znanstvenicima matematike koji su potekli iz naše sredine, dali veliki

⁷ Šenoa, August., *Branka*, Alfa, Zagreb, 2008.

⁸ Frank Mc Courta, *Učitelj (Teacher Man)*, Algoritam, Zagreb, 2008.

⁹ Sidney Poitier u ulozi učitelja Marka Thackeraya u engleskoj filmskoj drami, *To Sir, with Love* (1967).

¹⁰ Dragutin Tadijanović, *Da sam ja učiteljica* Pjesme i proza/Dragutin Tadijanović. Zagreb: Matica Hrvatska; Zora, 1969. str. 134.

doprinos u nastavi i popularizaciji matematike, a kojih više nema. Na prvom skupu govorili smo o životu i djelu prof. dr. sc. Borisa Pavkovića (1931. – 2006.). Na drugom smo skupu povodom stogodišnjice rođenja akademika Stanka Bilinskoga (1909. – 1998.) komentirali i analizirali njegov, i danas aktualni članak “Hoćemo li studirati matematiku?” objavljen 1959. u Matematičko-fizičkom listu, a kojim se učenici potiču na upis studija matematike. Na žalost, u zadnjih nekoliko godina Hrvatska je ostala bez niza uglednih matematičara koji su dali veliki doprinos nastavi matematike. Taj doprinos možemo grubo podijeliti u dvije bitno različite, ali ipak nerazdvojne cjeline. Dva su matematička velikana – prof. dr. Svetozar Kurepa (1929. – 2010.) i prof. dr. Krešimir Horvatić (1930. – 2008.) na ovim prostorima (Osijeku) pokrenuli, organizirali i koordinirali provedbu i realizaciju dopunskoga studija iz matematike nastavnoga smjera sedamdesetih i osamdesetih godina prošloga stoljeća ispred Matematičkoga odjela Prirodoslovno-matematičkoga fakulteta iz Zagreba. Dopunskim studijem stotinjak je polaznika steklo visoku stručnu spremu i diplomu učitelja matematike. Time je učinjen dalekosežni poticaj stručnom, znanstvenom i gospodarskom zamahu na prostorima Slavonije, Baranje i dijelu Bosne i Hercegovine. Drugu skupinu matematičara ponajprije ćemo pamtiti po sjajnim knjigama, časopisima, provedbi matematičkih natjecanja, kvizova te drugih aktivnosti s ciljem popularizacije matematike, ali i znanstvenoga doprinosa nastavi matematike. To su već spomenuti profesor Boris Pavković (1931. – 2006.) i akademik Bilinski (1909. – 1998.), zatim Dominik Palman (1924. – 2006.) i Zdravko Kurnik (1937. – 2010.), redom profesori s Matematičkoga odjela Prirodoslovno-matematičkoga fakulteta u Zagrebu. Posebna zahvala pripada i osebnjoj osobi – matematičaru, književniku i japanologu, koji je radio na Fakultetu strojarstva i brodogradnje prof. dr. sc. Vladimiru Devideu (1925. – 2010.). Zaljubljenik u matematiku i umjetnost uvijek je nalazio vremena pisati za najmlađe. U širim krugovima u svijetu odjeknula je i vijest da je svoja djela zaokružio i veliki popularizator matematike, Martin Gardner (1914. – 2010.).

Popularizacija matematike igra važnu ulogu u motivaciji učenika i studenata za bavljenje matematikom. Osobito je to važno i osjetljivo pitanje kada su u žarištu pozornosti djeca rane, predškolske i mlađe školske dobi. Stoga nas raduje što su jednu inicijativu pokrenuli sudionici *Prvoga međunarodnoga znanstvenoga kolokvija Matematika i dijete* i uspješno priveli kraju. Naš predloženi državne nagrade za znanost Ministarstva znanosti obrazovanja i športa u 2007. godini, dr. sc. Mirko Polonijo, redoviti profesor u trajnom zvanju Matematičkoga odjela Prirodoslovno-matematičkoga fakulteta Sveučilišta u Zagrebu, postao je dobitnik te nagrade! Raduje nas što je godišnja nagrada te 2007. godine dodijeljena profesoru Mirku Poloniju za popularizaciju i promidžbu matematike među najmlađima.

Organizacijski odbor Trećeg međunarodnoga kolokvija *Matematika i dijete* postavio je *Učitelja matematike* s aktualnostima iz njegova stručnoga, znanstvenog i životnog okruženja, središnjom temom skupa. Pri tome je važno znati da termin *učitelj* rabimo za osobu koja poučava, a iz konteksta svakoga rada razumjeti će se govori li se o učitelju u predškolskoj ustanovi, školi ili o sveučilišnom nastavniku.

Vjerujemo da će izlaganja, rasprava i izmjena iskustava domaćih i stranih izlagača na Trećem znanstvenom skupu *Matematika i dijete* potaknuti i ojačati

znanstvenu suradnju iz područja izobrazbe, djelovanja i položaja učitelja matematike u okvirima stručne, kulturne, akademske – jednom rječju, svekolike zajednice.

U ime organizacijskoga odbora i u svoje osobno ime zahvaljujem nazočnima na vremenu posvećenom razmjeni mišljenja i zajedničkom druženju. Također, čelnicima i vrijednim poduzetnicima lokalne zajednice te Ministarstvu znanosti, obrazovanja i športa RH zahvaljujemo na sponzorstvu za održavanje ovoga skupa.

U Osijeku, 18. ožujka 2011.

Margita Pavleković

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Contents

Preface	3
----------------------	---

1. New approaches to education of math teachers

<i>Rukavina, S.; Kovčević, D.; Bakić, M.</i> Why is there a shortage of mathematics teachers? The high-school students' views on the mathematics teacher profession	9
---	---

<i>Romstein, K.; Ajman, S.</i> Mathematics in programs of pre-school teachers' professional education	20
---	----

<i>Vulović, N.; Mihajlović-Kononov, A.; Egerić, M.</i> Contemporary Approaches in Teaching Mathematics at Teacher Education Faculties in Serbia	26
---	----

<i>Jukić, Lj.; Matić, I.</i> Educating future mathematics teachers: repeating mathematics from primary and secondary school	27
---	----

<i>Mrkonjić, I.; Topalovec, V.; Marinović, M.</i> Concept Maps in Maths Teachers' Education	35
--	----

<i>Stanić, M.</i> The Number Line as an Arithmetic Machine	36
---	----

2. Skills, knowledge and abilities of students

<i>Manfreda Kolar, V.; Hodnik Čadež, T.</i> Analysis of Inductive Reasoning in Mathematical Problem Solving among Primary Teacher Students	49
---	----

<i>Szilágyiné Szinger, I.</i> The study of the geometrical concepts of teacher training college students	64
--	----

<i>Petz Tiborné (Toth, Sz.)</i> The mathematics-knowledge of the teacher training' students in the first year	71
---	----

<i>Đeri, I.; Gregorović, Ž.; Moslavac, D.</i> Competence of teacher education students for giving lessons in advanced mathematics	79
---	----

<i>Čižmešija, A., Milin-Šipuš, Ž.</i> Spatial Reasoning of Students of Mathematics Education at the Department of Mathematics, Faculty of Science, University of Zagreb – Mental Cutting Ability	90
---	----

<i>Divjak, B.; Erjavec, Z.; Jakuš, M.; Žugec, P.</i> When technology influences learning?	92
--	----

3. How to achieve desirable learning outcomes within teacher studies

<i>Cotič, M.; Felda, D.</i> Between axioms and reality	103
---	-----

<i>Molnár, E.</i> Nice tilings, nice mathematics!?!	113
--	-----

<i>Munđar, D.; Horvat, D.</i> Use of technology for enhancement of teaching mathematics: perspective, problems, and criteria and selection method	120
---	-----

<i>Dobi Barišić, K.; Đeri, I.; Jukić, Lj.</i> What is the future of the integration of ICT in teaching mathematics	128
---	-----

<i>Đurđević, I.; Zekić-Sušac, M.; Pavleković, M.</i> The effect of students' learning style on the selection of elective modules	141
--	-----

<i>Brumec, M.; Divjak, B.; Žugec, P.</i> Self-efficacy of the first year ICT students	150
--	-----

<i>Brueckler, F., M.</i> Exploratory learning of mathematics in primary school: advantages and examples (poster)	158
--	-----

4. Teacher and students in teaching practice

<i>Brueckler, F., M.</i> Mathematical experiments	163
--	-----

<i>Petrović-Sočo, B.; Hajdin, Lj.; Balić, T.</i> Natural learning of mathematical terms in preschool	172
<i>Vekić-Kljajić, V.</i> Supporting development of mathematical skills of pre-school children through projects	174
<i>Munkácsy, K.</i> Real Objects and Problem Solving in Mathematics Education	180
<i>Pavleković, I.; Duvnjak, Krampač-Grljušić, A.</i> Adaptation of teaching units in mathematics for pupils with disabilities in the regular school system	185
<i>Basta, S.; Mesić, D.</i> Play – a way towards mathematical competence	188
<i>Juričić Devčić, M.</i> Didactic games in mathematics teaching	196
<i>Bjelanović Dijanić, Ž.</i> Discovery learning in mathematics by using dynamic geometry software <i>GeoGebra</i> – action research	198
 5. What difference can a math teacher make	
<i>Rešić, S.; Suljičić, A.</i> Low success in mathematics teaching: causes and consequences	215
<i>Mišurac Zorica, I.; Domazet, J.</i> Teachers' attitude toward mathematics	232
<i>Glasnović-Gracin, D.</i> Teacher and textbook in geometry education	234
<i>Agić, H.; Rešić, S.</i> Mathematics as a factor in the development of a well-educated individual	242
<i>Baranović, B.; Štibrić, M.</i> Mathematics education in Croatian elementary schools – the teachers' perspective	255
<i>Katalenić, A.; Tomić, D.</i> We would like to thank our distinguished mathematicians (poster)	257

Sadržaj

Predgovor	261
------------------------	-----

1. Novi pristupi u izobrazbi učitelja matematike

Rukavina, S.; Kovčević, D.; Bakić, M.

Zašto nedostaje profesora matematike? Stavovi gimnazijalaca o zanimanju “profesor matematike”	265
---	-----

Romstein, K.; Ajman, S.

Matematika u programima profesionalne edukacije odgojitelja	276
---	-----

Vulović, N.; Mihajlović-Kononov, A.; Egerić, M.

Savremeni pristupi učenja matematike na učiteljskim fakultetima u Srbiji	282
--	-----

Jukić, Lj.; Matić, I.

Obrazovanje budućih nastavnika matematike: Ponavljanje matematičkih sadržaja iz osnovne i srednje škole	293
---	-----

Mrkonjić, I.; Topalovec, V.; Marinović, M.

Konceptualne mape u obrazovanju učitelja matematike	302
---	-----

Stanić, M.

Brojeva crta kao aritmetički stroj	317
--	-----

2. Vještine, znanja i sposobnosti studenata učiteljskih studija

Manfreda Kolar, V.; Hodnik Čadež, T.

Analiza induktivnih strategija rješavanja matematičkih problema kod studenata primarne edukacije	331
--	-----

Szilágyiné Szinger, I.

Tanítójelöltek geometriai fogalmainak vizsgálata	333
--	-----

Petz Tiborné (Toth, Sz.)

Az elsőéves tanító szakos hallgatók matematikai ismeretei	340
---	-----

Đeri, I.; Gregorović, Ž.; Moslavac, D.

Kompetentnost studenata učiteljskih studija za izvođenje dodatne nastave iz matematike	348
--	-----

<i>Čižmešija, A., Milin-Šipuš, Ž.</i> Prostorno mišljenje studenata nastavnčkih studija matematike na Matematičkom odsjeku Prirodoslovno-matematičkog fakulteta Sveučilišta u Zagrebu – sposobnost mentalnog presijecanja	359
--	-----

<i>Divjak, B.; Erjavec, Z.; Jakuš, M.; Žugec, P.</i> Kada tehnologija utječe na učenje?	361
--	-----

3. Kako prema poželjnim ishodima učenja na učiteljskim studijima

<i>Cotič, M.; Felda, D.</i> Med aksiomi in stvarnostjo	371
---	-----

<i>Molnár, E.</i> Szép kövezések, szép matematika!?!	381
---	-----

<i>Mundar, D.; Horvat, D.</i> Upotreba tehnologije za unaprjeđenje nastave matematike: perspektiva, problemi, kriteriji i metoda odabira	388
--	-----

<i>Dobi Barišić, K.; Đeri, I.; Jukić, Lj.</i> Kakva je budućnost integracije ICT-a u poučavanje matematike	396
---	-----

<i>Đurđević, I.; Zekić-Sušac, M.; Pavleković, M.</i> Utjecaj stila učenja studenata na odabir izbornoga modula	409
---	-----

<i>Brumec, M.; Divjak, B.; Žugec, P.</i> Samoeфикаsnost IKT studenata prve godine	418
--	-----

<i>Brueckler, F., M.</i> Istraživačka nastava matematike u osnovnoj školi: prednosti i primjeri (poster)	426
--	-----

4. Učitelj i učenici u nastavnoj praksi

<i>Brueckler, F., M.</i> Matematički eksperimenti	431
--	-----

<i>Petrović-Sočo, B.; Hajdin, Lj.; Balić, T.</i> Prirodno učenje matematičkih pojmova u dječjem vrtiću	440
---	-----

<i>Vekić-Kljajić, V.</i> Poticanje razvoja matematičkih vještina kod djece predškolske dobi kroz projekte	451
---	-----

<i>Munkácsy, K.</i> A valódi tárgyak és a probléma megoldás szerepe a matematika tanulásában	457
--	-----

<i>Pavleković, I.; Duvnjak, Krampač-Grljušić, A.</i> Prilagodbe nastave matematike za učenike s teškoćama u redovnom školskom sustavu	462
---	-----

<i>Basta, S.; Mesić, D.</i> Igra– put prema matematičkoj kompetenciji	470
--	-----

<i>Juričić Devčić, M.</i> Didaktičke igre u nastavi matematike	478
---	-----

<i>Bjelanović Dijanić, Ž.</i> Učenje matematike otkrivanjem uz pomoć programa dinamičke geometrije <i>Geogebra</i> – Akcijsko istraživanje	491
--	-----

5. Što može promijeniti učitelj matematike

<i>Rešić, S.; Suljičić, A.</i> Slab uspjeh u nastavi matematike: uzroci i posljedice	507
---	-----

<i>Mišurac Zorica, I.; Domazet, J.</i> Stav učitelja razredne nastave prema matematici	526
---	-----

<i>Glasnović-Gracin, D.</i> Učitelj i udžbenik u nastavi geometrije	538
--	-----

<i>Agić, H.; Rešić, S.</i> Matematika u ulozi izgradnje dobro obrazovane ličnosti	547
--	-----

<i>Baranović, B.; Štibrić, M.</i> Nastava matematike u osnovnim školama u Hrvatskoj – Nastavnička perspektiva	560
---	-----

<i>Katalenić, A.; Tomić, D.</i> Zahvala našim uglednim matematičarima (poster)	562
---	-----

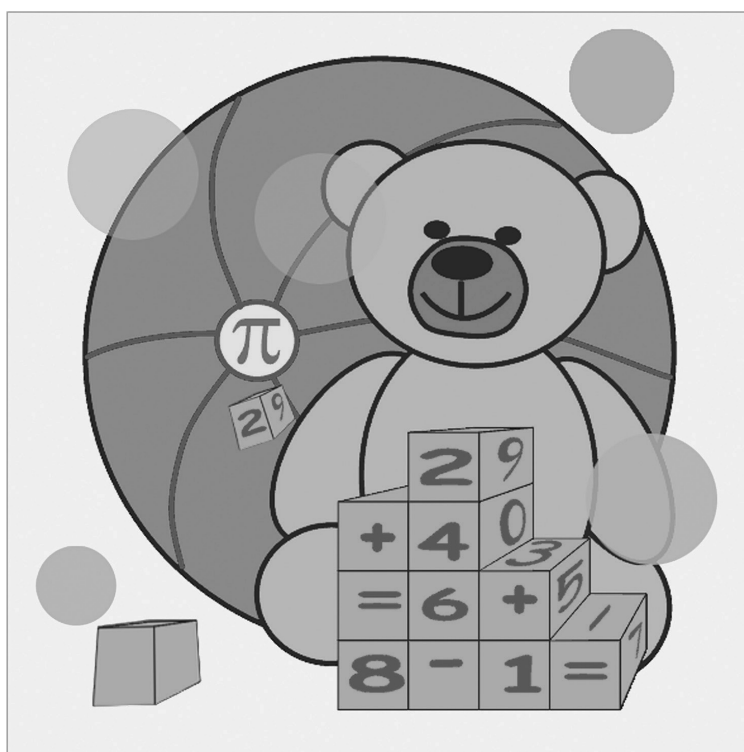
<i>Polonijo, M.; Varošanec, S.</i> Zdravko Kurnik (Travnik, 2. 2. 1937. – Zagreb, 25. 7. 2010.) svestrani hrvatski matematičar	563
--	-----

Index / Kazalo imena	571
-----------------------------------	------------

Acknowledgment of sponsors / Zahvala sponzorima	575
--	------------

The Third International Scientific Colloquium
MATHEMATICS AND CHILDREN
(The Math Teacher)

monography



Preface

In the monograph *The Math Teacher* the term *teacher* designates a person who teaches mathematics, and the context of each article reveals whether this implies the teacher at a pre-school institution, a school or a university instructor.

In their works the authors will critically assess study programmes used to educate teachers of mathematics. Furthermore, the results of research regarding levels of learning outcomes at faculties of teacher education in view of student achievements will be analysed. Finally, we will suggest solutions and answers to current issues concerning teaching practice.

Mathematics is one of nine academic areas with a finalised common framework in the first phase of the project *Tuning educational structures in Europe*¹ (stages/cycles, learning outcomes, general and specific competences, learning and teaching methods, implementation of ECTS system). In Croatia there is currently no consensus on the governmental level regarding the National framework curriculum for mathematics, although the objectives, learning outcomes and competences of pupils/students at the end of each term have been clearly defined by the professionals. We, therefore, encounter much vagueness when it comes to mathematical education of teachers, organising mathematics teaching across educational cycles, as well as lifelong education of teachers, and scientific promotion.

In the first chapter of this monograph titled *New approaches to education of math teachers*, the authors by means of their research offer answers to why this profession is in short supply. Moreover, in their articles the authors analyse the level of representation and the position of mathematics in study programmes which equip pre-school teachers of mathematics, as well as primary and secondary school teachers. Certain omissions and errors are indicated within programmes of teacher studies, and solutions are offered to revise them. New contents and original procedures are suggested in order to, according to the authors' opinion, achieve a higher quality education of teachers.

In the second chapter the authors present results of their research conducted among students of teacher studies. The research investigated levels of student achievements in important skills and knowledge relevant to the profession. We find particularly important the results of research implemented in order to familiarise ourselves better with student strategies of problem solving and their spatial thinking skills.

¹ Tuning educational structures in Europe. www.tuning.unideusto.org/tuningeu, first phase from 2001. – 2004.

In the third chapter of the monograph in their articles the authors answer the question *How to achieve desirable outcomes of mathematics learning*. Some of the authors place emphasis on the interactive constructivist approach to learning and teaching, and address the important question of popularising mathematics. Others are oriented towards information technology and research the attitude of students of teacher studies towards using ICT in the classroom, as well as critically assess relevant issues, criteria and methods for selecting the appropriate on-line materials for teaching mathematics. Modern learning conditions require a competent teacher and pupil/student for long distance learning and application of intelligent systems. It is therefore proposed that additional room be found for achieving such competences within reviewed programmes of teacher studies.

In the fourth chapter titled *Teacher and students in teaching practice*, in their contributions the authors speak of the results of research and/or success stories of strategies in mathematics teaching which have resulted in desirable outcomes of learning among children and students. The authors report the general awareness among parents, teachers and society of the importance of early mathematical education. Likewise, a growing number of parents of children with special needs are interested in integrating their children into regular public instruction, which is also suggested by the national development strategy of the Republic of Croatia². Thus, the future revision of programmes needs to include the development of curricula which would enable students to recognise, educate and support pupils with special needs.

In the fifth chapter of the monograph titled *What difference can a math teacher make* in their articles the authors comment on the general issues of accepting mathematics as a vital scientific field by the broader community. Certain research suggests that a child's attitude towards mathematics largely depends on the attitude of their parents, but also on the attitude of the teacher in the first four years of primary education. Issues connected to mathematics school books are brought up as well, and the chapter offers comments on the teacher perspective in Croatia.

From the articles we can conclude that math teachers face similar problems in Croatia as they do in the surrounding countries. They are not adequately equipped for their profession within the existing study programmes. They are not properly introduced to school mathematics. Due to lacking conditions regarding equipment at the university and in schools, they are not given the proper opportunity to master information technology and intelligent systems of learning. They mostly remain unprepared for professionally handling pupils with special needs. Finally, they show reluctance to deal with more serious problems of children and youth, such as drugs, violence and alcohol. They lack competences required to cooperate with parents and work in a team.

Governments systematically promote free movement of persons and the importance of professional and scientific development. There have been attempts to bring order into educational systems. Most teachers accept such proposals without complaints, even when they are somewhat ambiguous.

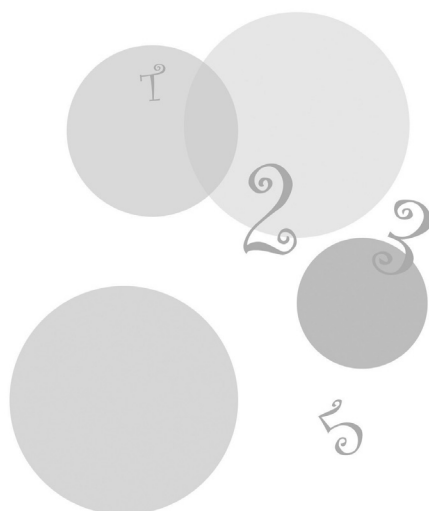
² Nacionalna strategija izjednačavanja mogućnosti za osobe s invaliditetom od 2007. do 2015., Vlada RH, Zagreb, 2007.

What remains is hope that the academic community in Croatia and its surroundings will find strength and competence to make and implement good and important decisions in science and education, and that under new circumstances the education of teachers and their lifelong development will be more open and constructive. Pupils and their teachers do not belong on the margins of society. Teachers must not be underpaid because their final product has always been and always will be the future of our society.

Margita Pavleković

1.

New approaches to education of math teachers



In this chapter the authors by means of their research offer an answer to a question of why the mathematics teaching profession is in short supply. Moreover, in their articles the authors analyse the level of representation and the position of mathematics in study programmes which equip pre-school teachers of mathematics, as well as primary and secondary school teachers. Since 2005 most of the students enrolled university within the newly-approved, so-called Bologna programme. Five years later, half-way through 2010 the students of the same generation are taking their graduation exams within integrated, five-year university teacher studies, and are in Croatia awarded the academic title *master of primary education* or *master of education in mathematics*. Certain omissions and errors are indicated within programmes of teacher studies, and solutions are offered to revise them. New contents and original procedures are suggested in order to, according to the authors' opinion, achieve a higher quality education of teachers.

Why is there a shortage of mathematics teachers? The high-school students' views on the mathematics teacher profession

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Abstract. The problem of the lack of Mathematics teachers at all levels of the school system in the Republic of Croatia is one of the most discussed issues among mathematicians nowadays. In spite of the fact that this shortage is well known to the general public not only from data provided by the HZZ (Croatian Employment Service), but also from frequent job vacancies for Mathematics teachers and scholarships for deficitary professions at both local and state levels, granted, among other professions, for Mathematics teachers, it is a fact that every year there are not enough students interested in choosing Mathematics Studies for Mathematics Teachers.

In order to answer the question why there is the lack of Mathematics teachers, we conducted a survey on general and mathematics high school students' views on being a Mathematics teacher. The aim of the survey is to find out what high school students think about the Mathematics teacher profession, what factors influence their views and at what extent, and whether their former Mathematics teacher has had any influence on their views. Furthermore, the aim is to find out how informed the students are regarding the interrelationship among certain professions, Mathematics teacher being one of them, based on the demand in the labour market and the relationship between average salaries for different professions, as well as to find out what they think about the interrelation between chosen professions according to their attractiveness.

Keywords: Mathematics teacher, high school students' views

Introduction

In the last few years the lack of Mathematics teachers was seen at all levels of the educational system. In the period from January to November 2010, 400 Mathematics teacher job vacancies were reported to the HZZ (Croatian Employment Service), and 553 job vacancies for Mathematics professors. On the other hand, there are four higher education institutions in Croatia that are offering those

professions, and according to their enrolment data, 250 new Mathematics teachers could graduate from those institutions every year. Nevertheless, this number is often halved. The lack of experts results in unprofessional mathematics substitutions that are often not able to transfer efficiently the necessary mathematics knowledge, nor to get students to love mathematics and be interested in it, which are significant factors in choosing the Mathematics teacher profession. Therefore, we face a vicious circle with no way out.

In order to try to give an answer to the question why there is the lack of Mathematics teachers, in November 2010 we conducted a survey on general and mathematics high school students' views on being a Mathematics teacher. The aim was to find out what factors influence their views and at what extent, and whether their former Mathematics teachers had had any influence. Furthermore, due to the assumption that high school students' views on being a Mathematics teacher, among others, are based upon the information about salaries, the demand in the labour market, and the overall situation in the Croatian educational system, etc., the survey embraced also their views on employment and salaries for several professions, Mathematics teacher being one of them.

Survey methodology and results

The survey was conducted through questionnaires given to students in four high schools. 412 general and mathematics high school students were given the questionnaires. Data analysis took into consideration 396 answers. Questionnaires that were not completely filled in were not taken into consideration. The sample included students from all high school grades, so the examinees' age range from 14 to 18 years of age. The survey included 91 student of the first grade, 86 students of the second grade, 83 students of the third grade, and 136 students of the fourth grade. The reason why students of those course of studies were chosen is the assumption that exactly they will be the majority of future mathematics students.

The distribution of subjects according to their course of studies and gender is shown in Table 1.

Subjects	General course of studies	Mathematical course of studies	Total
Schoolboys	83	82	165
Schoolgirls	192	39	231
Total	275	121	396

Table 1. Number of subjects according to gender and high school course of studies.

“I consider the profession of Mathematics teacher as an attractive profession.”

The analysis of the frequency of certain answers to the first question “I consider the profession of Mathematics teacher as an attractive profession” was done according to gender (Table 2.), high school course of studies (Table 3.) and grade (Table 4.) of the subjects. In order to ease data interpretation, in data preparation the answers “fully applies to me” and “mostly applies to me” were put together in

the category “applies to me”, and the answers “mostly does not apply to me” and “fully does not apply to me” in the category “does not apply to me”.

“I consider the profession of Mathematics teacher as an attractive profession.”		
Subjects' gender	Applies to me	Does not apply to me
Schoolboys	25, 45%	74, 55%
Schoolgirls	17, 31%	82, 69%
Total	20, 7%	79, 3%

Table 2. The frequency of certain answer categories to the question “I consider the profession of Mathematics teacher as an attractive profession” according to subjects' gender.

A mere 20,7% of subjects consider the profession of Mathematics teacher as an attractive profession, while the 79,3% of subjects consider it to be a non-attractive profession. Although HZZ (Croatian Employment Service) data show a high presence of women among mathematics teachers in the Republic of Croatia, a lower percentage of schoolgirls consider it as an attractive profession in relation to their male colleagues.

“I consider the profession of Mathematics teacher as an attractive profession.”		
Subjects' course of studies	Applies to me	Does not apply to me
General course of studies	17, 09%	82, 91%
Mathematical course of studies	28, 91%	71, 09%
Total	20, 7%	79, 3%

Table 3. The frequency of certain answer categories to the question “I consider the profession of Mathematics teacher as an attractive profession” according to subjects' course of studies.

The results show that students of the mathematical course of studies have a more positive attitude towards the profession of Mathematics teacher than the students of the general course of studies. Those results are in line with the choice of the course of studies attended by the students. The survey also shows that students who attend the mathematics course of studies have a more positive attitude towards mathematics as a school subject (Arambašić, Vlahović-Štetić and Severinac, 2005), so in that sense this result was expected.

“I consider the profession of Mathematics teacher as an attractive profession.”		
Subjects' grade	Applies to me	Does not apply to me
1. grade	30, 76%	69, 24%
2. grade	18, 59%	81, 41%
3. grade	25, 29%	74, 71%
4. grade	12, 49%	87, 51%
Total	20, 7%	79, 3%

Table 4. The frequency of certain answer categories to the question “I consider the profession of Mathematics teacher as an attractive profession” according to subjects' grade.

In Table 4. there is a significant decrease from “better” to “worse” attitude towards the attractiveness of the profession of Mathematics teacher among students of the fourth grade. It is possible that this is due to the fact that those are senior year students who are seriously considering their future profession, as well as positive and negative sides of potential professions.

“I consider the profession of Mathematics teacher as a potential future profession.”

The analysis of this question was done according to gender (Table 5.), high school course of studies (Table 6.) and the grade (Table 7.) As in the previous question, the answers “fully applies to me” and “mostly applies to me” were put together in the category “applies to me”, and the answers “mostly does not apply to me” and “fully does not apply to me” in the category “does not apply to me”.

“I consider the profession of Mathematics teacher as a potential future profession.”		
Subjects’ gender	Applies to me	Does not apply to me
Schoolboys	12, 12%	87, 88%
Schoolgirls	8, 65%	91, 35%
Total	10, 09%	89, 91%

Table 5. The frequency of certain answer categories to the question “I consider the profession of Mathematics teacher as a potential future profession.” according to subject’s gender.

According to the views on the attractiveness of being a Mathematics teacher, the obtained results were “worse” than expected. Namely, only 10, 09% of subjects consider the Mathematics teacher as a potential future profession, while the 89, 91% of subjects do not think about this profession as their potential future profession. As with the previous question, schoolgirls were less interested in this profession in relation to their male colleagues.

“I consider the profession of Mathematics teacher as a potential future profession.”		
Subjects’ course of studies	Applies to me	Does not apply to me
General course of studies	6, 17%	93, 83%
Mathematics course of studies	19%	81%
Total	10, 09%	89, 91%

Table 6. The frequency of certain answer categories to the question “I consider the profession of Mathematics teacher as a potential future profession.” according to subject’s course of studies.

Only one third of high school students attending the general course of studies who consider the Mathematics teacher profession to be attractive think about this profession as of a potential future profession, while the same applies to approximately two thirds of students with the same views that are attending the mathematics course of studies. It is likely that this thinking is influenced also by the conviction about one’s own preparation for obtaining the necessary mathematics education, in

relation to the number of hours of the Mathematics subject in the monitored course of studies.

“I consider the profession of Mathematics teacher as a potential future profession.”		
Subjects' grade	Applies to me	Does not apply to me
1. grade	23, 06%	76, 94%
2. grade	5, 8%	94, 2%
3. grade	10, 83%	89, 17%
4. grade	3, 67%	96, 33%
Total	10, 09%	18, 43%

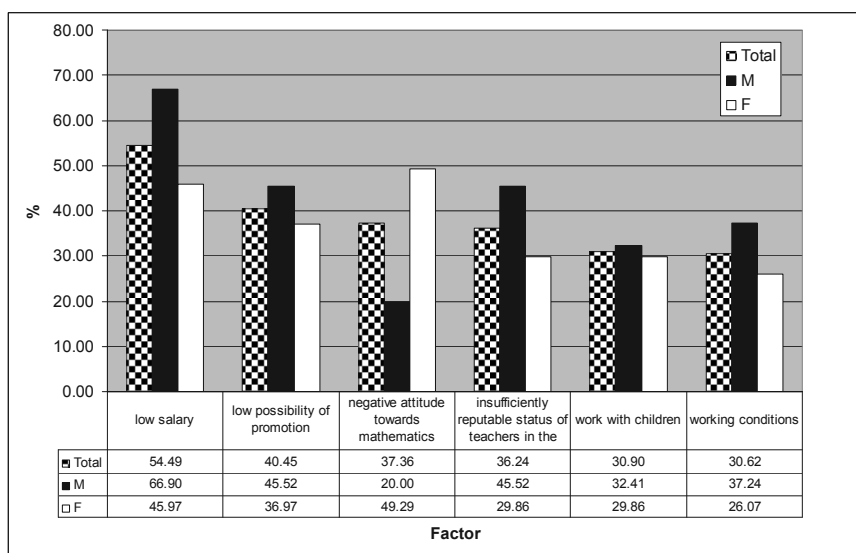
Table 7. The frequency of certain answer categories to the question “I consider the profession of Mathematics teacher as a potential future profession.” according to subject's grade.

75% of first grade students who consider the profession to be attractive think of the Mathematics teacher profession as of their future profession, while only 29, 41% of fourth grade students who consider the profession to be attractive think of it as of their potential future profession.

Factors influencing subjects' views on the Mathematics teacher profession

The third question tackled with the factors that influence subjects' views on the Mathematics teacher profession. Subjects were asked to select one or more factors, among thirteen given, that they consider to have had a major influence on their answers to the first two questions.

The given answers are as follows: assured state salary, high probability of employment, long vacation period, working hours, positive attitude towards mathematics, reputable status of teachers in the society, high social responsibility of the profession, low salary, low possibility of promotion, working conditions, work with children, negative attitude towards mathematics and insufficiently reputable status of teachers in the society. Moreover, subjects had the opportunity to add a factor that was not on the questionnaire list, and that has influenced their view. Since the primary objective of this survey was to find out why students do not choose the Mathematics teacher profession, the results of this question were analysed in relation to subjects' answers to the second question “I consider the profession of Mathematics teacher as a potential future profession.” The most frequent factors were determined from the answers of subjects who do not consider the Mathematics teacher as their potential future profession, as well as of those who consider it as their possible future profession. The first six most frequently chosen factors by those who do not consider the Mathematics teacher profession as their potential future profession, and according to gender, are shown in Picture 1.

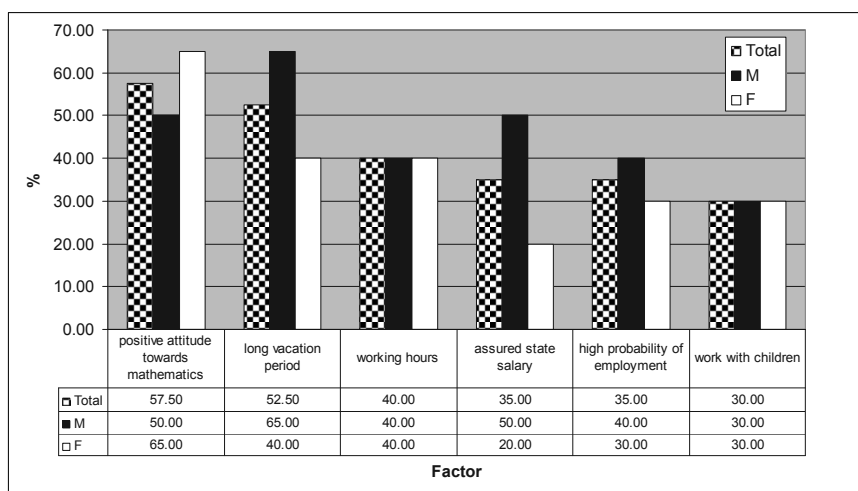


Picture 1. The first six most frequently chosen factors by those who do not consider the Mathematics teacher profession as their potential future profession, according to gender.

Among subjects who do not consider the Mathematics teacher profession as their potential future profession, 54,49% of them select the low salary as a factor that influences their views, and male subjects more frequently select this factor as one of the reasons for indifference towards the Mathematics teacher profession. This result can be partially explained by the divergence in gender roles, that is, traditional belief that men have to take care of the family. Schoolboys also more frequently than schoolgirls select as one of the reasons for their view the low possibility of promotion and the insufficiently reputable status of teacher in the society. A negative attitude towards the Mathematics subject is the reason for indifference for 37,36% of subjects, and this factor is more frequently selected by schoolgirls (49,29%) in relation to schoolboys (20%). Work with children and the working conditions are more frequently selected by schoolboys as a reason for their negative view.

The first six most frequently chosen factors by those who consider the Mathematics teacher profession as their potential future profession, and according to gender, are shown in Picture 2.

Subjects who consider the Mathematics teacher profession as their potential future profession, most frequently select as a factor that influences their view the positive attitude towards the Mathematics subject, as chosen by 57,5% of subjects from this category. The second most frequent actor is the long vacation period (52,5%). If we take into consideration also the fact that 40% of subjects from this category have chosen the working hours, we can conclude that there is a wrong perception about this profession among a large number of students who consider it as a potential future profession, and think of it as of a not demanding job with lots of free time. Other frequently selected factors are also the assured state salary (35%), high probability of employment (35%) and work with children (30%).



Picture 2. The first six most frequently chosen factors by those who consider the Mathematics teacher profession as their potential future profession, according to gender.

“My overall view on the Mathematics teacher profession was influenced by my former mathematics teachers.”

When analysing the frequency of answers to this question, answers “fully applies to me” and “mostly applies to me” were put together in the category “applies to me”, and the answers “mostly does not apply to me” and “fully does not apply to me” in the category “does not apply to me”. The analysis was carried out in relation to the subjects’ view on the Mathematics teacher profession, that is, according to the fact if they consider the Mathematics teacher profession as their potential future profession. Table 8. includes subjects who consider the Mathematics teacher profession as their potential future profession, and are marked with “positive attitude”, while the subjects who do not consider this profession as their potential future profession are marked with “negative attitude”.

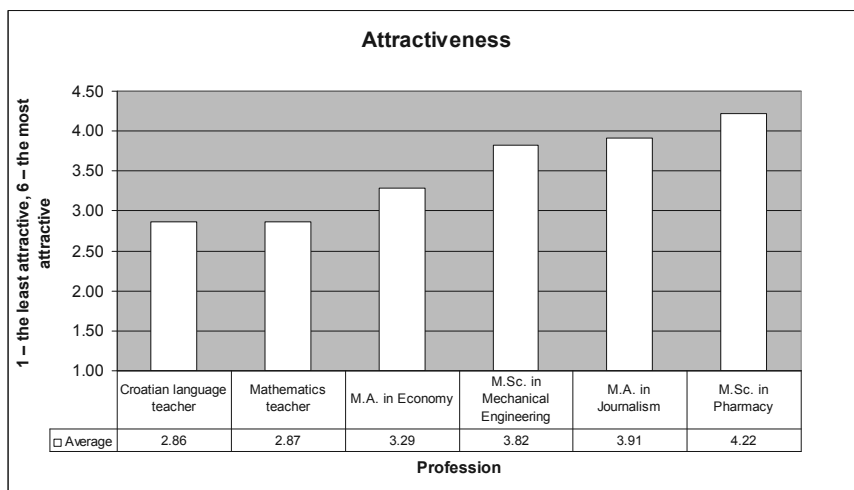
“My overall view on the Mathematics teacher profession was influenced by my former mathematics teachers.”		
View on the profession	Applies to me	Does not apply to me
Positive attitude	55%	45%
Negative attitude	36, 51%	63, 49%
Total	38, 37%	61, 63%

Table 8. The frequency of certain answer categories to the question “My overall view on the Mathematics teacher profession was influenced by my former mathematics teachers.” according to subjects’ views on the Mathematics teacher profession.

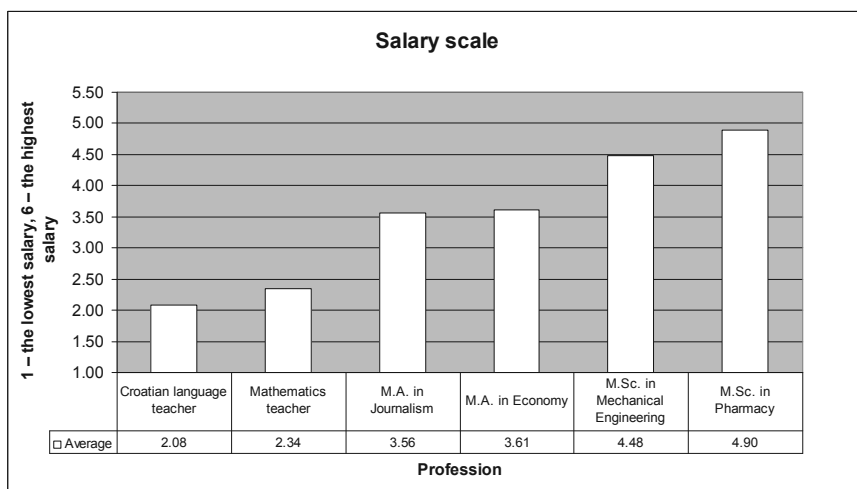
Former mathematics teachers influenced 38, 37% of subjects, and their influence is more significant on subjects with positive attitude, which means that those were teachers who have succeeded to make students love mathematics and consider it an interesting subject.

Comparison with other professions

Subjects were asked to classify, according to their own opinion, six given professions (Mathematics teacher, Croatian language teacher, M.A. in Economy, M.Sc. in Mechanical Engineering, M.Sc. in Pharmacy, M.A. in Journalism), by their attractiveness, salary scale and employment possibilities. The analysis is represented in Picture 3. (professions' attractiveness) Picture 4. (salary scale) and Picture 5. (employment).



Picture 3. Average marks for professions according to their attractiveness.

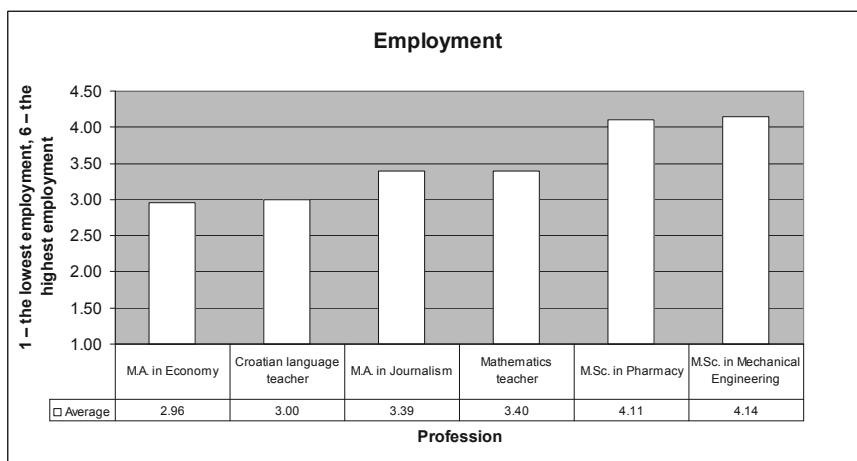


Picture 4. Average marks for professions according to salary scale.

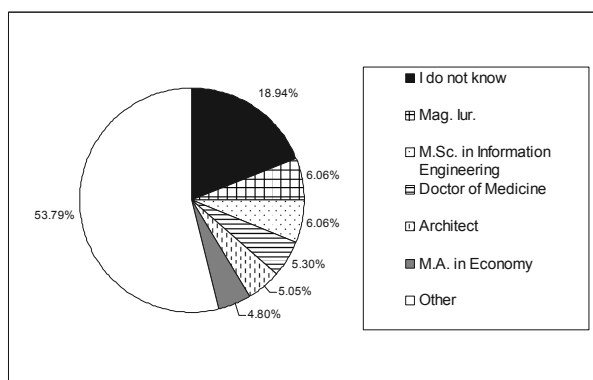
Among given professions, subjects consider the least attractive (almost equally) the Croatian language teacher and the Mathematics teacher professions. From this

figures we can conclude that subjects find the teacher professions not attractive in general, and not exclusively the Mathematics teacher profession.

The subjects' estimate about the salaries of six given professions is quite realistic (according to figures of the Croatian Bureau of Statistics), although the salaries of the M.A. in Journalism and M.A. in Economy can vary a lot. From those results we can conclude that there is the relations between the expected salary for a specific profession and its attractiveness, that is, the expected salary of a single profession has an influence on its attractiveness.



Picture 5. Average marks for professions according to the employment possibilities.



Picture 6. Professions subjects want to engage with in the future.

As already stated before, in the Republic of Croatia from January to November 2010 553 job vacancies for Mathematics professors were reported and 400 Mathematics teacher job vacancies. Furthermore, in the same period 172 vacancies for M.Sc. in Pharmacy were reported and 342 vacancies for M.Sc. in Mechanical Engineering. On the other hand, M.A. in Economics, Croatian language teacher and M.A. in Journalism are among ten professions with the highest rate of unem-

ployment. According to obtained figures, we can conclude that the subjects are not well informed about the situation.

The last question was an open one. Subjects had to write the name of the profession they would like to engage with in the future. The results are shown in Picture 6. where the six most attractive professions are marked.

Of 369 subjects, five of them answered they wanted to be Mathematics teachers, which is a 1,35% of all subjects.

Discussion and conclusion

The results of the conducted survey are in line with the current lack of Mathematics teachers. Namely, almost 80% of the subjects do not consider this profession as an attractive one, and only the 10% think of it as a potential future profession. It is surprising how, although women are more present in this profession, schoolgirls from the survey consider this profession less attractive in relation to their male colleagues, and more rarely consider it as their future profession. It is possible that schoolgirls have a more negative view on the Mathematics subject in relation to schoolboys, and that they transfer this view also on the Mathematics teacher profession. Numerous surveys have been carried out about the influence of gender on the view on Mathematics subject, and some of them confirm that it is relevant, and that it goes from smaller among younger subjects, towards stronger among high school students and university students. On the other hand, there are surveys that show how high school students have a neutral view on the Mathematics subject and they do not agree with the assumption that Mathematics is a male domain (Arambašić, Vlahović-Štetić and Severinac, 2005).

For Mathematics teachers in high schools it is interesting the fact that 75% of first grade students who consider this profession as attractive think about the Mathematics teacher profession as their future profession, while only a 29,41% of fourth grade students who consider this profession as attractive think about it as a potential future profession. If we take into consideration the fact that the percentage of students who consider this profession attractive is considerably lower in the fourth than in the first grade, it is obvious that additional efforts have to be done in order to decrease in high schools the negative trend about this profession.

The majority of subjects as a reason for their negative attitude state the salaries, which is to be taken into consideration if we want to avoid further negative trends. Bad results obtained also for the second given teacher profession (Croatian language teacher) underline the unattractiveness of work in the educational system in general. In addition, subjects are in general not aware of the fact that Mathematics teacher profession is the profession with the highest possibility of employment among six given professions. According to employment possibilities, students gave to this profession the average 3,4 mark, similar to the M.A. in Journalism profession (3.39) where there is a high number of highly educated unemployed people. The question is whether the results of the survey on high school students' views on Mathematics teacher profession would have been different if they were much more aware of the figures regarding the employment possibilities of this profession.

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Mathematics in programs of pre-school teachers' professional education

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Abstract. Mathematics is inherent part of culture. It can be found in works of culture (especially in architecture and art) and everyday life. Although, some results speak in favor of negative or neutral attitudes towards mathematics, people still use it on everyday basis. Phoning, paying bills, measuring etc. are some of many everyday activities which require mathematical knowledge. Functional use of mathematics is exactly the one that obtains great deal of attention in pre-school education. National curriculum for pre-school, obligatory primary and secondary education cite necessity of acquisition of mathematical competencies due to resolving everyday problems. In case of pre-school education, carrier (precisely designer and organizer) is pre-school teacher. Presuming that pre-school teachers' professional education is characterized by pragmatism, that mathematical knowledge is needed for everyday life and that early childhood is period of intensive learning about immediately environment and acquirement of tools of culture (language, symbols...), the question of mathematics in pre-school teacher professional education is arising. To gain an insight into the presence of mathematics in programs of pre-school teachers' professional education, programs from 5 Croatian universities have been analyzed. Results showed low presence of mathematics and reduction of mathematics onto professional tool. Regarding the fact that only two Croatian universities has launched new, five-years university class for pre-school teachers, this results can be used as frame for further inquiring of importance of mathematics in pre-school teachers' professional education and its place in institutional pre-school education.

Keywords: mathematics, programs of professional education, pre-school teachers, pragmatism, functionality

Introduction

Mathematics is inherent part of culture. It can be found in works of culture (especially in architecture and art) and everyday life. Although, some results speak in favor of negative or neutral attitudes towards mathematics (Witmann, 2006) people

still use it on everyday basis. Presence of functional use of numbers and mathematics can be found in early childhood. Importance of mathematics is described in many educational policies and documents of European Union and Republic of Croatia. So European Commission (2004) cited key competences for lifelong learning: communication in the mother tongue and in foreign languages, mathematical competence and basis competences in science and technology, information and communication technology (ICT) and digital competences, learning to learn, social and civic competences, sense of initiative and entrepreneurship, cultural awareness and expression. Croatia's frame for National curriculum for preschool education and obligatory primary and secondary education (MZOŠ, 2010) cites necessity of acquiring mathematical competences essential for solving everyday problems. In case of pre-school education, bearer (precisely, designer and organizer) of these activities is pre-school teacher. Future pre-school teachers in Republic of Croatia are educated upon professional three- and five-years educational studies to ensure successful (in terms of effectiveness) of pedagogical work. "Equipping" future pre-school teachers with adequate competences is main goal of these educational studies. During professional education, pre-school teachers "built their own value-system, based upon individual experience of socialization, personal participation in educational process and knowledge about "vocation" "(Irović, Romstein, 2007, 229). Analyzing pre-school teachers' effectiveness, Colker (2008) claims that effectiveness is under direct influence of pre-school teachers' knowledge, skills and personal characteristics. As important factor of pre-school teachers' professional success, students – future pre-school teachers cite professional education (Irović, Romstein, 2007). Regarding competences cited in international documents, and expected by Republic of Croatia, Lepičnik-Vodopivec (2007) after research conclude: the most significant competences (from the perspective of students – future pre-school teachers) are knowledge about developmental characteristics of children, differences and needs among children, knowledge about cognitive-developmental theories which are followed by competences from the field of methodology of performance. As it was already said before, presence of mathematics can be found in early childhood and on everyday basis.

Implementation of Bologna gave universities in Republic of Croatia an opportunity for conceptualizing education in regard of pre-school teachers' professional demands. To gain an insight into presence of mathematics and mathematical contents, 5 (three years) pre-school teachers' educational programs have been analyzed: Faculty of Teacher Education in Zagreb (proposal for three year graduated study for preschool teachers); Faculty of Teacher Education in Osijek (three year graduated study program for preschool teachers); Faculty of Teacher Education in Rijeka (three year graduated study program for preschool teachers); Department for education of preschool teachers and primary school teachers at University of Zadar (graduated study program for preschool teachers); and Faculty of Philosophy in Split (study program for preschool teachers).

Analyzed programs were from academic year 2004/ 2005, the 1st year of Bologna implementation. Presence of course (syllabus), goals and contents of the course and expected competencies were encompassed by analyses.

Analysis showed absence of course Mathematics and low representation of mathematical contents in educational programs, i.e. its reduction in direction of professional tool, which especially can be seen in courses such as Methodology of pedagogical research, Statistics and Informatics.

Presence of course Mathematics in preschool teachers' educational programs

Mathematical learning is possible in early childhood. The research results about mathematical learning in early childhood shows: mathematical learning with simple operation such as classification and conjoining has direct influence on success of mathematical learning during primary education (Bodrovski & Farkas, 2007), preschool children, very early, even before 5th year of life, have ability of understanding spatial relations, numbers, quantity and geometry which can be facilitated through construction games, especially building bricks (Sarama & Clements, 2004). Regarding mathematics can be learned in early childhood, its value enclosed in recent educational policies at international and national level, and importance of competences (from students' perspective) in field of performance, it is deservingly to make an insight into "offer" of mathematics in preschool teachers' educational programs.

From 5 analyzed non offered course Mathematics. But, there are courses with mathematical elements: at Faculty of Teacher Education in Zagreb courses are Methodology of preschool education 1 and 2; at Faculty of Teacher Education in Osijek students can encounter mathematics in courses Tabular calculator, Methodology of preschool education 4 and Methodology of pedagogical research; at Faculty of Teacher Education in Rijeka in courses Basic pedagogical methodology and Basic statistics; at Department for education of preschool teachers and primary school teachers at University of Zadar in courses Methodology of educational work 3 and Informatics 2; at Faculty of Philosophy in Split in courses Basic informatics, Methodology of preschool education 2 and Basic methodology of pedagogical research.

Analysis of courses with mathematical contents

Taking into an account that in preschool teachers' educational programs not even one course called Mathematics existed, we have focused our attention on analyzing courses with mathematical contents. Courses that, among others, offered mathematical contents are: Methodology of preschool education 1 and 2 (Faculty of Teacher Education in Zagreb); Tabular calculator, Methodology of preschool education 4 and Methodology of pedagogical research (Faculty of Teacher Education in Osijek); Basic pedagogical methodology and Basic statistics (Faculty of Teacher Education in Rijeka); Methodology of educational work 3 and Informatics 2 (Department for education of preschool teachers and primary school teachers at

University of Zadar); Basic informatics, Methodology of preschool education 2 and Basic methodology of pedagogical research (Faculty of Philosophy in Split).

Content analysis showed that offered mathematical elements within educational programs were not in function of pedagogical work. Mathematical contents in mentioned courses are focusing on research competences which are just one aspect of pedagogical work. This kind reduction of mathematics shows understandings of its everyday presence and importance insufficient. At the other hand, courses focused on statistics presume high level of students' mathematical knowledge, but covariance, correlation etc. preschool teachers' rarely use in their pedagogical practice. Although raising quality of pedagogical work presume preschool teachers' possession of "science competences" within field of methodology, statistics and pedagogical research (Matijević, 2007, 304) which facilitate designing developmentally appropriate practice, understanding of elemental mathematical concepts for higher mathematical operation is needed. Manifestly, there is parallel presence of reduction and high level demands towards students' knowledge. So, it is important to synchronize levels of demands towards students' achievement and to question contents relevance in applicability in their later professional work.

Analysis of particular courses showed anticoincidence and inconsistency. Goals (as starting points in course design) in particular courses (for example Methodology of preschool education 1 and 2, Methodology of educational work 3, Informatics) are absent (not known) so the question about evaluation is raised.

Due competences, two faculties did not offered competences at all, so the students' profits are vague. In fact, competences on international level are common principles in preschool teachers' professional education. Competences have function of "resolving actual and future global requests and anticipations" (Babić, 2007, 33) which is presumption of Bologna studying. But, certain courses (for example, Methodology of educational work 3) went one step further and offered students contents and competences about play, which is held as polygon for testing owns' mathematical competences (for example, "Play as fundamental method in field of development primary mathematical concepts") which require meta-competence in field of mathematics.

Ditto, it can be concluded that mathematical contents are not synchronized and information- insufficient due mathematics implementation in everyday pedagogical work. Additionally, mathematics is perceived as tool for successful research work which is, actually, just one narrow aspect of preschool teachers' professional work. Taking into account that mathematics is present in everyday life and that pragmatic value is one of most mentioned (Wittmann, 2006), it is important to think about possibilities of its' implementation into new, MA, preschool teachers' educational programs.

Analysis of preschool teachers' educational programs showed absence of course named Mathematics. Mathematical contents/ elements are comprised within other courses such as Statistics, Methodology and Informatics, which points

on low level of value perception for direct pedagogical work with preschool children.

Analysis of courses revealed low level of representation of mathematics and its reduction in direction of professional tool, especially within courses methodology of pedagogical research, statistics and Informatics. Although, competences of autonomous research are important aspect of preschool teachers' work, actually, they are props in developmentally appropriate practice design. Many authors (Liebeck, 1994; Wittmann, 2006; Bodrovski i Farkas, 2007) remind that mathematics in early childhood is often taken into account solely as "measure" for school readiness, but practical use of mathematics is its main value which can be marked in early childhood.

Actually in Republic of Croatia preschool teachers' educational programs are converting into MA level, which is unique opportunity for questioning actual programs and competences. Entry data shows that only Faculty of Teacher Education in Osijek offered mathematics for students – Mathematical culture and communication and Mathematics in play and amusement. That kind of courses able student wider diapason of mathematical contents and acquisition of relevant competences needed for later successful professional work.

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Contemporary Approaches in Teaching Mathematics at Teacher Education Faculties in Serbia

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Abstract. In this paper we give a brief overview of mathematics curriculum at faculties of teacher education in Serbia and put special emphasis on ways of developing creativity and careful lesson planning as a precondition for successful teaching in practice lessons.

Keywords: teaching, mathematics, creativity, active learning

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Educating future mathematics teachers: repeating mathematics from primary and secondary school

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Abstract. The preparation of student teachers for their future profession is complex and difficult task. In this preparation, three components of can be identified: mathematics knowledge, didactics of mathematics and psychology of learning (Hodgson, 2001). Mathematics knowledge is widely acknowledged as one of the critical attributes of mathematics teachers, thus, many studies investigated deficiencies in knowledge of in-service and pre-service mathematics teachers (Ponte & Chapman, 2010). Many of these studies have been motivated by Klein's "Doppelte Diskontinuität" (double discontinuity). In 1924 Felix Klein (2004ab) used term double discontinuity to describe two breaches (discontinuities) in the education of mathematics teachers. He was motivated by the discontinuity between school and university mathematics: first, university mathematics bore little relationship to the experiences students had had in schools; second, university graduates returning to schools used little of what they had experienced in university in their teaching lives. Almost century has passed since Klein' paradigm and the double discontinuity still exists (Barton, 2008).

In this paper, we will report the results of the survey conducted among future mathematics teachers who were given questionnaire containing questions from primary school mathematics. This survey was motivated by following questions: What do we expect that our mathematics students know form primary school? What does "know" include?

Keywords: student teachers, mathematics knowledge

Introduction

The preparation of student teachers for their future profession is complex and difficult task. In this preparation, three important components can be identified: mathematics content knowledge, didactics of mathematics and psychology of learning (Hodgson, 2001). Mathematics knowledge is widely acknowledged as one of

the critical attributes of mathematics teachers, thus, many studies investigated deficiencies in knowledge of in-service and pre-service mathematics teachers. Many of these studies have been motivated by Klein's "Doppelte Diskontinuität" (double discontinuity). In 1924 Felix Klein used term double discontinuity to describe two breaches (discontinuities) in the education of mathematics teachers. He was motivated by the discontinuity between school and university mathematics: first, university mathematics bore little relationship to the experiences students had in schools; second, returning to schools university graduates used little of what they had experienced in university in their teaching lives. Almost century has passed since Klein formulated this paradigm and the double discontinuity still exists (Barton, 2008). Studies still report the evidence of "vicious circle": student teachers enrol in university without sufficient understanding of school mathematics, experience little instruction on the mathematics they will teach and then enter to the classroom unprepared to teach mathematics to new generations (e.g. Chick, 2004; Ponte & Chapman, 2008).

Theoretical framework

Many studies showed that teachers' mathematics knowledge is generally problematic, in terms of what teachers know and how they hold knowledge of mathematical concepts or processes (Ponte & Chapman, 2008). It was discovered that teachers do not always possess a deep, broad, and thorough understanding of the content they are to teach. Steinberg et al. (1985) found that novice secondary school teachers use various coping strategies when they lack content knowledge, including relying heavily on "the textbook" as a convenient source of information. A general perspective on content knowledge in relation to mathematics teaching usually is characterized by the attitude that secondary teachers already have it and elementary teachers need very little of it (Rowland, 2006). But there is evidence from the UK, USA, Australia and beyond to refute both parts of that statement (e.g. Alexander & et al., 1992; Ma, 1999, Chick, 2002).

When considering teacher's knowledge, Schulman (1981) had identified seven categories: subject matter knowledge, pedagogical content knowledge, knowledge of students' thinking, curricular knowledge, knowledge of classroom management (general pedagogical knowledge), knowledge of educational contexts and knowledge of educational ends, purposes and values. Three of them have explicit focus on content knowledge: subject matter knowledge, pedagogical content knowledge and curricular knowledge. Subject matter knowledge is knowledge of the content of the discipline per se (Shulman, 1986, p. 9), consisting of the key facts, concepts, principles and explanatory frameworks in a discipline. Curricular knowledge includes the scope and sequence of teaching programmes and the materials used in them. Pedagogical content knowledge incorporates *how* subject matter knowledge is used in teaching, what implies knowing ways of explaining and representing concepts.

Rowland (2006) investigated the mathematics content knowledge of pre-service elementary teachers in the UK, and its relation to classroom teaching

performance. The effective classroom teaching was associated with secure subject matter knowledge, and many pre-service teachers experienced problems with the generalization and proofs. Wilson et al. (2009) compared mathematical knowledge of pre-service and in-service teachers, and discovered there were significant differences in the amount of details, vocabulary, and depth of explanation generated by the two groups, whereas the knowledge of broad mathematical concepts was similar. Southwell & Penglase (2005) found deficiencies in pre-service mathematics teachers' understanding in operations with common fractions, multiplication of decimal fractions and measurement. Chick (2004) described pre-service mathematics teachers' struggle to explain equivalence of $\frac{3}{8}$ and $\frac{37}{5\%}$ and to justify the procedure of adding zero to the end of whole number when multiplying with 10. Examining knowledge of secondary pre-service teachers, Wilburne & Long (2010) found significant content weakness in data analysis, algebra, and pre-calculus concepts.

Even though there has been much research in pre-service and in-service teacher subject matter and pedagogical content knowledge in the last decades, there are still unanswered questions that demand further investigation (Chick, 2003). Guided with this implication, we examined knowledge of mathematics students with focus on the primary school mathematics. This survey was motivated by following questions: What do we expect that our mathematics students know from primary school? What does "knowing" include?

Methodology

Two groups of mathematics students participated in this study. First group consisted of first year students who enrolled in the five year study programme of mathematics, with intention to become mathematics teachers for either primary or secondary school. The second group of students consisted of fourth year students who had studied tertiary-level mathematics as major subject and after third year of their study programme, they have chosen to finish their studies as mathematics teachers. Our sample consists of 71 students: 40 first year students and 31 fourth year students. Participation in the questionnaire was voluntary, thus some students decided not to answer given questions. Those students are not included in the sample number. The same questionnaire was administered to the both groups of students at the beginning of academic year 2010/2011, during lectures in mathematics courses. Also, time for filling questionnaire was not limited. The questionnaire contained ten items that covered mathematics subject matter in the primary school, ranging from fifth to the eight grade. Items were designed to ascertain whether the participants had the necessary mathematical knowledge on topics they were expected to teach. In some items students were asked to explain certain concepts in mathematics, while some items required solution to a mathematical problem. Responses were categorized by the number of distinctly different explanations/solutions, and whether the explanations/solutions were correct. Here we will not address all categories for each item due to the lack of the space, but we will address to the most surprising results that emerged from students' responses. All questions can be found in Appendix.

Results and discussion

Results of conducted survey revealed that no item was answered correctly in both groups of students. Further more, item 7. had no correct responses at all. Low rate of correct answers can be also found in other items as well, e.g. in item 5. and item 6.a. The items with most correct answers were 4.b and 6.b. Other result on correctness of each item can be found in the following table.

Questions	First year N = 40	Fourth year N = 31
Item 1.	10	7
Item 2.	25	17
Item 3.	10	1
Item 4.a	23	25
Item 4.b	28	22
Item 5.	3	2
Item 6.a	7	5
Item 6.b	25	25
Item 7.	0	0
Item 8.	17	15
Item 9.	3	4
Item 10.	21	15

Table 1. Number of correct responses in two groups of students.

As written above, we will not address all response categories for each item, but we will report the most interesting results and findings that emerged from this questionnaire.

Item 1. The first item in this survey asked students to explain how one finds the greatest common divisor of two or more natural numbers. Only quarter of surveyed students gave correct response (Table 2). The terms as prime divisor rarely emerged in their explanations. Some students used correct method for finding greatest common divisor of two numbers as an example, but without explanation, so their responses were regarded as incorrect. The striking outcome of this item was not the number of empty responses, but the number of students who have written “nonsense” in their answer. This nonsense consist of students’ explanation that the greatest common divisor can be found by multiplying both numbers.

Categories	First year N = 40	Fourth year N =31
Correct	10	7
Empty	14	9
Nonsense	12	11
Used example, but no explanation	4	4

Table 2. Categories of students’ responses to **Item 1.**

Item 5. Both groups had difficulty with writing clear explanations how one adds two whole numbers. The extent of their difficulties can be seen in the Table

3. One of the striking outcomes is the similarity in the performance of both groups where a third of first and fourth year students did not even attempt to write the explanation.

Categories	First year N = 40	Fourth year N = 31
Correct	3	2
Empty	13	11
Incorrect for numbers with different signs	6	4
Nonsense	4	14
Used example, but no explanation	4	0

Table 3. Categories of students' responses to **Item 5**.

Students who gave explanation rarely used the conception of subtracting smaller absolute value from greater in their explanations. Responses of students who used examples to show how one adds two whole numbers is regarded as incorrect since the item asked students to verbalize the procedure e.g. to explain what one is actually doing. Overall, it seems that it was difficult for students to use appropriate wording to describe what was happening in the calculations. Results of this item do not indicate that students can not add whole numbers, but that they lack fluency with the language and conceptual understanding of the underlying procedure. Similar results have been found Thompson & Thompson (1996) and Chick (2003), while investigating pre-service teacher in USA and in Australia. This suggests that our finding is not just isolated case of Croatian education systems, but goes in hand with world problem of teacher education.

Language in mathematics is another area that needs special attention. Even though mathematics is seen as universal language, the assumption that students who meet mathematical environments every day will be able to understand terminology and syntax of problems is not true. There can be found many studies that refuted this assumption (e.g. Chick, 2001; Southwell & Penglase, 2005). In our survey, this can be seen in item 6.a where students calculated the exact number of pupils who play football, even though item asked for the part of pupils in the relation to the whole number of pupils. Similarly, in the item 7. many students just listed types of quadrilaterals they knew without any attempt of classification.

The overusing of proportional strategy can be seen in the item 9. Students applied the proportion algorithm without considering "common sense" when posing the algorithm. Similar findings where students sought security in proportion algorithm without even considering given situation can be found in Cramer et al. (1993) and Lim (2008). Result of item 9. can be connected with overall students' performance. In their responses, students showed strong desire to calculate, what indicates severely procedural orientation to the mathematical problems. Students connect their knowledge with the process of calculation. This was found even in the item 1. and the item 5., where written explanations were sought. Strong procedural knowledge was found among the wrong responses in the item 8. Student posed system of equations, calculated numbers which should represent years of sister and brother, but did not check if these solutions satisfied conditions listed in the

problem. Following Davis & McGowen (2001), this kind of severely procedural orientation to mathematics requires the remedy in form of shifting focus to conceptualization of mathematics as discipline rich in relationship, not just discipline of formulas and right and wrong answers.

Results of this questionnaire indicate that students have forgotten large portion of primary school mathematics (Table 1). One can ask is it fair to make such claim. We have investigated two groups of students with different backgrounds. First year students had state graduation exam, and in order to enrol in the mathematics studies, they were required to pass A-level exam. On the other hand, fourth year students were involved with studying mathematics for three years. Therefore, one should think the mathematical knowledge of both groups is sufficient enough to deal with all or majority of given items. But if we look these results in terms of mathematics education across the world, then our results are far from surprising (e.g. Southwell & Penglase, 2005; Wilburne & Long, 2010).

Conclusion

There is strong evidence that completion of mathematics courses is not sufficient for developing the mathematical knowledge needed for teaching mathematics (Monk, 1994). In the last decade many studies emerged researching mathematical knowledge of pre-service teachers and emphasizing that without sound mathematical knowledge many pedagogical processes are of little benefit (e.g. Davis & McGowen; 2001, Chick, 2001; Southwell & Penglase, 2005). Those studies also addressed the need of designing mathematics education programs which will make connections between university content courses, primary and secondary methods courses, and primary and secondary mathematics content across the curriculum. Those courses would prepare pre-service teachers for teaching school mathematics; they would address fundamental mathematical concepts, help pre-service teachers to master procedures and concepts and also develop their ability to articulate them.

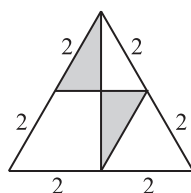
At the Department of mathematics, University of Osijek, school mathematics is repeated within the course Methods of teaching mathematics 1 through several colloquiums. Some questions from the questionnaire were given in the first colloquium, thus we are able to compare results from the questionnaire and the colloquium. Results improved significantly for all questions except for the 6.a, where students still calculated exact number of pupils that played football. However, amount of time spent for school mathematics is still inadequate, since we expect that students would go through school mathematics themselves and repeat required subject matter. Thus we support the idea of course in school mathematics, to address “forgotten” mathematical concepts and engage students’ knowledge in different context; context of teaching and not learning.

Appendix

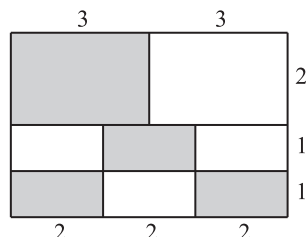
1. Explain determining of the greatest common divisor of two or more numbers.

2. Four sailboats meet in the sea around the island: the first comes every 16 days, others every 8 days, third ever 12 and fourth every 6 days. When will sailboats meet again in the sea around the island?
3. Draw all the axes of symmetry of the following figures: rectangle, square, rhombus, isosceles triangle and equilateral triangle.
4. Using fractions express the part of figure that is:

a)



b)



5. Explain the addition of two whole numbers.
6. Two-fifths of pupils of some school are engaged in sports. Of those involved in sports, three sevenths of them play football.
 - a) What part of the pupils is in the school football team?
 - b) If the school has 805 pupils, how many of them participate in sports?
7. How do we classify quadrilaterals with respect to parallelism of their sides?
8. Sister is 8 years older than her brother. In 5 years she will be two times older. How old is the sister and how old is the brother?
9. 42 workers may build the dam for 25 days. After 5 days, 7 workers were absent from work. How long will it take the rest of the workers to build the dam?
10. If a is a real number, what is $\sqrt{a^2}$ equal to?

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Concept Maps in Maths Teachers' Education

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Abstract. The demands for new approaches in teaching and educating Maths teachers are getting higher. There are more and more talks about different approaches to knowledge representation and then to assessment and evaluation in the process of Maths learning and teaching.

Concept maps, mental maps, knowledge maps, as well as other similar approaches, give a good base for effective and lasting learning for both teachers and students, with the aim of improving their skills and abilities.

This paper discusses the influence, strategy and trends in using concept maps in educating Maths teachers.

Keywords: concept maps, mental maps, knowledge maps, education

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The Number Line as an Arithmetic Machine

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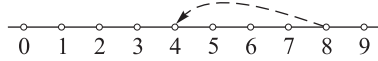
Abstract. This article attempts to present arithmetic operations using a number line in the form of an arithmetic machine, using only “mechanic” commands such as the length of the step and whether it is directed leftward or rightward, i.e. towards or away from the zero, etc. The operations will be presented with the usual visual objects used in lessons, such as arrows, lines, numbers etc., thus the machine can be understood as a visual syntax in the vein of a logic diagram. The mechanical result is a visual figure on the number line, in the form of a stop action programme on a machine that corresponds to numerical operations in an arithmetic structure, in the sense of logical soundness and completeness. Numerical operations, with particular emphasis on division and the zero, can be simulated as a programme on the number line. On the other hand, the machine can be used in mathematics lessons as a manifest learning tool for teaching abstract arithmetic operations based on the pupils’ immediate intuition in school lessons. The processing of the operations on the number line is, in its general form, displayed dynamically, with a small-scale computer programme.

Keywords: arithmetic operations, number line, diagram syntax, computer programme

With this article, I wish to present a visual depiction of arithmetic operations on a number line. The model of the number line is present in virtually every junior grade classroom. Croatian math textbooks for the second and third grades of primary school illustrate arithmetic operations as “jumps” on the number line. It is precisely this approach that I took as the foundation for the depiction of an arithmetic machine capable of explaining arithmetic operations well and in full. The machine is usually portrayed as a dyadic tree with commands that are, basically, the same as those used in computer programming, such as COPY, DELETE, GOTO etc. A complete visual depiction of abstract mathematical content is in the vein of the ideas expounded in the article [2] and in the books [1] and [4]. A methodical approach to number line lessons is enriched with a small-scale computer programme that aims to present the theoretical machines generated in the article.

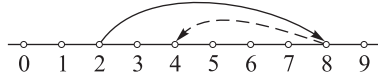
Graphical Objects

1. The carrier $\eta = (\text{————})$ is a continuous one-dimensional object in the form of either an open or a closed line. $P \subset \eta$ is a subset of reference points on the carrier η .
2. Let us define the structure of the *number line* $\mathbf{b} = (t_0, f \circ I)$. t_0 is the selected point, and $f \circ I$ is the function that associates each graphical object on the line with its *name*. $I : P \rightarrow D$ is the *orientation* function. For the set $D \subseteq \mathbf{N} \times \mathbf{N}$ it applies that it is a well-defined set and that
 1. $\{(n, 0) | n \in \mathbf{N}\} \subset D$.
 2. Let us select the point $t \in P$, so that $I(t) = (0, 0), f(I(t)) = 0$, that is, marked as t_0 in short.
 3. For the point $v \in P$ we can say that it is *visuall* between t_0 and w iff it is $(I(w) \neq I(v))$ and $(I(w) < I(v))$. $f : I(P) \rightarrow T$ is the *assignment* function. T is the set of the *names* of the graphical objects ($\mathbf{N} \subseteq T$). If $n, m \in I(P)$ ($n < m$), then $(f(n) < f(m))$.
3. The *small circles*: $t_n = (\circ)_{f(n)}$, $n \in \mathbf{N} \times \mathbf{N}$ are the basic objects on the number line.
4. The *jumps* $l_{ch}(\gamma, \tau) = (\overset{\curvearrowright}{\gamma} \tau)(\gamma, \tau)$ and $l_{ph}(\gamma, \tau) = (\overset{\curvearrowleft}{\gamma} \tau)(\gamma, \tau)$. γ is the *head* of the jump and τ is the *tail* of the jump, $h \in \mathbf{N}$ is the *name* of the jump.
5. Generating a *jump on the number line* is an expansion of $\mathbf{b}$ with the jump: $l_{ih}(n_{\gamma, i, h}, n_{\tau, i, h})$ ($n_{\gamma, i, h}, n_{\tau, i, h} \in T$) ($f \circ I(\gamma) = n_{\gamma, i, h}$, $(f \circ I)(\tau) = n_{\tau, i, h}$) added onto the number line $\mathbf{b} \cup l_{ih}$.
6. The point t_n is contained within the jump l_{ih} : $(\forall m, k \in \mathbf{N} \times \mathbf{N}) (f(m) = n_{\gamma, i, h} \wedge f(k) = n_{\tau, i, h}) \implies (m \leq n \leq k) \vee (k \leq n \leq m)$



t_6 is contained within l_c

7. The *intersection* $\sqsupset$ is the relation: $l_{i, h}(\gamma, \tau) \sqsupset l_{j, k}(\gamma, \tau)$ $i, j \in \{c, p\}$ iff $(\forall m, n \in \mathbf{N} \times \mathbf{N}), f(m) = n_{\gamma, j, k}, f(n) = n_{\tau, j, k} \implies (t_n \text{ and } t_m \text{ contained within the jump } l_{ih})$.



8. *Copying* – the COPY operation: $\text{COPY}_n l_{ih}(\gamma, \tau) = l_{ih}(\gamma, \tau)(n_{\gamma, i, h} = f(n))$
9. *Removing* – the DELETE operacija: $\text{DELETE } l_{ih}(\gamma, \tau)$ is the number line $\mathbf{b}/l_{ih}(\gamma, \tau)$
10. *Doubling* $l_{jd}(\gamma, \tau)$ ($j \in \{c, p\}$) is: $l_{jm}(\gamma, \tau)$ so that $l_{jd+1}(\gamma, \tau) = \text{COPY}_{n_{\tau, j, d}} l_{jd}(\gamma, \tau)(n_{\gamma, j, m} = n_{\gamma, j, d}, n_{\tau, j, m} = n_{\tau, j, d+1})$
11. The *midpoint* between points t_α and t_β is point t_x , which is such that $l_{jd}(\gamma, \tau)$ ($n_{\gamma, ji} = f(\alpha)$ and $n_{\tau, jd} = f(\beta)$) doubling $l_{jh}(\gamma, \tau)$ ($n_{\gamma, hi} = f(\alpha)$ and $n_{\tau, hd} = f(x)$).

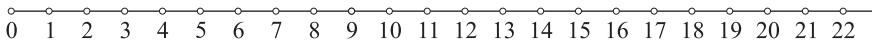
12. *Rotation (inversion) $\mathcal{J}$* – an operation on the jumps replacing γ and τ .

$$\mathcal{J}l_{ch}(\gamma, \tau) = \mathcal{J}(\overset{\circ}{\gamma} \overset{\circ}{\tau}) = l'_{ch}(\overset{\circ}{\tau} \overset{\circ}{\gamma})(\tau, \gamma)$$

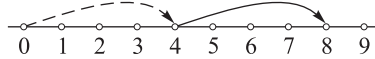
Machines for Numerical Operations in $\mathbf{N}$

The points on the number line are assigned the names of natural numbers $f : D = \{(n, 0), n \in \mathbf{N}\} \rightarrow n$, ($T = N$), graphical objects (small circles on the line) are equated with their names, i.e.: $t_n = n$.

Let us define the arithmetic operations on the number half-line. $t_0 = 0$ is the initial point of the half-line.

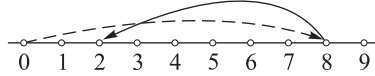


1. *Adding $a + b$ is:*



1. START: $l_c(\gamma, \tau), n_{\gamma,c} = 0$ and $n_{\tau,c} = a$ jump from 0 to the left addend;
2. Generate *jump* $l_{p1}(\gamma, \tau) n_{\gamma,p1}=0, n_{\tau,p1}=b, l_p(\gamma, \tau)=\text{COPY } n_{\tau,c} \mathcal{J}l_{p1}(\gamma, \tau)$;
3. $n_{\tau,p}$ is the result STOP.

2. *Subtracting $a - b$ is:*



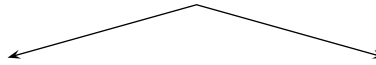
1. START: $l_c(\gamma, \tau), n_{\gamma,c} = 0$ i $n_{\tau,c} = a$ jump from 0 to the minuend;
2. Generate *jump* $l_{p1}(\gamma, \tau) n_{\gamma,p1}=0, n_{\tau,p1}=b, l_p(\gamma, \tau)=\text{COPY } n_{\tau,c} \mathcal{J}l_{p1}(\gamma, \tau)$;
3. $n_{\tau,p}$ is the result STOP.

If $b > a$, the result cannot be displayed on the number line.

3. *Multiplication: $a \times b = c$:*

1. START: generate on the number line $l_{pi}(\gamma, \tau), n_{\gamma,p,i} = 0$ i $n_{\tau,p,i} = 0$,
 $l_{p1}(\gamma, \tau), n_{\gamma,p,1} = 0$ i $n_{\tau,p,1} = a$;

2.

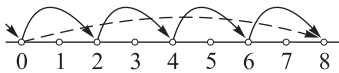


$i > b$

DELETE $l_{pi}(\gamma, \tau)$, $c = n_{\tau,p,b}$
 $l_c(\gamma, \tau), n_{\gamma,c} = 0$, $n_{\tau,c} = n_{\tau,p,b}$,
 $c = n_{\tau,c,1}$, result, STOP

$i \leq b$

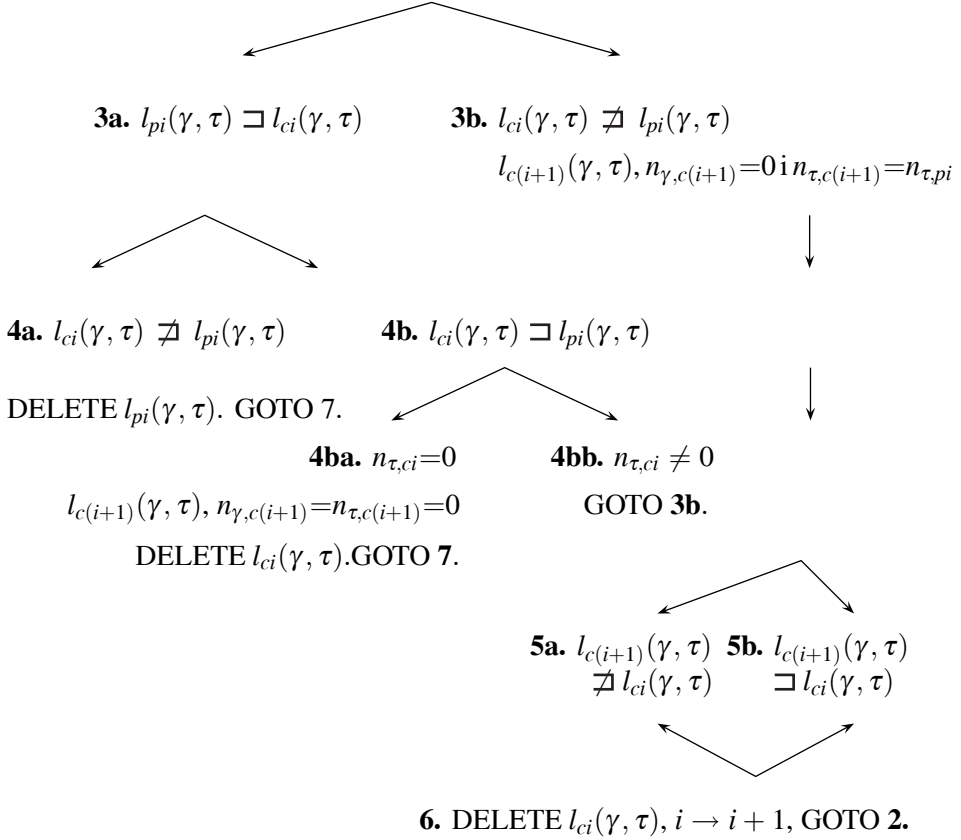
$l_{p,i+1}(\gamma, \tau) = \text{COPY } n_{\tau,p,i} l_{p,i}(\gamma, \tau)$,
 $i \rightarrow i + 1$, GOTO 2.



4. Division: $a : b$ in $\mathbf{Z}$:

1. START: $l_{ci}(\gamma, \tau), n_{\gamma, ci}=0$ i $n_{\tau, ci}=a$, (ime: $i=0$)

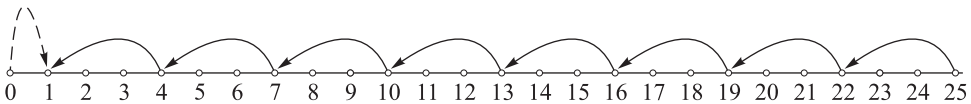
2. $l_{pi'}(\gamma, \tau), n_{\gamma, p, i'}=0$ i $n_{\tau, p, i'}=b, l_{pi}(\gamma, \tau)=\text{COPY}_{\tau, c, i} \mathcal{J}l_{pi'}(\gamma, \tau), \text{DELETE}l_{pi'}(\gamma, \tau)$



7. Result: the quotient is the name of the last blue jump i and the remainder $n_{\tau, c, i+1}$. STOP.

Let us check the procedure.

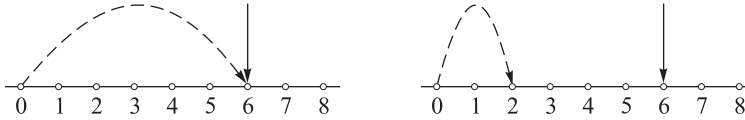
Example 1. $25 : 3 = 8$ and r. 1



We can select only this row of junctions in the division machine's tree: 1., 2., 3b., 5a., 6., 2. The same procedure is repeated 8 times. The ninth time, we turn towards 3a. 4a. and reach STOP.

Example 2. $6 : 0 =$

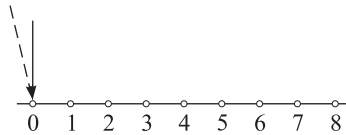
1. 1. START (draw red jump) and 2. Draw blue jump 3. 3a. reject, 3b. accept
4. 5b. reject 5. 6. accept and return to 2.



The cycle repeats itself and never reaches the STOP command. The machine cannot stop. The task is insoluble!

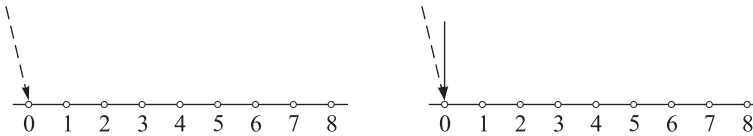
Example 3. $0 : 0$

1. START (draw red jump) $i = 0$ 2. apply 2. Draw blue jump 3. 3a. and 3b. are acceptable, let us select 3a.

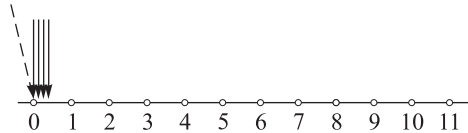


4. we can apply only 4b. 5. only 4ba. is acceptable. 6. we reach stop STOP. or 31. we accept 3b.

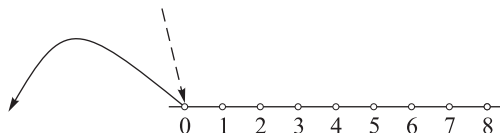
41. we can apply only 5b. 51. we apply 6. 61. we apply 2. 71. we can accept 3a. and reach the STOP command via either 3b. or 4bb. The cycle can be repeated many times, according to wish.



The term is undefined.

**Example 4.** $0 : 6 =$

1. START (draw red jump) $i = 0$ 2. apply 2. draw blue jump

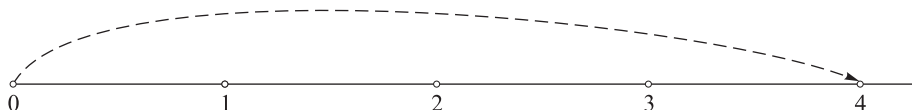


3. we can accept only 3a. 4. only 4a. is acceptable and we reach the STOP command. The result is 0.

Visualising Division and Rational Numbers

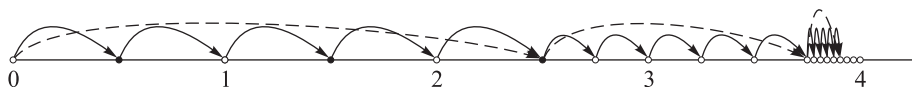
Is it possible to, by expanding the number line with new points in the existing visual system, justify the equivalence formula: $a : b = c \iff a = b \cdot c$? The visual task is to find a jump (associated with c) of such a length with which we can, by starting from 0 and consecutively repeating the same jump for a b number of times, reach the dividend on the number line.

Let us observe example $4 : 5$.



The task is to find a point between 0 and 1 so that a jump from 0 to that point, when sequentially repeated 5 times, reaches point 4. The jump should have a length smaller than 1. Is it at least $\frac{1}{2}$?

Let us draw the midpoints of the segment units into the segment from 0 to 4. Therefore, we have doubled the number of points (green points) in the observed segment. Do we have a sufficient number of points for at least 5 jumps? Yes. Let us mark the point that we have reached in red. However, we have not reached 4. Can we extend the jump with another halving of the points existing from the red point to point 4? Purple points. We have completed 5 jumps, but this is still insufficient to reach point 4.



Can we continue the process? That is, let us draw in new midpoints (yellow points) in the quotient from the point reached to the dividend. Insufficient for new jumps! We need at least 5 points! Let us repeat the process with another halving (blue points). With this division, we have generated an amount of points sufficient for 5 new jumps! Let us observe that, after the last division, 3 more possible jumps remain at our disposal, just like in the first division (the green points). Therefore, the process will be repeated and continued into infinity.



The graph above displays the length of the requested jump (the dotted yellow jump). If we had continued the process, we would have ended up closer to the dividend, but thus compromising visual readability. The blue jumps drawn show that we could have reached the same result by looking for only one jump the length of the dividend at a time! Let us write out the conclusions that clearly emanate from a visualisation of the problem: 1. Halving the segment between the point reached and the divisor is a visual operation of doubling the number of points 2 A jump is

14. Let us expand with new graphical objects, *small circles*:

$$t_{b-} = t_{g(n)} = (\circ)(g(n) = g(n-1)0),$$

$$t_{c-} = t_{g(n)} = (\bullet), (g(n) = g(n-1)1), n \in \mathbf{N}.$$

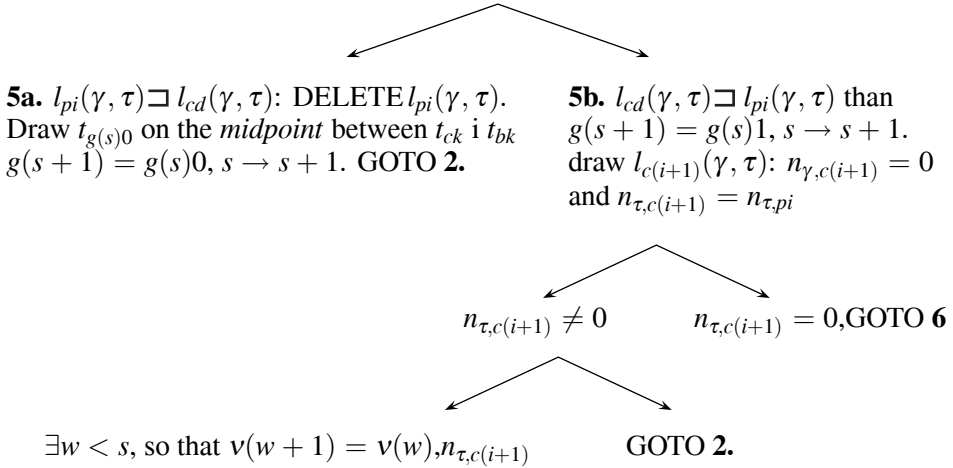
We start from division within the $\mathbf{N}$ set, which we have described in the text above. a is the dividend, b is the divisor, q is the number of blue jumps r is the length of the dotted red jump. Let us suppose that $r > 0$.

1. START: $s = 0$ (0 state) $f(q) = g(0) = g(-1)1$ and $f(q+1) = g(1) = g(0)0$, draw $t_{c,g(0)}$, $t_{b,g(1)}$ and $l_{c0}(\gamma, \tau)$, $n_{\gamma,c0} = 0$ and $n_{\tau,c0} = r$. Let us write out the sequences: $v(1) = r$.

2. Let $s = i$. For $k = \min_x (0 \leq x \leq s)g(s-x) = g(s-x-1)1$ (if at least $x = s$), there is one single point $t_{ck} = t_{g(s-k)}$. For $h = \min_z (0 \leq z \leq s)g(s-z) = g(s-z-1)1$ (if at least $z = s-1$), there is one single point $t_{bh} = t_{g(s-h)}$.

3. Let us double the red jump $l_{ci}(\gamma, \tau)$, forming the jump $l_{cd}(\gamma, \tau)$, $v(s+1) = v(s)$, $n_{\tau,cd}$.

4. generate jump $l_p(\gamma, \tau)$, $n_{\gamma,pi} = 0$, $n_{\tau,p} = b$. $l_{pi}(\gamma, \tau) = \text{COPY } n_{\tau,cd} \mathcal{J}l_p(\gamma, \tau)$. There are only two options.



Then we will say “The process repeats itself into infinity. Select point $t_{g(s)1}$ as the last and GOTO 6. or continue and GOTO 2. Concerning the following points of the $t_{g(s)1}$ type, the length of the result q will be better in the sense that $b \cdot q$ will be closer a .”

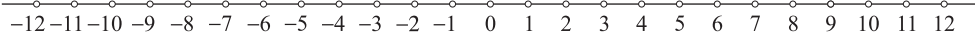
6. Draw (green) $l_z(\gamma, \tau)$, so that $n_{\gamma,z} = 0$ and $n_{\tau,pi+1} = t_{g(s)1}$, STOP.

If we name the points on the number line within the binary system, then the green jump represents a rational number decimally written as: $q.g'(s+1)$ so that $g(s+1) = g(1)g'(s+1)$.

Whole Numbers

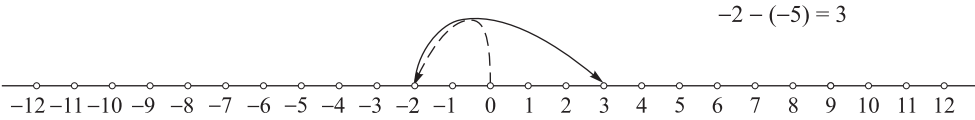
We commence from the $\mathbf{N}$ number line. Function I (assignment) remains the same. Let us *voluntarily* choose a number n according to our needs. Let us form a new identity function f ,

$$f(k) = \begin{cases} k - n, & k \geq n \\ -(n - k), & k < n \end{cases}$$



The image displays a representation of the number line $\mathbf{Z}$ for $n \geq 12$. Subtraction: $a - b$

1. START: generate on the number line $l_c(\gamma, \tau)$, $n_{\gamma,c} = 0$ i $n_{\tau,c} = a$, $l_{p1}(\gamma, \tau)$, $n_{\gamma,p,1} = 0$ i $n_{\tau,p,1} = b$
2. $l_{p2}(\gamma, \tau) = \mathcal{J}l_{p1}(\gamma, \tau)$, DELETE $l_{p1}(\gamma, \tau)$
3. $l_{p3}(\gamma, \tau) = \text{COPY } n_{\tau,c} \ l_{p2}(\gamma, \tau)$, DELETE $l_{p2}(\gamma, \tau)$
4. $n_{\tau,p,2} = c$ result of the sum. STOP.



Multiplication: $a \times b = c$

1. START: generate on the number line $l_{pi}(\gamma, \tau)$, $n_{\gamma,p,i} = 0$ i $n_{\tau,p,i} = 0$, $l_{p1}(\gamma, \tau)$, $n_{\gamma,p,1} = 0$ i $n_{\tau,p,1} = a$,

2.

$$i > |b|$$

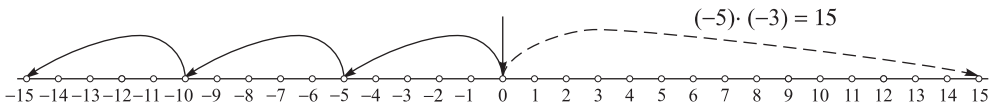
DELETE $l_{pi}(\gamma, \tau)$, $c = n_{\tau,p,|b|}$

$$i \leq |b|$$

$l_{p,i+1}(\gamma, \tau) = \text{COPY } n_{\tau,p,i} l_{p,i}(\gamma, \tau)$,
 $i \rightarrow i + 1$, GOTO 2.

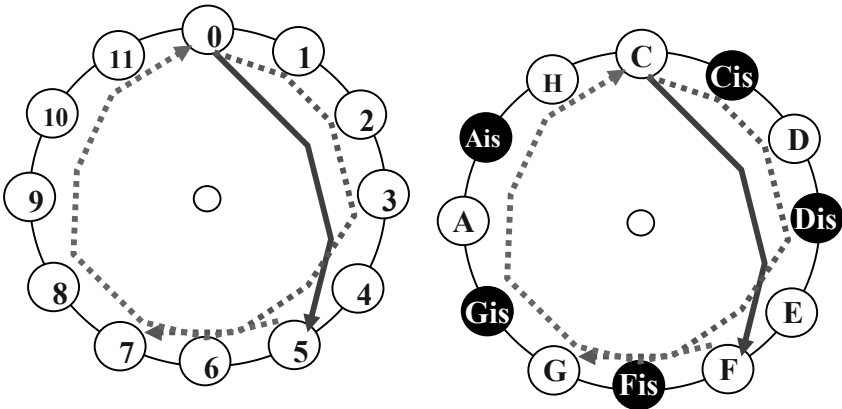
$b < 0$, $l_c(\gamma, \tau)$,
 $n_{\gamma,c} = 0$, $n_{\tau,c} = n_{\tau,p,|b|}$
 $l_{p1}(\gamma, \tau) = \text{COPY}_0 l_c(\gamma, \tau)$,
DELETE $l_c(\gamma, \tau)$
 $c = n_{\tau,c,1}$, result, STOP.

$b \geq 0$, $c = \text{result}$, STOP



Number Line on a Circle

Let us place the number line on an object topologically different from the line and other constructions topologically equivalent to it. Let us place it on a circle. The arithmetic machines, basically, remain the same, but the pupils can be shown that the results become unexpectedly different. For example, that the sum of two numbers larger than 0 equals 0. ($5 + 7 = 0$, as the image below with 12 selected points from the P set shows)



The names of semitones within the chromatic C major scale can also be drawn into the circles with numbers. In this, the possibility of connecting the methodics of music education and of mathematics is opened. The image above shows the addition of $E + Cis = F$ on a C major scale. The “bizarre” table below is a multiplication table of flat semitones.

X	C	Des	D	Es	E	F	Ges	G	As	A	Hes	H
C	C	C	C	C	C	C	C	C	C	C	C	C
Des	C	Des	D	Es	E	F	Ges	G	As	A	Hes	H
D	C	D	E	Ges	As	Hes	C	D	E	Ges	As	Hes
Es	C	Es	Ges	A	C	Es	Ges	A	C	Es	Ges	A
E	C	E	As	C	E	As	C	E	As	C	E	As
F	C	F	Hes	Es	As	Des	Ges	H	E	A	D	G
Ges	C	Ges	C	Ges	C	Ges	C	Ges	C	Ges	C	Ges
G	C	G	D	A	E	H	Ges	Des	As	Es	Hes	F
As	C	As	E	C	As	E	C	As	E	C	As	E
A	C	A	Ges	Es	C	A	Ges	Es	C	A	Ges	Es
Hes	C	Hes	As	Ges	E	D	C	Hes	As	Ges	E	D
H	C	H	Hes	A	As	G	Ges	F	E	Es	D	Des

By successively adding 5 or 7 semitones to the semitones within the C major scale, an array of scales with the sharp and flat symbols continuously changing will be formed. In music, this is known as a circle of fifths or fourths, which is not bizarre but is, in its essence, arithmetic on a circle. Musical examples could, in a methodic sense, be used to expand on the definition of the number line and to cast a new light on arithmetic realised through the manipulation of computer graphical objects.

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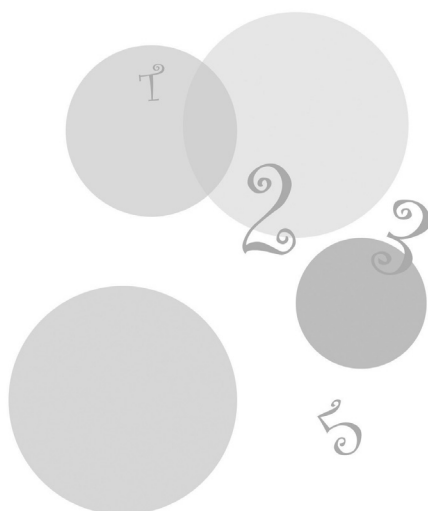
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2.

Skills, knowledge and abilities of students



In the second chapter the authors present results of their research conducted among students of teacher studies. The research investigated levels of student achievements in important skills and knowledge relevant to the profession. We find particularly important the results of research implemented in order to familiarise ourselves better with student strategies of problem solving and their spatial thinking skills.

Analysis of Inductive Reasoning in Mathematical Problem Solving among Primary Teacher Students

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Abstract. Inductive reasoning is part of the discovery process, whereby the observation of special cases leads one to suspect very strongly (though not know with absolute logical certainty) that some general principle is true. Deductive reasoning, on the other hand, is the method one would use to demonstrate with logical certainty that the principle is true.

Both are necessary parts of mathematical thinking. When we develop mathematical thinking in the early school years, we mainly deal with the situations in which children have to reason inductively. Inductive reasoning is used as a strategy in teaching basic mathematical concepts, as well as in problem solving situations. When educating primary teachers, the emphasis is also on developing problem solving skills based on inductive reasoning. We believe that in mathematics only those teachers who have competences in inductive reasoning can create and deal with the situations in the classroom which contribute to the development of those competences in children, as well.

In the paper the results of the study on primary teacher students' competences in inductive reasoning are presented. The students were posed a mathematical problem which provided for the use of inductive reasoning in order to reach the solution and make generalizations. Their results were analysed from different perspectives: from the perspective of understanding the problem situation, from the perspective of the problem solving depth, and from the perspective of the applied strategies. Further, we analysed the relationship between the depth and the strategy of problem solving and established that not all strategies were equally effective at searching for problem generalizations. Students more often decided on the strategies which did not draw on the basic problem context; moreover, preserving the problem context even turned out to be less effective than the mere operating with numbers at the transition into higher, more abstract stages of problem solving.

Introduction

In many cases the researchers related the inductive reasoning process to the problem solving context (e.g. Christou & Papageorgiou, 2007; Küchemann &

Hoyles, 2005; Stacey, 1989). These examinations pay attention to the cognitive process, as well as to the general strategies, that students use to solve the posed problems. Problem solving is considered a highly formative activity in mathematics education fostering various kinds of reasoning, more specifically, inductive reasoning.

The manner in which a student addresses the problem, the manner of his solving it, the conclusions he manages to reach, his reflections on his findings, his testing of applying reasoning in new situations – all these are the key elements of mathematical reasoning. Reasoning, explanation and proof at an appropriate level of sophistication should therefore be a prominent part of learning mathematics of students of all ages.

In literature terminology of various kinds is used when addressing reasoning in mathematics: deductive reasoning, inductive reasoning, mathematical induction, inductive inferring, reasoning and proving. *Deductive reasoning* is unique in that it is the process of inferring conclusions from the known information (premises) based on formal logic rules, where conclusions are necessarily derived from the given information, and there is no need to validate them by experiments (Ayalon & Even, 2008). Although there are also other accepted forms of mathematical proving, a deductive proof is still considered as the preferred tool in the mathematics community for verifying mathematical statements and showing their universality (Hanna, 1990; Mariotti, 2006; Yackel & Hanna, 2003). On the other hand, *inductive reasoning* is also a very prominent manner of scientific thinking, providing for mathematically valid truths on the basis of concrete cases. Pólya (1967) indicates that inductive reasoning is a method of discovering properties from phenomena and of finding regularities in a logical way, whereby it is crucial to distinguish between inductive reasoning and mathematical induction. *Mathematical induction* (MI) is a formal method of proof based more on deductive than on inductive reasoning. Some processes of inductive reasoning are completed with MI, but this is not always the case (Canadas & Castro, 2007). It is useful to distinguish between *inductive reasoning* and *inductive inferring* (Klauer & Phye, 2008). The former is aimed at detecting generalizations, rules, or regularities, an inductive inference, whereas the latter extends the generalization beyond the scope of experience by asserting something about a nonobserved or even nonobservable universe of objects (Klauer & Phye, 2008). Stylianides (2008, 2008a) uses the term *reasoning-and-proving* (RP) to describe the overarching activity that encompasses the following major activities that are frequently involved in the process of making sense of and in establishing mathematical knowledge: identifying patterns, making conjectures, providing non-proof arguments, and providing proofs. Given that RP is central to doing mathematics, many researchers and curriculum frameworks in different countries, especially in the United States, noted that a viable school mathematics curriculum should provide for the activities that comprise RP central to all students' mathematical experiences, across all grade levels and content areas (Ball & Bass, 2003; Schoenfeld, 1994; Yackel & Hanna, 2003).

From our curricular perspective, we believe that at elementary school level inductive reasoning is more appropriate than deductive reasoning in mathematics teaching and learning, whereas at the secondary level the basic goal of mathematics is to develop also a certain level of deductive reasoning, mathematical induction and abstraction. At the end of the secondary school students should master operations

of abstract thinking allowing them to understand conjecture formulation, observation of particular cases, experimentation, hypothesis validation, and to elaborate explanations. This knowledge is further delved into at the study of mathematics at the Faculty of Mathematics and Physics, whereas with other study courses the complexity of mathematics is in line with the particular study programme competences. The primary teacher students, who participated in our research, were presented with rather concrete mathematics study contents, comprising certain abstract elements, such as mathematical induction, deductive reasoning and inductive reasoning.

Inductive Reasoning

As our research shall be dedicated to inductive reasoning, this will be specified from the perspectives of various theories and practices. Glaser and Pellegrino (1982, p. 200) identified inductive reasoning, as follows: “All inductive reasoning tasks have the same basic form or generic property requiring that the individual induces a rule governing a set of elements.” There is general agreement that tasks such as classifications, analogies, incomplete series, and matrices require inductive reasoning, and that they are widely accepted as typical inductive reasoning tasks (Büchel & Scharnhorst, 1993). It is commonly accepted that these four types of tasks require the detection of a rule or, more generally, of a regularity (Klauer & Phye, 2008).

Inductive reasoning tasks can be solved either by applying the *analytic strategy or the heuristics strategy* (Klauer & Phye, 2008). The former enables one to solve every kind of an inductive reasoning problem. Its basic core would be the comparison procedure. The objects (or, in case of correlations, the pairs, triples, etc., of objects) would be checked systematically, predicate by predicate (attribute by attribute or relation by relation), in order to establish commonalities and/or diversities. However, the solution seekers generally tend to resort to the heuristics strategy, at which a participant starts with a more global task inspection and constructs a hypothesis, which can then be tested, so that the solution might be found more quickly, depending of the quality of the hypothesis. We believe that problem solving in mathematics is based on both strategies, with pupils, who learn mathematics, as well with scientists, who can reach new cognitions by applying either the analytic strategy or the heuristics one.

There are various theories as to the detailed identification of the stages of inductive reasoning. Pólya (1967) indicates four steps of the inductive reasoning process: observation of particular cases, conjecture formulation, based on previous particular cases, generalization and conjecture verification with new particular cases. Reid (2002) describes the following stages: observation of a pattern, the conjecturing (with doubt) that this pattern applies generally, the testing of the conjecture, and the generalization of the conjecture. Cañadas and Castro (2007) consider seven stages of the inductive reasoning process: observation of particular cases, organization of particular cases, search and prediction of patterns, conjecture formulation, conjecture validation, conjecture generalization, general conjectures justification. There are some commonalities among the mentioned classifications: Reid (2002) believes the process to complete with generalization, Pólya adds the stage of “conjecture verification”, as well as Cañadas and Castro (2007), who

name the final stage “general conjectures justification”. In their opinions general conjecture is not enough to justify the generalization. It is necessary to give reasons that explain the conjecture with the intent to convince another person that the generalization is justified. Cañadas and Castro (2007) divided the Polya’s stage of conjecture formulation into two stages: search and prediction of patterns and conjecture formulation. Also Stylianides (2008, 2008a) refers to different stages of inductive reasoning, which are first divided in a very complex manner into Dimension 1 and Dimension 2, within which the following stages are identified: Dimension 1 uses (1) the notion of “making mathematical generalizations” – the transporting of mathematical relations from the given sets to new sets, for which the original sets are subsets (Polya, 1954) – to capture two of the activities that comprise RP (“identifying a pattern” and “making a conjecture”), and (2) the notion of “providing support to mathematical claims” to capture the other two activities that comprise RP (“providing a proof” and “providing a nonproof argument”). Dimension 2 is complementary to Dimension 1 and concerns the purposes (functions) that patterns, conjectures, and proofs may serve in students’ engagement in RP.

The above stages can be thought of as levels from particular cases to the general case beyond the inductive reasoning process. Not all these levels are necessarily present, there are a lot of factors involved in their reaching.

Empirical part

Problem Definition and Methodology

In the empirical part of the study conducted with primary teacher students the aim was to explore their competences in inductive reasoning. In the early school years inductive reasoning is often used as a strategy to teach the basic mathematical concepts, as well as to solve problem situations. In the very research the focus was on the use of inductive reasoning at solving a mathematical problem. We believe that in mathematics only teachers who have competences in problem solving can create and deal with the situations in the classroom which contribute to the development of those competences in children.

The empirical study was based on the descriptive, non-experimental method of pedagogical research.

Research Questions The aim of the study was to answer the following research questions:

1. Is the posed problem situation perceived as a problem by the students?
2. Do the students possess adequate knowledge to solve the problem by applying the inductive reasoning strategy?
3. How much do the students delve into problem solving, i.e. which step in the process of inductive reasoning do they manage to take?
4. Which strategies are used by the students at their search for problem generalizations?
5. Are all the applied strategies equally effective for making generalizations?

6. To what extent do the used strategies of problem solving preserve the context of the source problem situation?

Sample Description

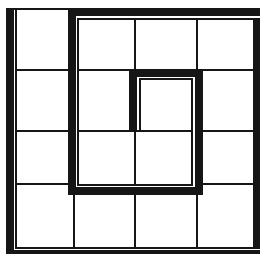
The study was conducted at the Faculty of Education, University of Ljubljana, Ljubljana, Slovenia in May 2010. It encompassed 89 third-year students of the Primary Teaching Programme.

Data Processing Procedure

The students were posed a mathematical problem which provided for the use of inductive reasoning in order to reach a solution and make generalizations. The problem was, as follows:

The students were solving the problem individually, they were simultaneously noting down their deliberations and findings, they were also aided with a blank square paper sheet of, so they could delve into the problem by drawing new spirals.

On the picture below the shaping of the spiral in the square of 4×4 is presented. Explore the problem of the spiral length in squares of different dimensions.



The data gathered from solving the mathematical problem were statistically processed by employing descriptive statistical methods. The students' solutions were analysed from different perspectives: from the perspective of understanding the problem, from the perspective of the problem solving depth, and from the perspective of the applied strategies. As some students tested various problem solving strategies, thus contributing more than one solution to the result analysis, the decision was made to use the number of the received solutions and not the number of the participating students as the basis for the analysis of the problem solving depth and of the strategies of solving. We received 95 solutions, i.e. 6 students contributed two different approaches to problem solving.

Results and Interpretation

In continuation the results are shown, which are analysed as to various observation aspects.

a) *Understanding of the instructions*

The instructions to the problem read to explore the length of the spirals. Our first interest was in what way the students understood this particular instruction, i.e.

whether they were aware of the fact that the verb “explore” means that one should not only deal with the given example, but also extend the case to similar situations and make generalizations. The corresponding results are shown in Tabele 1.

Understanding of the instruction “Explore”		
Solves the isolated example	3	3,38%
Explores more cases	86	96,62%
Total	89	100,00%

Table 1. Presentation of the results from the perspective of the students’ understanding the instructions to the problem.

The results show that the majority of the respondents perceived the posed problem situation as a problem, which had also been expected, as during their studies the students encountered with inductive reasoning many a time.

b) The solving problem depth

Those students who correctly interpreted the instruction of the problem and extended the situation to other cases with spirals of different dimensions, further underwent the examination of their level of depth at dealing with the situation. The received solutions were classified into many levels, which were graded as to the achieved problem solving depth:

Level 1: only the pictures of the spirals,

Level 2: calculations of the lengths of the spirals,

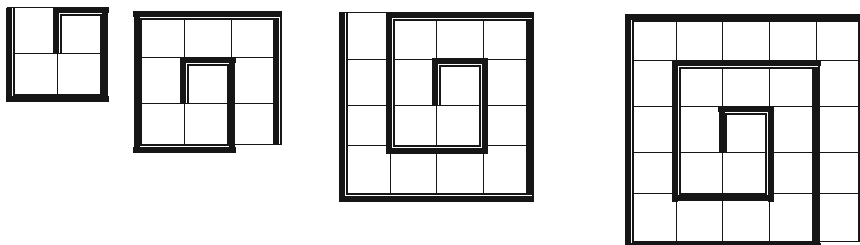
Level 3: structured records of the lengths of the spirals, but only for those cases, that are graphically presented,

Level 4: structured records of the lengths of the spirals and the prediction of the result for the case, which is not graphically presented,

Level 5: making generalizations for the spiral of any length.

The following examples show the differences among the levels:

The level 1 example:



The level 2 example: the record contains the drawn spirals for the square dimensions ranging from 2×2 to 6×6 , and the corresponding calculations:

Square dimensions	Spiral length
2×2	8
3×3	15
4×4	24
5×5	35
6×6	48

The level 3 example: The record contains the drawn spirals for the square dimensions ranging from 2×2 to 6×6 , the corresponding calculations and the structured record of the length:

Square dimensions	Spiral length	Structured record of the length
2×2	8	$2 \times 2 + 2 \times 2$
3×3	15	$3 \times 3 + 2 \times 3$
4×4	24	$4 \times 4 + 2 \times 4$
5×5	35	$5 \times 5 + 2 \times 5$
6×6	48	$6 \times 6 + 2 \times 6$

The level 4 example: The record contains the drawn spirals for the square dimensions ranging from 2×2 to 6×6 , and the corresponding calculations as for the level 3 case; in addition, the student predicted the result for the spiral of the 7×7 dimension, which he did not present graphically. In this case it was only possible for him to determine the length of the spiral on the basis of his prior calculation of the structured record, and not directly from the picture.

Square dimensions	Structured record of the length	Spiral length
7×7	$7 \times 7 + 2 \times 7$	63

The level 5 example: apart from the pictures and concrete calculations encompassed in the level 3 example, the student added his prediction of the record for the general case:

Square dimensions	Structured record of the length
Generalization to $n \times n$	$n^2 + 2n$

As obvious the transformation of the problem from the geometric to the arithmetic one, and consequently operating with numbers and not only with pictures of the spirals is witnessed not until one has reached the level 2. Taking into account the stages in inductive reasoning (Polya, 1967, Reid, 2002, Canadas and Castro, 2007) we can also state that all the students at the levels from 1 do 5 reached the stage “observation of particular cases”, yet they were not equally successful in the process of searching and predicting of patterns. Mere drawings of spirals and

calculations of their lengths (the levels 1 and 2) did not provide for a deeper insight into the nature of the problem and for making a generalization for the spiral of any dimension. The level 3 may be considered a transitional stage. These students already knew that mere calculations would not suffice, so they tried to structure them, i.e. they analysed the calculated numbers, and tried to define a certain pattern and a rule, respectively. However, they considered this to be enough and did not try to make a rule for the “ n ”-number of times-steps. In these cases students were deliberating on a possible pattern just for the cases they were observing. In comparison with them the level 4 students were already thinking about a possible pattern for a non-observing case, but they were still not thinking about applying their pattern to all cases. According to Reid (2002) the students at the level 4 reached the stage of conjecture (with doubt). They were convinced about the right of their conjecture for those specific cases, but not for other ones (see also Canadas and Castro, 2007). Only those students who achieved the level 5 can be considered to have reached the stage called “generalization of the conjecture” according to Reid (2002). In the opinions of Canadas and Castro (2007) generalization is by no means the final stage in the inductive reasoning process. The final stage - general conjectures justification – includes a formal proof that guarantees the veracity of the conjecture, namely. Similar to the research conducted by Canadas and Castro (2007), also in our research none of the students recognised the necessity to justify the results. They interpreted the results as an evident consequence of particular cases, with no need of any additional justification to be convinced of its truth.

Table 2 shows the distribution of responses regarding the achieved problem solving depth.

Depth	Number of responses	Responses in percentage
Level 1	6	6,32%
Level 2	19	20,00%
Level 3	24	25,26%
Level 4	11	11,58%
Level 5	32	33,69%
Other	3	3,16%
Total	95	100,00%

Table 2. Distribution of the responses regarding the achieved problem solving depth.

As can be inferred from the table, more than one third of the students achieved the stage of making a generalization, whereby it should be pointed out that two ways of making generalizations were considered in this group: 30 students did it symbolically with records for “ n ”-number of times-steps, whereas two students generally created rules in a descriptive manner with words, e.g. two lengths of the side are added to the square of the side length. The “Other” group comprises the responses of three students who were eliminated from further analysis of the problem solving procedures due to their non-understanding of the instructions.

During the problem solving procedure also failures were caused, mainly of three types:

- failures at drawing spirals: the inappropriate picture of spirals of larger dimensions (7 responses);

- failures at interpretation of the concept of the length of the spiral: the student equals the concept of the length of the spiral with the number of squares covered by a spiral instead of with the length of the line (2 responses);
- miscalculations/mismeasurements of the length of the spiral (7 responses).

If we eliminate all the responses containing some failure during the problem solving procedure, 14 responses at the level 2 (14,73%), 18 responses at the level 3 (18,95%) and 11 responses at the level 4 (11,58%) remain; although these students did not make any mistake during solving procedure, they still did not manage to make generalizations. For the students at the level 2 it is assumed that among the collected data they did not notice any structure, which prevented them from further exploration. On the other hand, the students of the levels 3 and 4, who did notice the structure anyhow (total 30,53%), most likely either did not know how to write their findings in a general form or they did not feel the need to upgrade their concrete findings with a general record. Nevertheless, we think that the percentage of those students is quite high, and may reflect the orientation of primary teacher education with focus on dealing with concrete situations.

c) Problem solving strategies

The analysis of the modes of reasoning that the students applied at their search for generalizations revealed that it was possible to perceive the posed problem from various perspectives. Various problem perception modes are addressed as various solving strategies in continuation, out of which the ones that were encountered among the students' solutions are presented:

Strategy denotation	Strategy description	Generalization record
1 – “squares” strategy	It is observed that the values of the lengths are obtained by squaring the lengths of the consecutive square (e.g. $15 = 16 - 1$)	$(n + 1)^2 - 1$
2 – “product” strategy	It is observed that the length of the spiral is equal to the product of two numbers that differ for 2 (e.g. $15 = 5 \times 3$)	$n(n + 2)$
3 – “binomial” strategy	It is observed that the length of the spiral is calculated by adding the double length to the square of the square length (e.g. $15 = 3 \times 3 + 2 \times 3$)	$n^2 + 2n$
4 – “difference” strategy	When observing the differences among the lengths of the spirals, it is obvious that the result is the sequence of odd numbers (e.g. from 1×1 square onwards the lengths of the spirals increase by 5, 7, 9, 11, 13, 15. ...)	The difference between the spiral in the square with $n \times n$ dimensions and the consecutive spiral is $2n + 1$ or in a recursive manner: $d_{n \times n} = d_{(n-1) \times (n-1)} + (d_{(n-1) \times (n-1)} - d_{(n-2) \times (n-2)} + 2)$, whereby the denotation $d_{n \times n}$ stands for the length of the spiral in the square with $n \times n$ dimensions.

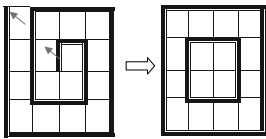
5 – “sum” strategy	It is observed that the length of the spiral can be presented as the sum of individual even sections of the spiral (e.g. $15 = 1 + 1 + 2 + 2 + 3 + 3 + 3$).	$3n + 2(n - 1) - 2(n - 2) \dots + 2 \times 2 + 2 \times 1$
6 – “transformation strategy”	It is observed that in cases when the dimension of the square is an even number, spirals can be transformed in squares, the perimeters of which can be calculated.	$4n + 4(n - 2) + 4(n - 4) \dots + 4 \times 2;$ $n = 2k, k \in \mathbb{N}$ 

Table 3. Description of the applied problem solving strategies.

In continuation the students' selection of the strategies is presented. The strategy was evaluated only with the responses, achieving the depth of the levels 3, 4. or 5., i.e. of those students, who noted the length of the spiral in a structured record, as it was possible to define the applied strategy and the mode of reasoning, respectively, only with this record.

Strategy	Number of responses	Responses in percentage
1 – squares	2	2,11%
2 – product	8	8,42%
3 – binomial	12	12,63%
4 – difference	28	29,47%
5 – sum	16	16,84%
6 – transformation	1	1,05%
Other	28	29,47%
Total	95	100,00%

Table 4. Distribution of the responses as regards the applied problem solving strategy.

According to Klauer & Phye (2008) it can be stated that the majority of the students approached problem solving by applying the analytic strategy, including a systematic analysis of individual cases, hence the search for potential patterns and generalizations. In two cases the strategy of insight or the heuristics strategy could be considered. Both students first established the rule for the “ n ”-number of times-case $(3n + 2(n - 1) + 2(n - 2) + \dots + 2 \times 1)$ and in the follow-up they tested their rule on concrete cases with spirals. It can be assumed that as early as at the analysis of the given case the students figured out the spiral structure, the fact which led them directly to the general record.

Let us have a closer look of the results presented in Table 4. As can be inferred the strategy prevails, with which the students focused on the difference between the neighbouring spirals (29,47%). The “sum” and “binomial” strategies are often used; however, only 2 students noticed that there was a correlation between the

lengths of the spirals and the squares of the natural numbers. In the “Other” column the responses were placed at which it was not possible to consider the selected strategy (all of the students who did not reach the level 3).

Other questions worth addressing can be posed at the analysis of the solving strategies, such as:

1. Were all the strategies equally effective when searching for generalizations?
2. What triggers the development of each individual strategy: relation to the problem context or mere operating with numbers, separated from the source context?

The following table provides for the answer to the 1st question, clarifying the relation between the selected strategy and the problem solving depth:

Strategy	Level 3	Level 4	Level 5	Total	Percentage of responses at the level 5
1 – squares	0	0	2	2	100,0%
2 – product	2	1	5	8	62,5%
3 - binomial	1	1	10	12	83,3%
4 – difference	17	7	4	28	14,3%
5 – sum	3	2	11	16	68,8%
6 – transformation	1	0	0	1	0,0%

Table 5. Problem solving depths in relation to the problem solving strategy.

The values in the last column attest to the percentage of the responses pertaining to the selected strategy of those students who managed to reach the final level, i.e. the generalization. According to the results one of the applied strategies was substantially less effective than the other ones, i.e. the strategy 4 – strategy “differences”, but it was the most often used strategy of all (see table 4). Thus, a conclusion can be reached that the percentage of responses including a generalization to a common case was largely influenced by the selected strategy. From this perspective some of them (e.g. strategies 2, 3 and 5) were much more useful than the other ones (the strategy 4).

When analyzing strategies from the viewpoint of the context preservation each response was considered separately trying to determine whether the record was analysed on the basis of observing the spiral construction or on the basis of “operating with numbers”. The latter category included all the cases at which the student first calculated the length of the spirals and then tried to define commonalities of the gained results, which he further analysed, rearranged, whereby eliminating the source problem context.

The relation between the strategy and the context preservation is shown in the following table:

Strategy	Operating with numbsrs	Context preservation	Total
1 – squares	2		2
2 – product	8		8
3 – binomial	12		12
4 – difference	27	1	28
5 – sum		16	16
6 – transformation		1	1
Total	49	18	67

Table 6. Relation between the problem solving strategy and the context preservation.

Obviously only one of the strategies was explicitly problem context related, i.e. the “sum” strategy. In this case when defining the total length the students based their procedure on the fact, that the total length is the sum of the lengths of individual sides of the spiral. A conclusion can be reached that the students more often applied the strategies in which the context was not paid attention to at the stage of their searching for generalizations.

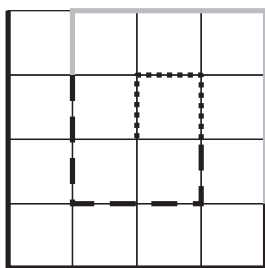
Let us take an example showing that one can either consider the context or eliminate it with the application of the same strategy. As it can be inferred from the table 6, one such case was encountered with the application of the strategy 4, i.e. when the focus was on the differences among elements (members). The students’ records were, as follows:

Case 1: “operating with numbers”:

$$\begin{array}{rcl}
 4 \times 4 & \longrightarrow & 24 \\
 & & \downarrow +11 \\
 5 \times 5 & \longrightarrow & 35 \\
 & & \downarrow +13 \\
 6 \times 6 & \longrightarrow & 48 \\
 & & \downarrow +15 \\
 7 \times 7 & \longrightarrow & 63
 \end{array}$$

Case 2: Context preservation

The square of 1×1 : 1 should be subtracted, because one side is missing: $4 - 1 = 3$. This is the length of the shortest snake (the dotted line).



The square of 2×2 (perimeter of 8 units): 1 should be subtracted, because one side of the square is not part of the spiral; at the same time 2 more units are to be subtracted, because they were already considered at the previous, red spiral. Hence: $8 - 3 = 5$ (the dashed line).

The square of 3×3 (perimeter of 12 units): the sides that are not part of the spiral or were already considered in the blue spiral, should be subtracted. Hence: $12 - 5 = 7$ (the gray line).

Similar reasoning leads one to determine the length of the line in the square of 4×4 to be $16 - 7 = 9$ (the black line).

The summing up: $3 + 5 + 7 + 9 = 24$ of the gained results equals the length of the spiral in the square of 4×4 . In order to calculate the length of each consecutive spiral, the consecutive odd numbers of the sequence should be added (e.g. 9 is to be added up to get the spiral of 5×5).

The analysis of the responses from the perspective of the failures in the procedure proves an interesting fact: among 49 cases with “operating with numbers” the procedure failure was observed in 4 cases, totalling 8,16%, whereas among 18 cases with “the context preservation” 3 failures were recorded, amounting to 16,67%. These results indicate that the context preservation in the procedure of problem solving by inductive reasoning leads to more failures in calculations than the transformation of the problem to the mere operating with numbers. In the process of searching for generalizations the reliance on the concrete problem content can be more of a hindrance than stimulation; generalizations are noticed more quickly if the problem is transformed into the mathematical problem, irrespective of the context.

Summary

In the course of their studies at the Faculty of Education one of the important competences to be developed with primary education students is to qualify them to solve mathematical problems. We are aware of the fact that this field of expertise is often neglected in our primary schools, mostly in favour of consolidating the learning contents by calculations and attending to classical word problems. We believe that students – future primary teachers are the ones, to whom we should start to bring about changes of this mindset, and introduce the role of the problem situations as an indispensable part of mathematics lessons in elementary schools. The presented research provided us with some important responses as to the qualification of students for problem solving by inductive reasoning. It was established that the

majority of the students usually perceive the given situation as a problem, however, their abilities to delve into the problem are rather different: based on the stages of inductive reasoning according to Polya (1967), Reid (2002) and Castaneda and Castro (2007) it can be inferred that the students' responses were mainly pertaining to the following three stages: observation of particular cases, searching for pattern and prediction, as well as generalization. We find it important to establish that the stage an individual student manages to reach is largely influenced by his strategy selection. Some strategies in the process solving proved to be more effective than the other ones, from the perspective of making generalizations. Further, we established that students tend to select the strategies, enabling them to abandon dealing with the context at the stage of their search for generalizations and pass over to merely operating with numbers. And, what is more, the results even revealed that the context preservation in the procedure of problem solving by inductive reasoning leads to more calculation failures than the transformation of the problem to the mere operating with numbers. Nevertheless, this finding is not a surprising one: according to Piaget (Piaget in Inhelder, 1956) the generalization stage is the stage of formal-logical thinking, at which one no longer operates with concrete models, but deliberates on relations, abstract schemes, which are either upgraded or the new ones are developed. In this process of abstract thinking, leaning on concreteness can inhibit one at developing ideas. The participating students approached the problem situation in a creative manner, as they applied six strategies of different quality, and they were highly motivated to deal with such problem situations; both facts seem to be extremely encouraging from the perspective of their later role as teachers of mathematics to the youngest children.

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The study of the geometrical concepts of teacher training college students

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Abstract. It is a common experience with students of teacher training colleges that they rather dislike geometry. It happens quite often that students who passed GCSE examination in mathematics are not familiar with basic geometrical concepts such as rectangular, square, parallel and perpendicular and axial symmetry. Some students consider general parallelogram as rectangle and some of them believe the diagonals of general parallelogram are symmetry axes and sometimes they are not aware of the fact that squares are actually rectangles. They consider the neighbouring sides of the squares parallel (probably they misinterpret the term 'neighbouring') etc.

The causes of these problems probably can be found in primary school education. For this reason we carried out a developmental teaching experience with learners in grade four in order to study the formation of these concepts. The aim of this educational experiment was to develop the concepts of square, rectangle, parallel, perpendicular and symmetry by means of putting the van Hiele model of teaching geometry into practice and to justify the use of the levels of development according to van Hiele in the lower primary geometry teaching in Hungary.

The teaching experiment was conducted in the class 4. c of the practice school of József Eötvös College in Baja in 2006 May and June, and then it was repeated in class 4.b in 2008 May and June.

The developmental teaching experiments were completed by evaluation worksheets. Beside the classes participating in the teaching experiment the worksheets were also filled in by the pupils of parallel classes in 2006 and 2008.

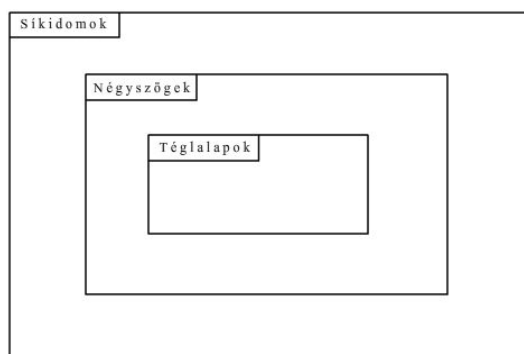
The post-test used in 2008 was also done by 38 regular students of the teacher training college. Twenty four of them were fourth year students and 14 of them were second year students. The test results of the would- be teachers will be shown below and their results are compared with that of the 56 pupils of class 4 gained in 2008.

Keywords: methodology of teaching mathematics, concepts of geometry, van Hiele's levels

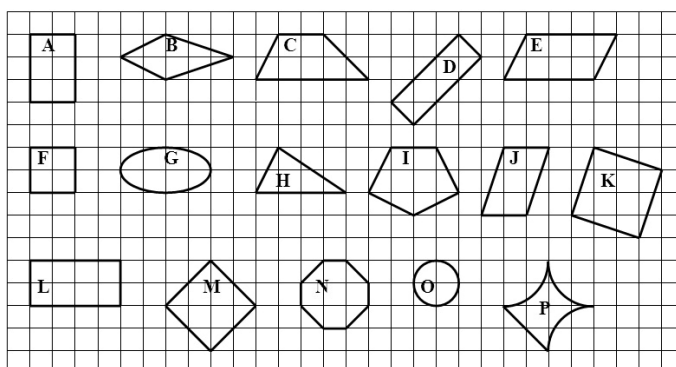
1. Tasks related to square and rectangular

In task 1 of the post-test, which is related to the identification of the concept of quadrangles, rectangles and squares, out of 16 plane figures

- the quadrangles;
- the rectangles;
- the squares had to be selected;
- and the letter identifying the plane figure had to be put in the right place.

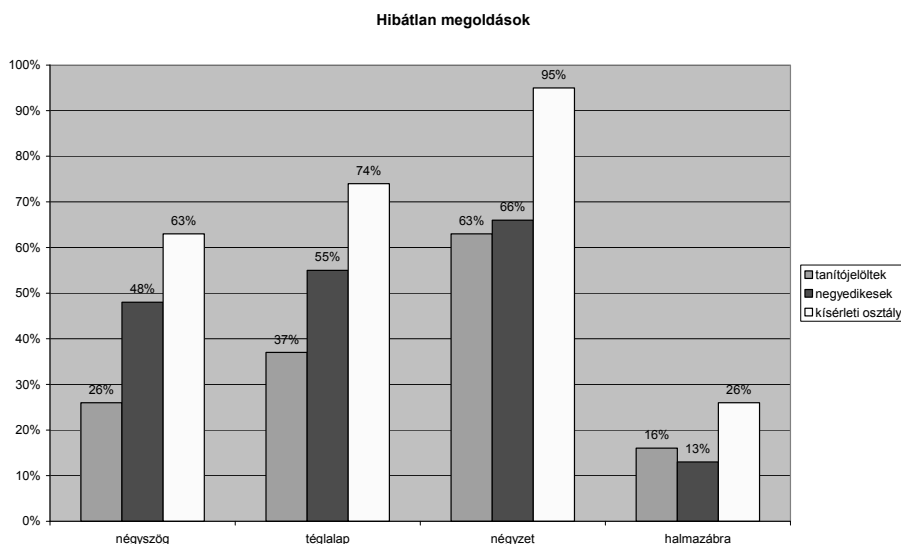


This set had to be arranged:



The results of the task are shown in the following chart.

When recognizing quadrangles 68% of the students identified plane figure P as quadrangles, which was wrong. It was 29% of the pupils that made the same mistake.



The geometrical thinking of the majority of children in class four reaches van Hiele's level 2. At this level children do not recognize the relationships between the figures. They take notice of the characteristics the figures share e.g. that both squares and rectangles have four vertices and four sides and their opposite sides are parallel and their neighbouring sides are perpendicular, and their angles are right angles. However they still do not come to the conclusion that squares are actually rectangles. Being aware of the characteristics is necessary only to make difference between the figures. This is why in case of four grader pupils we considered it as an acceptable solution when children did not rank squares among rectangles if otherwise they did not make any mistake. However in case of teacher training college students we expected the ranking of squares among rectangles. It was a typical mistake ranking general parallelograms among rectangles, as it happened in case of 41 % of four grade pupils and 21 % in the experimental class and 37 % of teacher training college students.

When sorting out squares the mistake 29% of both the children and the students made was due to the fact that they did not recognize squares on their vertex.

Arranging the set of plane figures proved to be the most demanding task. A common mistake was that letters designating plane figures were placed next to several plane figures in case of 21 % of the children and 34% of the students. Multiple classification seems to be a too demanding and abstract task for the children, which goes beyond van Hiele's level 2. Thus the poor results produced by the children was in line with our expectations but not in the case of the students.

In the worksheet of the survey on the characteristics of squares and rectangles the task was to underline the statements that were true for

a) squares

Its opposite sides are parallel.

Its opposite sides are perpendicular.

The neighbouring sides are parallel.

The neighbouring sides are perpendicular.
Every angle is right angle.
Not every angle is right angle.
It has exactly two symmetry axes.
It has four symmetry axes.
It has eight symmetry axes.
Every side is of the same length.
The opposite sides are of the same length.

b) rectangles

Its opposite sides are parallel.
Its opposite sides are perpendicular.
Its neighbouring sides are parallel.
Its neighbouring sides are perpendicular
Its angles are all right angles.
Its angles are not all right angles.
It has four symmetry axes.
It has two symmetry axes.
The diagonals are symmetry axes.
Its every side is of the same length.
The opposite sides are of the same length.

In the evaluation of the task we focused on only the results without any mistake. All the true statements related to the characteristics of the square were chosen by 61% of the children and 45% of the students. In case of rectangle the results were as follows: 57% of the children and 47% of the students got it right. In class four the mistakes can be due to the misconceptions of the terms opposite and neighbouring and also due to the fact that the concepts of parallel and perpendicular were not firmly established. Students made the most mistakes in defining the numbers of symmetry axes of squares and rectangles, 21% and 42% respectively.

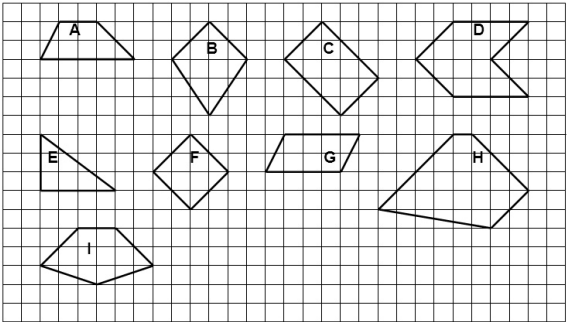
2. Tasks related to parallel and perpendicular

In the evaluation worksheet task 2 and task 5 are connected with the establishment of the concepts of parallel and perpendicular. In task 5 parallel and perpendicular are involved with the recognition of the characteristics of squares and rectangles (see above).

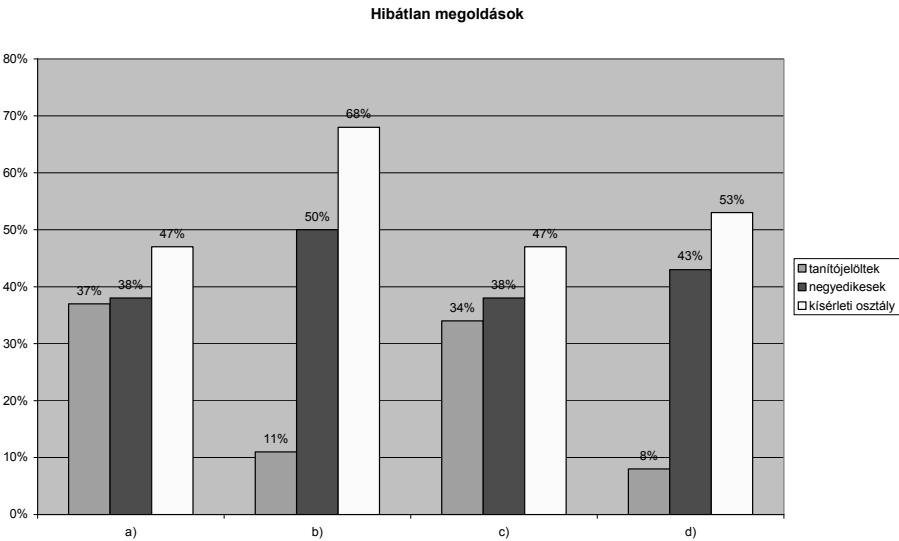
In task 2 related to the recognition of the parallel and perpendicular lateral pairs of the sides of polygons nine shapes had to be examined. They also did the following tasks:

- a) colouring the parallel sides using the same colour;
- b) colouring the right angles red;
- c) enlisting the letters designating the plane figures which have parallel sides;
- d) enlisting the letters designating the plane figures which have perpendicular sides.

The following polygons have been examined:



The results are shown in the following chart:



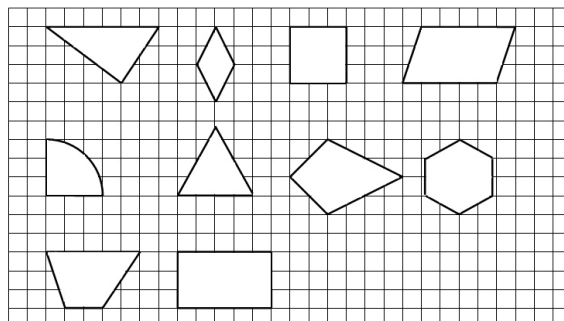
The incidence rate of typical mistakes:

	students	four graders	experimental class
The opposite sides deltoid B were seen parallel by	8%	14%	26%
The parallelism of the inclined straight lines of hexagon D was not marked by	13%	34%	26%
The parallel pair of straight lines of pentagon H was not noticed by	34%	34%	21%
Not only one right angle was found in deltoid B by	8%	14%	11%
The right angle was not marked in hexagon D	82%	23%	11%
The right angle of pentagon H was not recognized by	45%	34%	16%

3. Task related to axial symmetry

In task 3 of the evaluation worksheet the children and the students had to decide which of the given plane figures were symmetrical and also they had to mark the place of the symmetry axes red.

The following plane figures have been examined :



The evaluation of the results of task 3:

	students	four graders	experimental class
Correct solution	5%	20%	47%
Mistake in one plane figure	13%	27%	32%
Two or fewer than 2 symmetry axes were drawn in the square by	37%	18%	0%
In the general rhombus only one or no symmetry axis was drawn by	13%	7%	0%
In the general parallelogram symmetry axis was drawn by	34%	7%	0%
In the equilateral triangle one symmetry axis or not even one was marked by	63%	73%	37%
In the quarter cycle no symmetry axis was drawn by	18%	30%	11%
In the regular hexagon not all the symmetry axes were found by	90%	55%	37%
In the rectangle the diagonals were also seen as symmetry axes	13%	4%	0%

In the experimental group nobody made mistake in case of general rhombus, parallelogram and the rectangle. Most mistakes were made by the teacher training college students.

We regret to say that the results of the students almost in every task were poorer than that of the fourth graders. These finding are related only the samples examined, which were not representative, which is why no statistical trials were conducted. The tasks of the worksheet are in line with the curriculum requirements of class 4. This is the teaching material the teachers are supposed to pass on to children, but how can they teach it, if the would-be teachers have not mastered it

properly? We are wondering which of the van-Hiele level of geometrical thinking is reached by the students.

In our opinion it is a realistic expectation that students who gained the secondary school-leaving certificate should reach van Hiele's level 4, the level described by *the efforts to reach the complete logical set-up* and the *formal deduction*. At this level the learners are able

- to formulate simple theorems
- to grasp the essence and relevance of axioms, definitions and theorems
- to understand how to determine and to figure out the role of necessary and sufficient conditions.

Even at level 3, the *level of local logical arrangement and informal deduction* learners are able to cope with multiple classification and they understand the classification of various quadrangles according to concepts. They perceive the relationships between the characteristics of a given shape or among various shapes. At this level the square is seen as a rectangle, even a parallelogram.

The multiple classification was carried out properly by 16 % of the students.

Thus in teaching geometry to would-be teachers we need to return to van Hiele's level 2 on several occasions and their geometrical concepts should be developed by using even more activities than before such as measuring, folding, clipping, modelling and applying mirrors in order to reach the required level.

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The mathematics-knowledge of the teacher training' students in the first year

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Abstract. The Hungarian mathematics school-leaving examination of Mai 2010 finished with an saddening result (the average of the country takes 2,91 at intermediate level). This result shows that the subject mathematics still makes the biggest difficulty for the school-leaving students. That's why I would like to survey the mathematics knowledge of the teacher training' students at the first year within the help of a self-made test. The test consists on with mathematics concepts given standard exercises that are practice-oriented and true to life. The questions that I search answers of: in how much percent do the teachers solve the exercises? Because the competence of the teacher training' students is very important for the following generations. Then I examine how big the difference is in the number of students who solves an ordinary mathematics exercise and/or with a text conceived exercise of an other branch of science. Because in this fast world the life-like of mathematics plays a significant role at the manpower-market. I also analyze whether it influences the mark of the school-leaving examination that a student has come from a grammar school or a vocational secondary school. Because the results of the school-leaving examination show that the students from the vocational secondary school can mostly solve the exercises only for that the knowledge brought from the primary school is enough.

“The aim of teaching is not the dates, but giving of worths.” (W. R. Inge)
This quotation could be the motto of my dissertation. The Hungarian education has not changed completely so much, as the establishment of the new school-leaving examination would expect. The main front of classworking, the little opportunity of introduction, the uninteresting material take the joy of students from the mathematics. So long at primary school for 6-10-year -olds mathematics is in the three most favorite subjects, then by passing of time it goes down “on the hierarchy”. The mechanical learnt knowledge discourage students from mathematics, but there is a danger, they cram a lot of things that they don't understand., The Hungarian school-leaving examination of Mai 2010 had a very sad result(2,91 the average at middle level). This result shows that mathematics still causes the biggest difficulty for school-leaving students. The most saddest consequence of school-leaving

exam-results, that vocational secondary school's students can only solve exercises for which the at the primary school got knowledge is enough.

Middle level	Average percent:	45,97
	Class average:	2,91
Middle level	Number of exams	87422



Figure 1. Country-results of Mai 2010

Results in Győr became a bit better. It can be thanked that region of Győr is in the leader-regions of West-Hungary. And Győr is in a lucky situation in relation of schools, especially secondary schools. It has more secondary grammar-schools that are in leading groups at country-level and there is a number of students which visit mathematics—competitions and study on at well-known universities.

Middle level	Average percent:	54,64
	Average mark:	3,3
Middle level:	Number of exams:	2371



Figure 2. Results of Mai 2010

Watched the results of school-leaving exams, I examined the mathematics-knowledge of teacher-training students at the first year with a self-made test. The test is life-true, practice oriented and it contains standard exercises given with mathematics concepts. Questions of which I searched answers: in which percent the students solve the exercises. Competence of mathematics is very important for the following generations. Then I saw how big the solving-difference is by students between traditional mathematics exercises and with text-concepted exercises (used also in other sciences), because in this "fast" world the life-truth of mathematics has a very important part in manpower-market. The test was written by 58 students. Their school-leaving exam's mark is 2,89 which is similar to the country-average. The average of surveying 2,62 is under the school-leaving exam' average.

The exercises of the test:

1. Which is the number, from that we subtract its 37%, we get 2586?
2. Draw and characterize (range, zero place, monotonicity, extreme value) the following function into the set of real numbers!

$$f(x) = (x + 2)^2 - 1$$

3. The three sides of an triangle are 3 cm, 4 cm, és 6 cm. How big are the sides of a similar triangle, if its shortest side is 10,5 cm?
4. In a circle with 24 cm radius to a 30 cm long chord how big is the central angle?
5. A sheep grazes on a rectangle field which has 48m long fence. One side of the land is directly on the wall of the house. How shall we choose the measure of the rectangle if we would like that the sheep eats on the biggest landsurface?

In advertising of a glasses-company: a customer can reduce the price of the bought glasses-frame how old the customer is.

- a) How much percent can a 17-year-old customer reduce the price of his ready glasses if the chosen lens cost 60 000 Ft and the frame costs 25 000 Ft?
- b) How old should be that customer that would like to reduce the price of the ready glasses with 17% in case of another glasses like the previous?
- c) How worth should be that frame with that the 17-year-old customer can reach that the price of his glasses reduces by 10% of the whole price, if he needs lens worth of 60 000 Ft?
6. The flying-high of a plane is 8000 metres, flying-speed is 700 km/h. How long does it stay for a spectator above the horizon if it arrives from the Nord and goes to the south? (We shall take the Earth like a globe of 6378 km radius.)
7. Suzanne plans a tour with his friends. Surveyed on the tourist-map the planned route is 57 km. How long does it take for the group to go along the chosen tour-route, if they go average 3,8 km-t per hour, and the map of Suzanne is on the scale of 1:40 000?
8. Energie-need

This exercise's maintenance is that we must choose such meals which adjust to the needs of an Zedcountry's inhabitant.. The next table shows how much is the suggested energie-need in kilojoule (kJ) in case of different people.

AZ AJÁNLOTT NAPI ENERGIASZÜKSÉGLET FELNÖTTEK SZÁMÁRA			
Életkor (években)	Aktivitási szint	FÉRFIAK	NŐK
		Szükséges energiamennyiség (kJ)	Szükséges energiamennyiség (kJ)
18-től 29-ig	Könnyű	10660	8360
	Mérsékelt	11080	8780
	Nehéz	14420	9820
30-tól 59-ig	Könnyű	10450	8570
	Mérsékelt	12120	8990
	Nehéz	14210	9790
60 és afölött	Könnyű	8780	7500
	Mérsékelt	10240	7940
	Nehéz	11910	8780

EGYES FOGLALKOZÁSOKNAK MEGFELELŐ AKTIVITÁSI SZINT		
Könnyű:	Mérsékelt:	Nehéz:
Bolti eladó	Tanár	Építőmunkás
Irodai dolgozó	Üzletkötő	Fizikai munkás
Háztartásbeli	Ápoló	Sportoló

a. *exercise*

Károly Molnár is a 45 years old teacher. How much is his suggested energie-need every day in kJ?

b. *exercise*

Edit Szalay is a 19 years old high-jumper. One evening a few of her friends invite her to a restaurant. There is the menu:

ETLAP		1 adag energiatartalma Edit becslése szerint (kJ)
Levesek:	Paradicsomleves	355
	GombakréMLEves	585
Főételek:	Mexikói csirke	960
	Karibi gyömbéres csirke	795
	Zsályás sertéspörkölt	920
Saláták:	Burgonyasaláta	750
	Vegyes zöldségsaláta	335
	Kuszkusz saláta	480
Desszertek:	Almás-málnás lepény	1380
	Túrótorta	1005
	Répatorta	565
Tejes turmixok:	Csokoládé	1590
	Vanília	1470

At the restaurant you can get one-price menu, too.

Menü	
ára 50 zed	
Paradicsomleves	
Karibi gyömbéres csirke	
Répatorta	

Edit makes notices every day about that she had eaten..That day the eaten foods till supper suit 7520 kJ energie-consumption. Edit would like to reach that all her energie-consumption **should be not more than 500 kJ or it should not stay under the for her suggested day-quantity**. Decide wether Edit stays in the ± 500 kJ-limit with the consumption of the one-price menu!

The result's appraisal:

I took in paars the standard exercises with the practice-oriented exercises. The result is the following on the ground of this:

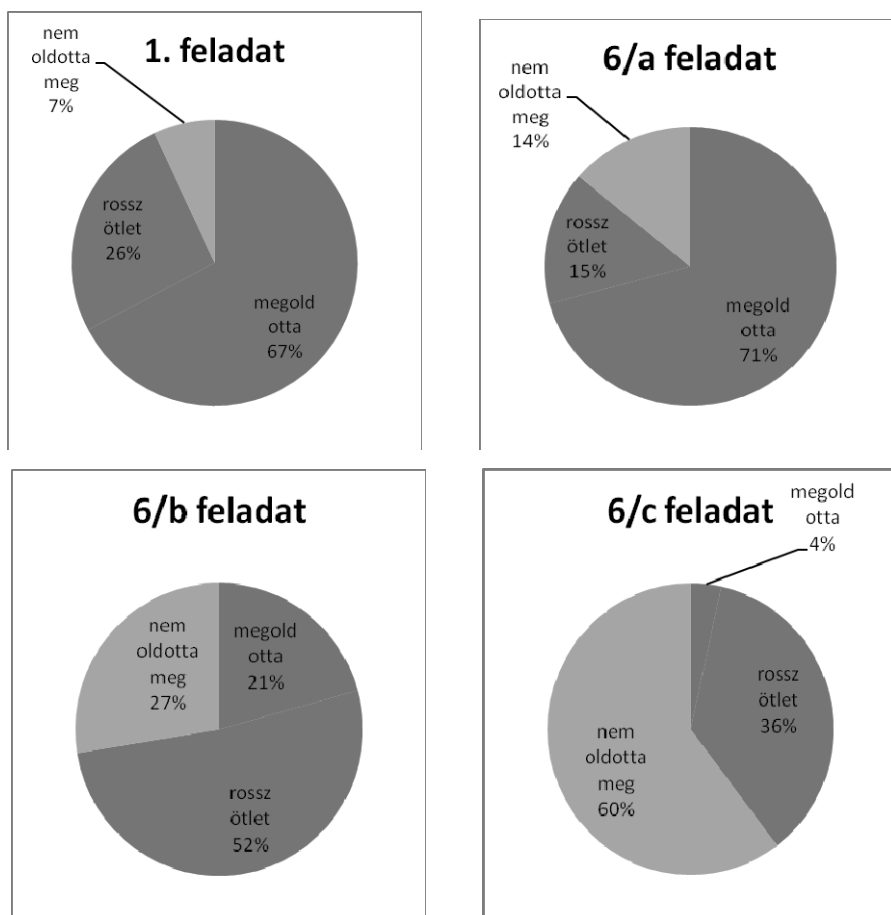


Figure 3. The success of the 1 and 6 exercises (megoldotta=solved, nem oldotta meg=not solved, rossz ötlet=bad idea)

The more the calculation of percentage'exercises are more difficult the less students can solve them. We can usually meet such sales in the ophthalmology shops like in the 6th exercise. It is a goog example whether we could count how much reduction we get in fact.

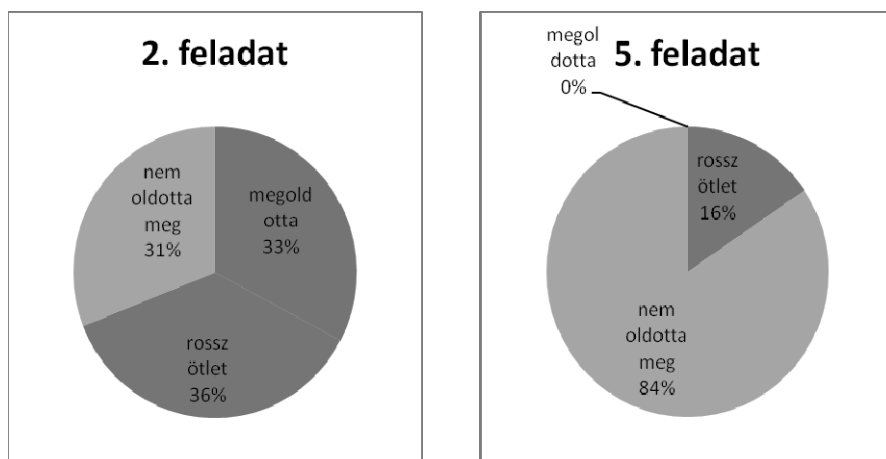


Figure 4. The success of exercises 2 and 5 (megoldotta=solved, nem oldotta meg=not solved, rossz ötlet=bad idea)

Generally students can solve the determination of the place and value of function's extrem point, but if they meet an exercise where they have to interpret this, they get in trouble again. For a lot of them it was the same how much is the length of the sides by given circumference to get a maximal surface

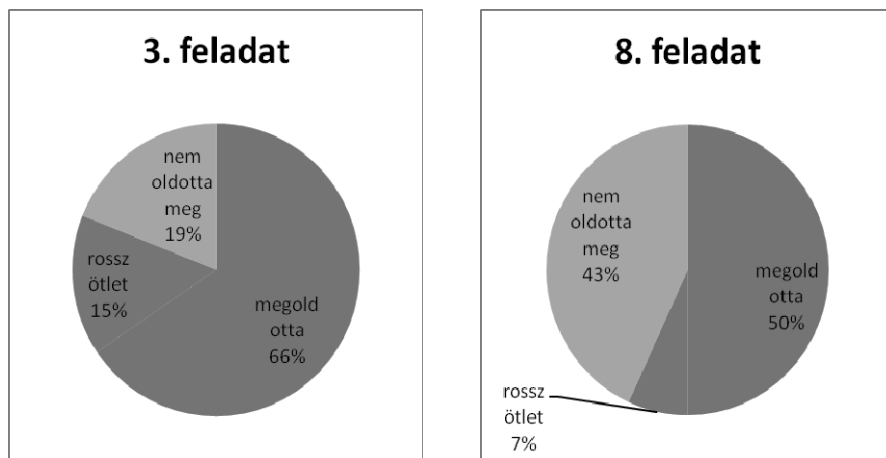


Figure 5. The success of the exercises 3 and 8 (megoldotta=solved, nem oldotta meg=not solved, rossz ötlet=bad idea)

The easier exercise in connection with similarity don't cause problems, this shows the solving-percentage of exercise 3. But I also wondered that in the case of the "map"-exercise became worse the solving-proportion, however little children also learn reading and searching from/on the map at the primary school in the subject of environment.

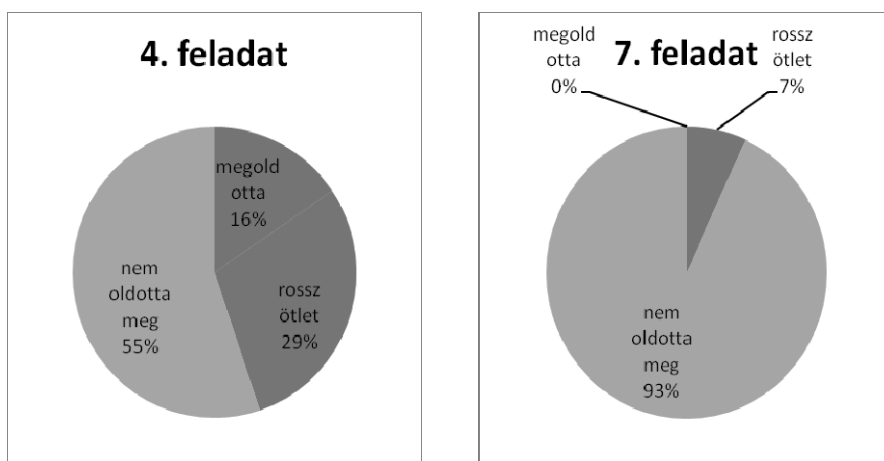


Figure 6. The success of the exercises 4 and 7 (megoldotta=solved, nem oldotta meg=not solved, rossz ötlet=bad idea)

Trigonometry does not belong to the favorite topic at the secondary school..The students still dealt with the standard exercises a bit, but they couldn't even draw the with text given exercises.

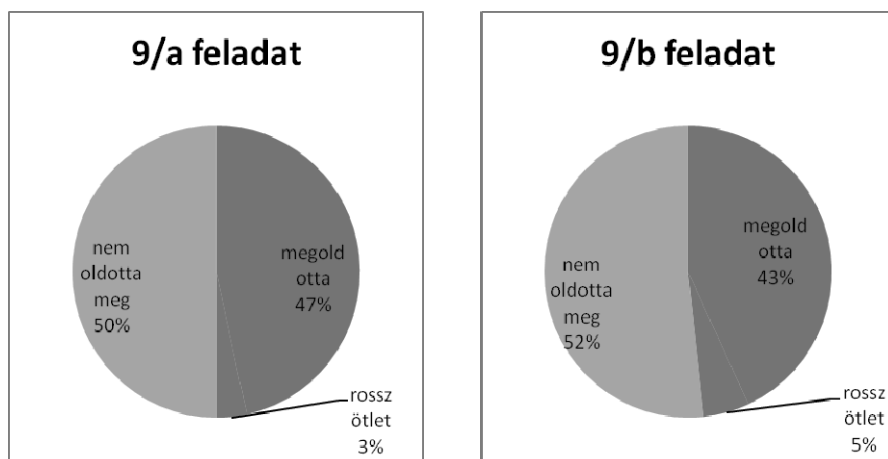


Figure 7. The success of the exercises 9 (megoldotta=solved, nem oldotta meg=not solved, rossz ötlet=bad idea)

This exercise can be found on the homepage of PISA, and not between mathematics exercises, but at the topic of problemsolving. You don't need a lot of mathematics knowledge to these, rather you must interpret the text. Surprising results were born.

Appraising the results it can be told, that students, that go on to universities/colleges, don't have right knowledges from mathematics. The with text given

examples cause them bigger problems. Also exercises that you must search from the table and that you don't need mathematics knowledge to. It turned out from the consulting with the students that they had left the 9th exercise, because the text was so long and there still was a table, that's why they had thought so that it must be very difficult. And this raises the question whether that causes the majority of mathematics problems that students – not only mines – don't understand the exercises' text? Because by exercises like these the students have to do the model-forming on the ground of the text's informations. To this it is essential that they must understand what they read.

What can be the problem?

Children at the primary school for 6-10 year-old-students can solve comprehension-exercises quite good, but later in case of PISA they performed the worst even in this part.. After the fourth class we don't develop certain skills – neither reading skills. On the other hand the importance is set on a more special subject' knowledge. Ending the development of reading, functional illiteracy will be in the population in large numbers.

The other important question is, what and how we should teach in the mathematics lessons?

It isn't good to teach only practical knowledge without more serious lexical knowledge. If there is only the concept of definition at the students, but not the forming of concepts, and they couldn't bind examples to these, then they will have only cramed knowledge without meaning. On the other hand, if we make efforts to take life-true examples in the lessons, we can help the students with understanding. This can be an experience for them to examine real problems, familiarizing themselves with mathematics relations this way. It is easier to get definitions from the lang-lasting memory, if we can contact them to a given situation. The school-work by us doesn't adjust to the characteristics of the development and to previous knowledge. Students are usually forced to learn something that they don't understand. If they can be victorious about these problems, perhaps we can reach that students have the right attitude to mathematics again.

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Competence of teacher education students for giving lessons in advanced mathematics

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Abstract. Elementary school teachers choose on their own, which extracurricular activities they want to lead. We are interested in their choice of advanced mathematics as extracurricular activity. Corresponding research addresses the teacher education students enrolled in final year of study, who will in future be responsible for the advanced mathematics courses in the first four grades of elementary schools.

This paper explores the ability of the teacher education students enrolled in final year of study to prepare the first four grades of elementary school pupils for mathematics competitions. Students' efficacy in solving exercises from the highest ranked mathematics competition in Croatia for fourth grade elementary schools pupils is tested and compared with fourth grade pupils' efficacy in solving those exercises.

Research is conducted at four Croatian universities (Osijek, Zadar, Split, Zagreb) on the sample of students enrolled in final year of teaching primary education programme. The obtained outcome indicates that pupils are more successful in solving those mathematical problems than students. Furthermore, the amount of mathematical subjects in teaching primary education programme has an effect on tested students' efficacy.

Keywords: teacher education students, mathematics competitions, lessons in advanced mathematics program, mathematical subjects in teaching primary education programme

Introduction

Each educational institution should organize appropriate extracurricular activities in order to meet requirements and interests of their pupils. Croatian law on education in primary and secondary schools demands that extracurricular activities should be denoted in schools curriculum and annual plan and programme. Leaders of extracurricular activities of pupils enrolled in one of first four grades of elementary schools are teachers who choose on their own the area of interest of extracurricular lessons that they will give to their pupils. Considering described

manner of choice the question if teachers are competent enough to give lessons in independently elected extracurricular activity arises.

Our research addresses the teacher education students enrolled in final years of study who will in future be responsible for the advanced mathematics courses as the extracurricular activity in the first four grades of elementary schools. For the purpose of this paper we consider that extracurricular activity in mathematics means preparing the pupils for mathematics competitions.

We assumed that students enrolled in final years of teaching primary education programme are able to prepare the first four grades of elementary schools pupils for mathematics competitions and that the amount of mathematical subjects at the teacher education study has an effect on it. Considering appointed assumptions we conducted the research by testing students' efficacy in solving exercises from the highest ranked mathematics competition in Croatia for fourth grade elementary schools pupils and afterwards comparing it with the fourth grade elementary schools pupils' efficacy in solving those exercises. Highest ranked mathematics competition in Croatia for fourth grade elementary schools pupils is called regional competition. It is the level of mathematics competition considered for the purpose of this paper.

Research is conducted at four Croatian universities (Osijek, Zadar, Split, Zagreb) on the sample ($n = 195$) of students enrolled in final years of teaching primary education programme. Hereafter, the term student denotes the student enrolled in one of final years of teaching primary education programme study, while the term pupil denotes the pupil enrolled in the fourth grade of elementary school.

Research framework

Extracurricular activities can take place in school and beyond, and can be divided according to purpose (educational, artistic, informative- educational) and according to the teaching field (physical education and sports, science and ecology, language and artistic field, information and mathematical area) (Jurčić, 2008).

Conducting extra-curricular activities needs to be addressed seriously, because without the necessary human competence, as well as good concept and organization of the activities, effective implementation cannot be achieved (Cardy, Selvarajan, 2006).

As extra-curricular activities pupils choose independently, their degree of motivation is extremely high, so in these activities they acquire new knowledge much easier and in much larger scale. Guided by the teacher- motivator pupils explore, discover and acquire teamwork skills, develop a sense of responsibility, self-esteem and work habits (Šiljković et al, 2007). Teacher's role in the conduct of extracurricular activities is crucial. The teacher must have the support of the school administration as well as the freedom to choose extracurricular activities that will lead. In addition, teachers must be competent to choose pupils who will participate in extracurricular activities according to their own interests and abilities, and be qualified to conduct the activities. The latter should be focused in two directions: (i) understanding the purpose and goal of extracurricular activities, (ii) the ability

to design pedagogical and methodical program of extracurricular activities with a different approach compared to regular classes (Jurčić, 2008).

As already stated, one of the teacher's competence and responsibility is the selection of pupils who will participate in extracurricular activities. Given the current situation in the classroom, we can see that gifted pupils are often ignored rather than teachers deal with them. The teachers think more about regular teaching than about their additional education and developing their competence in working with gifted pupils (Reis, Renzulli, 2010). Thinking about mathematically gifted pupils, they differ in three characteristics related to mathematics: (i) learning rate, (ii) the depth of understanding, (iii) interest in the subject. In additional teaching of mathematics teachers should prepare pupils for mathematical competitions, the place where students test their skills in comparison to their peers. The current system of competition for fourth grade pupils in elementary schools in the Republic of Croatia, which we will consider in this paper, includes the following levels of competition: school competitions, for which teachers prepare the school assignments, the municipal or city events, county and regional competition, for which the tasks are prepared by the state commission so the tasks and criteria are unique to each category of participants (Elezović, 2005). At the international level, fourth-grade pupils in elementary schools in the Republic of Croatia participate in competition "Math Kangaroo contest", where students from 40-odd countries compete in different categories.

The sample — data and variables

After we identified the problem of questionable competence of the teacher education students for giving lessons in advanced mathematics, we appointed two directions of the research on the sample of students of the population of interest: (i) efficacy in solving exercises from the highest ranked mathematics competition in Croatia for fourth grade elementary schools pupils, (ii) the effect of the amount of mathematical subjects in teaching primary education programme on tested students' efficacy. Taking research directions into account we designed the research and conducted it at four Croatian universities (Osijek, Zadar, Split, Zagreb) on the sample ($n=195$) of students enrolled in final years of teaching primary education programme in the period from October till December 2010.

Testing the efficacy in solving exercises from the regional competition of the pupils enrolled in fourth grade of elementary schools in Croatia is conducted on exercises adopted from regional mathematics competition held in May 2010. Students were solving the exercises exactly 90 minutes supervised by mathematics university teachers. This research included 195 respondents. Furthermore, we downloaded public data of fourth grade elementary schools pupils' efficacy in solving exercises from mentioned regional competition from the official web site of the Croatian mathematical society, where the results of this mathematics competition for the year 2010 are published. Pupils were solving five exercises for 180 minutes supervised by examination committee consisting of elected mathematics teachers. 153 pupils participated in this regional competition in 2010.

Structures of previously mentioned samples of students and pupils with respect to the number of points achieved in solution of particular exercise are depicted in tables below (Table 1 — Table 5).

EX1	STUDENTS ($n = 195$)		PUPILS ($n = 153$)	
Number of achieved point	Frequency	Relative frequency (%)	Frequency	Relative frequency (%)
0	56	28,71795	21	13,72549
1	0	0,00000	12	7,84314
2	19	9,74359	4	2,61438
3	0	0,00000	1	0,65359
4	7	3,58974	7	4,57516
5	2	1,02564	3	1,96078
6	6	3,07692	3	1,96078
7	2	1,02564	1	0,65359
8	4	2,05128	2	1,30719
9	4	2,05128	8	5,22876
10	95	48,71795	91	59,47712

Table 1. Structure of the samples with respect to the number of points achieved in solution of first exercise (EX1).

EX2	STUDENTS ($n = 195$)		PUPILS ($n = 153$)	
Number of achieved point	Frequency	Relative frequency (%)	Frequency	Relative frequency (%)
0	104	53,33333	27	17,64706
1	0	0,00000	11	7,18954
2	2	1,02564	28	18,30065
3	1	0,51282	1	0,65359
4	0	0,00000	2	1,30719
5	8	4,10256	4	2,61438
6	2	1,02564	1	0,65359
7	14	7,17949	2	1,30719
8	13	6,66667	2	1,30719
9	0	0,00000	1	0,65359
10	51	26,15385	74	48,36601

Table 2. Structure of the samples with respect to the number of points achieved in solution of second exercise (EX2).

EX3	STUDENTS ($n = 195$)		PUPILS ($n = 153$)	
Number of achieved point	Frequency	Relative frequency (%)	Frequency	Relative frequency (%)
0	156	80,00000	28	18,30065
1	0	0,00000	16	10,45752

2	5	5,56410	25	16,33987
3	0	0,00000	2	1,30719
4	3	1,53846	11	7,18954
5	2	1,02564	2	1,30719
6	2	1,02564	1	0,65359
7	1	0,51282	1	0,65359
8	5	2,56410	4	2,61438
9	0	0,00000	2	1,30719
10	21	10,76923	61	39,86928

Table 3. Structure of the samples with respect to the number of points achieved in solution of third exercise (EX3).

EX4	STUDENTS ($n = 195$)		PUPILS ($n = 153$)	
Number of achieved point	Frequency	Relative frequency (%)	Frequency	Relative frequency (%)
0	75	38,46154	24	15,68627
1	7	3,58974	20	13,07190
2	14	7,17949	8	5,22876
3	12	6,15385	4	2,61438
4	19	9,74359	2	1,30719
5	12	6,15385	3	1,96078
6	20	10,25641	7	4,57516
7	6	3,07692	3	1,96078
8	12	6,15385	8	5,22876
9	6	3,07692	3	1,96078
10	12	6,15385	71	46,40523

Table 4. Structure of the samples with respect to the number of points achieved in solution of fourth exercise (EX4).

EX5	STUDENTS ($n = 195$)		PUPILS ($n = 153$)	
Number of achieved point	Frequency	Relative frequency (%)	Frequency	Relative frequency (%)
0	53	27,17949	16	10,45752
1	1	0,51282	11	7,18954
2	10	5,12821	8	5,22876
3	1	0,51282	6	3,92157
4	5	2,56410	2	1,30719
5	6	3,07692	5	3,26797
6	6	3,07692	6	3,92157
7	6	3,07692	0	0
8	7	3,58974	15	9,80392
9	6	3,07692	3	1,96078
10	94	48,20513	81	52,94118

Table 5. Structure of the samples with respect to the number of points achieved in solution of fifth exercise (EX5).

The structure of the sample of students on which the research is conducted is depicted in table below (Table 6).

Characteristic	Category	STUDENTS ($n = 195$)	
		Frequency	Relative frequency (%)
GENDER	Female	188	96,41026
	Male	7	3,58974
ATTENDING GRAMMAR SCHOOL	Yes	141	72,30769
	No	54	27,69231
ECTS CREDITS IN SO FAR ATTENDED MATHEMATICS SUBJECTS	19	86	44,10256
	16	58	29,74359
	15	38	19,48718
	11	13	6,66667

Table 6. Structure of the sample of students.

Methodology

After previously expounded data collecting, we conducted statistical analysis using Soft STATISTICA 7.0 with purpose of estimation population parameter utilizing confidence intervals and testing statistical hypothesis derived from research hypotheses. In that manner we made conclusions about populations of interest.

We estimated population mean based on a sample data from the population of interest. For the purpose of this paper we used 95% confidence level for the interval estimate that ensures that randomly selected confidence interval encloses population parameter with probability of 0,95 (McClave et al., 2001).

Test of hypothesis is conducted by implementing following steps (McClave et al., 2001): (1) defining null hypotheses H_0 which represents status quo, (2) defining alternative hypotheses H_1 , (3) selecting the appropriate test statistic which will be applied at deciding about rejecting null hypotheses, (4) determining the rejection region with respect to the level of significance α of the test, (5) data collecting, (6) calculating test statistic, (7) decision making about rejecting null hypotheses, (8) making inference about population. Level of significance α utilized in hypothesis-testing process for the purpose of this paper is 0,05. In order to compare two population means we conducted test for testing hypothesis about difference between two population means utilizing z-statistic for large-sample case.

Results

Employing information from the samples for the number of points achieved in solution of particular exercise, we computed sample numerical descriptive measures and estimated mean using 95% confidence interval. Obtained calculations depicted in table below (Table 7) indicate the difference in the mean for the number of points achieved in solution of particular exercise between students and pupils

in favor of pupils. It is important to emphasize that pupils and students applied solving methods appropriated for the pupils of younger school age.

	STUDENTS ($n = 195$)				PUPILS ($n = 153$)			
	Descriptive statistics of the sample		95% confidence interval for mean estimation		Descriptive statistics of the sample		95% confidence interval for mean estimation	
	Mean	Standard deviation	Lower bound	Upper bound	Mean	Standard deviation	Lower bound	Upper bound
EX1	5,86667	4,49543	5,23174	6,50159	7,11765	4,06030	6,46911	7,76618
EX2	3,95385	4,45978	3,32396	4,58373	5,77124	4,37622	5,07225	6,47023
EX3	1,54359	3,36322	1,06858	2,01860	5,22222	4,28465	4,53786	5,90659
EX4	3,27692	3,32830	2,80684	3,74700	6,11111	4,24195	5,43356	6,78866
EX5	6,16410	4,37586	5,54607	6,78214	7,00000	3,86958	6,38193	7,61807
TOTAL	20,80513	12,89502	18,98387	22,62638	31,22222	12,03461	29,29999	33,14446

Table 7. Descriptive statistics of the samples and the estimation of mean.

Collected numerical data for the sample of students and the sample of pupils are presented graphically utilizing box-plot based on median for the number of points achieved in solution of particular exercise with respect to exercise solvers (Figure 1, Figure 2). This examination also indicates the difference in the number of points achieved in solution of particular exercise between students and pupils in favor of pupils.

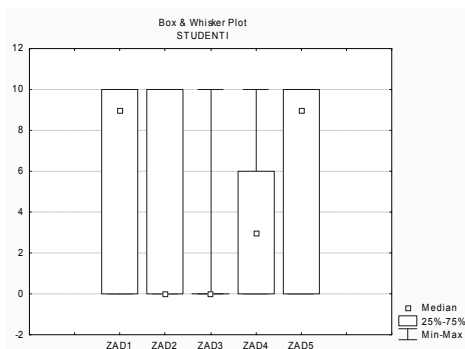


Figure 1. Box-plot for the number of points achieved in solution of particular exercise for students.

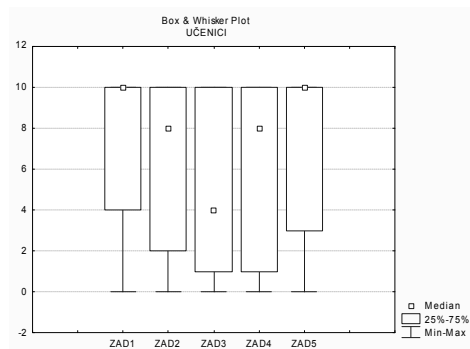


Figure 2. Box-plot for the number of points achieved in solution of particular exercise for pupils.

Furthermore, we conducted one-tailed large-sample test of hypothesis about difference in the number of points achieved in solution of particular exercise between students and pupils, where alternative hypothesis represented the existence of the difference in the number of points achieved in solution of particular exercise in favor of pupils ($H_1 : \mu_1 > \mu_2$). Design of this alternative hypothesis is based on previous analysis. Test results depicted in table (Table 8) indicate that at the $\alpha = 0,05$ level of significance there is a statistically significant difference in

means of the number of points achieved in solution of particular exercise between pupils' and students' solutions in favor of pupils. Ultimately, it makes pupils more successful in solving all exercises at 5% level of statistical significance.

Considered characteristic	p -value	$\alpha = 0,05$	z	$z_{\alpha} = 1,644854$	H_0
EX1	0,0032553383143	$p < \alpha$	2,720888253	$z > z_{\alpha}$	reject
EX2	0,0000686269	$p < \alpha$	3,81306549	$z > z_{\alpha}$	reject
EX3	0	$p < \alpha$	8,71933034	$z > z_{\alpha}$	reject
EX4	$5,75 \dots 10^{-12}$	$p < \alpha$	6,786322167	$z > z_{\alpha}$	reject
EX5	0,02952666	$p < \alpha$	1,887796191	$z > z_{\alpha}$	reject
UKUPNO	$4,10782519 \dots 10^{-15}$	$p < \alpha$	7,765870014	$z > z_{\alpha}$	reject

Table 8. Results of one-tailed test of hypothesis about difference in the number of points achieved in solution of particular exercise with respect to solvers.

In addition, we explored the impact of the so far attended mathematical subjects in teaching primary education programme study on tested students' efficacy in solving given exercises, where so far attended mathematical subjects are presented by the sum of the number of ECTS credits (Table 6). Descriptive statistics of the sample of students with respect to the total number of ECTS credits in the so far attended mathematical subjects and the mean estimation of the number of points achieved in solution of particular exercise (Table 9, Figure 3 — Figure 5) point out that students having the greatest number of ECTS credits in the so far attended mathematical subjects achieve the best results.

ECTS	n			EX1	EX2	EX3	EX4	EX5	TOTAL
19	86	Descriptive statistic of the sample	Mean	7,220930	6,709302	3,186047	4,534884	7,976744	29,62791
			Standard deviation	4,02463	4,30285	4,27994	3,59967	3,49782	11,21768
		95% confidence interval for the estimation mean	Lower bound	6,35805	5,78677	2,26842	3,76311	7,22681	27,22283
			Upper bound	8,08381	7,63183	4,10367	5,30665	8,72668	32,03298
16	109	Descriptive statistic of the sample	Mean	4,79817	1,77982	0,24771	2,28440	4,73394	13,84404
			Standard deviation	4,576065	3,215581	1,434728	2,728762	4,481737	9,421385
		95% confidence interval for the estimation mean	Lower bound	3,92936	1,16931	-0,02469	1,76633	3,88305	12,05531
			Upper bound	5,66697	2,39032	0,52010	2,80248	5,58484	15,63276

Table 9. Descriptive statistics of the sample of students and the mean estimation.

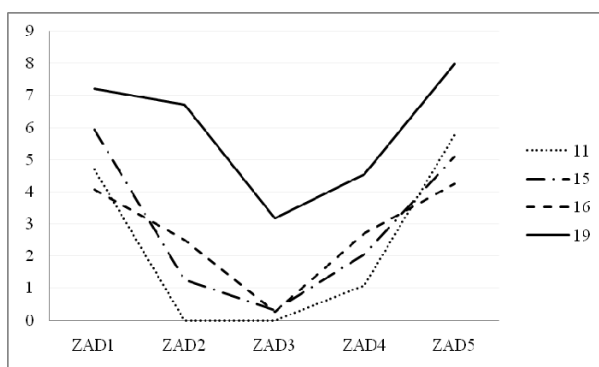


Figure 3. Line chart for the mean of the students' number of points achieved in solution of particular exercise with respect to the number of ECTS credits achieved in the so far attended mathematical subjects.

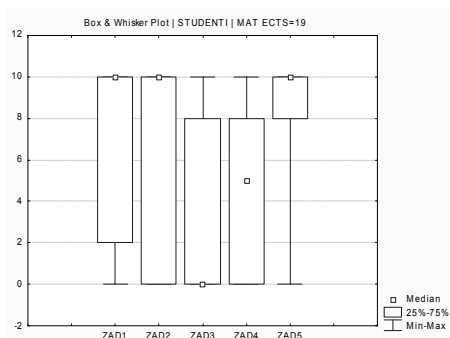


Figure 4. Box-plot for the number of points achieved in solution of particular exercise for students who achieved 19 ECTS credits in the so far attended mathematical subjects.

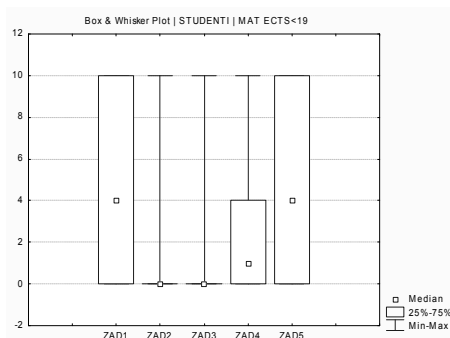


Figure 5. Box-plot for the number of points achieved in solution of particular exercise for students who achieved less than 19 ECTS credits in the so far attended mathematical subjects.

Finally, we conducted one-tailed test for hypothesis for comparing means of the students' number of points achieved in solution of particular exercise with respect to achieved number of ECTS credits in the so far attended mathematical subjects. The alternative hypothesis represented the existence of the difference in the number of points achieved in the solution of particular exercise in favor of students who achieved 19 ECTS credits in mathematical subjects ($H_1 : \mu_1 > \mu_2$). Design of this alternative hypothesis is based on previous analysis. Test results depicted in table (Table 10) indicate that at the $\alpha = 0,05$ level of significance there is a statistically significant difference in means of the number of points achieved in solution of particular exercise between students, who achieved 19 ECTS credits in so far attended mathematical subjects, and students, who achieved less than 19 ECTS credits in so far attended mathematical subjects, in favor of former. Ultimately, this finding makes students, who achieved described 19 ECTS credits, more successful than other students in solving all exercises at 5% level of statistical significance.

Considered characteristic	p -value	$\alpha = 0,05$	z	$z_{\alpha} = 1,644854$	H_0
EX1	0,000042849868	$p < \alpha$	3,927874986	$z > z_{\alpha}$	odbaciti
EX2	0	$p < \alpha$	8,851520848	$z > z_{\alpha}$	odbaciti
EX3	$5,24 \dots 10^{-10}$	$p < \alpha$	6,101923784	$z > z_{\alpha}$	odbaciti
EX4	$7,58 \dots 10^{-7}$	$p < \alpha$	4,809169076	$z > z_{\alpha}$	odbaciti
EX5	0,0000000069416	$p < \alpha$	5,674822095	$z > z_{\alpha}$	odbaciti

Table 10. Results of the one-tailed test for hypothesis for comparing two population means.

Conclusion

From the results obtained by conducting research we determined current state of the ability of the teacher education students enrolled in final years of study in Croatia to prepare the first four grades of elementary school pupils for mathematics competitions at extracurricular mathematics lessons.

Obtained outcomes indicate that pupils, who participated at regional mathematics competition, are more successful in solving exercises from regional mathematics competition than students. Therefore we believe that current generations of students enrolled in the final years of teacher education study in Croatia are not competent enough for preparing the first four grades of elementary school pupils for mathematics competitions in their future teaching. Besides, this research revealed that the amount of mathematical subjects at the teacher education study has a positive effect on tested students' efficacy. This result should certainly be a guideline for teacher education studies in Croatia for reviewing their programmes in order to provide quality education of future elementary schools' teachers who should be able for giving lessons at all weight levels of teaching appropriate to the pupils enrolled in the first four grades of elementary schools. We plan to conduct similar research in the next generations of teacher education students and thus evident possible change.

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Spatial Reasoning of Students of Mathematics Education at the Department of Mathematics, Faculty of Science, University of Zagreb – Mental Cutting Ability

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Abstract. In July 2010 the Ministry of science, education and sports of the Republic of Croatia has issued the *National framework curriculum for preschool education and compulsory general and secondary education* (NFC), with mathematical area as one of the eight educational areas in all educational cycles and compulsory for all students. In addition to aims and general goals of learning and teaching mathematics which are harmonized with the definition of the mathematical competence in *Key Competences for Lifelong Learning – A European Framework*¹, the document states expected learning outcomes for mathematics at the end of each cycle. They are organized in two dimensions – conceptual (mathematical concepts) and procedural (mathematical processes).

Since geometric reasoning and spatial reasoning particularly is important component of mathematical competence, its development has been rethought and reestablished by learning outcomes in domains Shape and space and Measuring in NFC. Implementation of this part of mathematical curriculum certainly requires highly developed spatial abilities of teachers. It is therefore necessary to examine the level of spatial ability of students of mathematics education programme.

The survey, whose results we present, is a continuation of our previous research of spatial abilities of students of mathematics education at the Department of Mathematics, Faculty of Science, University of Zagreb. The aim was to determine the level of student ability of visualization of spatial relationships, particularly of cross sections of spatial shapes.

¹ The document *Key Competences for Lifelong Learning – A European Framework* is a supplement to the document *Recommendation of the European Parliament and of the Council of 18 December for lifelong learning* (2006/962/EC).

Students have undergone the standardized Mental Cutting Test and their results were analyzed and compared with the results of similar student populations from neighbouring countries (Germany, Hungary, Austria). Besides, on a sample of first-year students, we have conducted a student activity of recognition and mathematical description of shapes formed by water in a box when the box is rotated about the shortest edge of its base. In his talk we present the results of the specified testing by the standardized test and we describe the aim and progress of the student activity, as well as the qualitative analysis of its results.

Keywords: National framework curriculum, mathematical competence, spatial reasoning, visualization, mental cut, standardized Mental Cutting Test, students of mathematics education programmes

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When technology influences learning?

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Abstract. It is very common to talk today about technology supported learning or e-learning. But there is very limited number of research answering the question if technology really enhances learning and quality of teaching as well as achievements of learning outcomes. Let us emphasize that we advocate approach to teaching and learning that is based on the paradigm that pedagogy precedes technology and that setting educational goals comes before looking for appropriate ICT tools or other media. The dilemma about the appropriate usage and influence of media on learning is not something new. There was the great “media debate” started in the 1990s by Richard Clark and Robert Kozma, when Clark (Clark, 1983) presented the analogy of technology’s role in learning as having no more effect than a grocery truck contributes to the quality of the produce it carries and Kozma (Kozma, 1994) stressed that the usage of computer technology is the most effective when it supports active engagement within the curriculum. In our paper we will analyse literature that deals with the question from the title, further we will present our own teaching and learning experience and results of qualitative and quantitative survey we performed.

Keywords: technology, learning

Introduction

From the very beginning of systematically organized learning process, people used all sorts of supplementary instruments to enhance productivity of the same. Availability and, after all, the existence of those supplementary instruments were dictated by the timeline itself. What does that mean? Let us use photography for example: it plays an important role in learning process and was black and white at first, but in years it became colored. Afterwards, mankind has managed to put it in motion and we got motion picture. And nowadays, instead of just watching that motion picture, you can interact with it and create your own diverse environments and conditions to obtain new knowledge. Technology progresses rapidly, hence time has a crucial role in discussion about technology enhanced learning. Not so long ago a chalk and a blackboard, along with books, were basic instruments

for teaching, and today you cannot imagine a lesson without introducing digital presentations on the canvas or screen. It is very common to talk about technology supported learning or e-learning today and the term that is used in society for those overall technologies is information and communication technologies. Among those, the computer technologies are the ones that have expanded extremely in the last few decades. Because of that expansion, people have started to wonder if they really influence, i.e. enhance learning and in this respect this questions still lay open. Computer technologies have been in our classrooms for nearly 30 years and yet we still ask questions about their effect on learning process. In this paper, we shall recall that famous debate between Richard E. Clark (Clark, 1983) and Robert Kozma (Kozma, 1994) about the technology influence and afterwards get acquainted with the latest overall summary of the achievements in higher education (CSLP, 2009) concerning the question from the title. In addition, we shall try to compare that situation with the one at our faculty, the Faculty of organization and informatics, University of Zagreb.

The “great media debate”

The “great media debate” was initiated by Clark’s article (Clark, 1983). He declares in his article that technology’s role in learning is and always will be minimal. What is more, he claims that instructional design and teaching practices (instructional methods) are the core of learning in formal settings, whether instruction is offered as classroom instruction, through distance education, or a combination of both (i.e. blended or hybrid). He said that “. . . media are mere vehicles that deliver instruction but do not influence student achievements any more than the truck that delivers our groceries causes changes in our nutrition”. On the other hand, Kozma (Kozma, 1994) was aware of the computer technology expansion and said that we must not ask ourselves a question “Do media influence learning?” but rather “In what ways can we use the capabilities of media to influence learning?” We have to emphasize that this was in 1994, almost 20 years ago and 10 years after Clark’s article. Kozma didn’t separate media from the instructional methods but considered them as having more integral relationship. He concluded that the usage of computer technology is most effective when students are actively engaged within the curriculum. In his article, students themselves have worked on computers, doing physics, and testing their problem solutions in different conditions. Those students have achieved better test results than the ones who attended traditional lessons. Clark would say that in that physics case computers were a part of a teaching method and the difference is in method you choose. We have to give credits to both of them and say their arguments make a good point, even today. Especially if we take into account that articles were written in early 1980s, when personal computers were still in diapers, so it is understandable why Clark considered technology and media as a part of educational methods. Because of the computer technology infestation in learning process today, one could agree that the majority of today’s modern teaching methods include computer technology and it is all about choosing the right method. We have got to ask ourselves whether technology has really enhanced and improved the learning process with its outcomes or it has just enhanced the availability, accessibility and volume of learning material? And if both of these

statements are correct, in what ratio do they correspond? This was the key note for our simple survey among students at Faculty of organization and informatics that we will talk about later in this paper.

Computer-based technologies

Centre for the Study of Learning and Performance (CSLP) at Concordia University, Canada, has performed an extensive literature search among papers published since 1990 about computer-based technology in higher education (CSLP, 2009). They wanted to explore the achievement effects of the computer-based technology use in higher education classrooms (non-distance education). The search revealed more than 6000 potentially relevant empirical studies which were reduced to a sample of 231 representative studies. We will not go so much into details of this research but rather discuss the results. The year 1990 was chosen because the computer-based technologies are more current technologies. Overall, the results suggested that using technology is better than not using it, but said nothing about *how much* technology improves learning achievement or *how* technologies function best to promote educational outcomes. They managed to find three additional pieces of evidence suggesting how technology use relates to learning in higher education environments. First, in analyzing the overall effects of technology applications in higher education, they found that technology use appears to have its limits when it comes to affecting learning achievement. So, we can keep increasing the quantity of technology applications, but at some point the learning achievement will reach its limit. It won't rise unconditionally. Secondly, they found that technologies *supporting cognition*, broadly defined, produce significantly better results than technologies used to *present or deliver content*. It is arguable that more engaging technology applications outperform less engaging applications related to receiving and internalizing content and this seems to support Kozma's (1994) claim for technology advantages. Although, it does not discount Clark's argument that design is at the root of learning success and that achievements might have been promoted otherwise by non-technology means. Third, they found a differentiation between the levels of technology saturation. Conditions of low and moderate technology saturation led to larger average effects than more highly saturated classroom uses. This suggests that there may be a downside to technology use. Guided by these three before mentioned major results of the CSLP study, we formed a questionnaire for students at the Faculty of organization and informatics which is the topic of our next section.

Survey at Faculty of organization and informatics (FOI), University of Zagreb

At first, let us explain the context we are dealing with. The major aim of the Faculty of organization and informatics is to provide students a first class education in the field of information sciences as well as information technologies. The presence of the word 'technology' in the curriculum implies in a way imperative of using technology enhanced learning during undergraduate and graduate education

(“use technology to teach about technology”). Besides using ICT (Information and Communication Technologies) in classrooms according to teachers’ decisions, e-learning has also been implemented at the Faculty level since 2006, based on strategy and systematic approach (Strategy FOI). The LMS (Learning Management System) that is being used since then is Moodle, which replaces several LMSs that were in use before, but not in systematic way. More about teaching and learning environment at FOI in (Divjak & Erjavec 2007, Divjak & Ostroski, 2009, Divjak et al 2010).

What about the students that Faculty enrolls? We can assume that they are well disposed towards computer technology. It is a very specific situation that is worth comparing with the results of the study about computer-based technologies described in the previous section. We wanted to see what the students’ observations concerning this subject are. Therefore, the questionnaire was conducted among 192 first year students (72% of all) and a sample of 17 students (85% of course enrolled) on the course Discrete mathematics with graph theory (5th semester undergraduate and 1st semester graduate). By the time they enroll Discrete structures and graph theory, students become familiar with much more e-learning courses and diversity of approaches to teaching and learning than the freshmen. Besides that, the mentioned course is well equipped with qualitative ICT materials and tools, so it is very useful to compare the results of the two observed student groups. We are aware that there is a considerable difference in sample size and type and therefore we interpret these data and especially differences only in qualitative way. Hereafter, we shall call freshmen the “younger students” and the second group the “older students”. For the start, we wanted to find out their experience with classroom ICT use in secondary school (Table 1).

%	Ppt presentations	Overhead transparencies	E-learning system (LMS)	Computers	No technology
Younger <i>N</i> = 192	77,1	76,1	7,8	29,2	10
Older <i>N</i> = 17	47,1	76,5	0	23,5	23,5

Table 1. High school experience (multiple choice question).

Overhead transparencies are still something standard in secondary school but are slowly being replaced by PowerPoint presentations as we were saying before. In two- or three-year period (the average age difference between the groups) the percentage of Ppt presentations use has increased considerably. It seems that the LMSs are also infiltrating themselves into secondary schools, what was not the case with the older group. After the experience, we wanted to be sure about their attitude towards technology. Almost 85% of younger students and all older students agreed or strongly agreed that the use of ICT enables easier adoption of teaching content. This confirms our claim about pro-technology oriented students (Figure 1).

Next, we gave them an open question to specify a single reason why they agreed with the previous statement. About 75% of younger students and more than 80% of older students wrote down variations of *teaching material availability*. This goes mainly in favour of Clark’s thesis: today we just have newer trucks to

transfer groceries. Afterwards, students had another similar statement to finish by choosing only one out of tree bidden endings (Table 2).

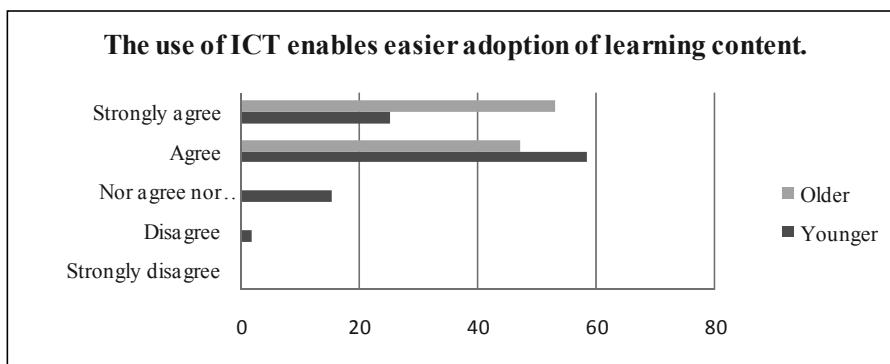


Figure 1. Attitude towards technology (results in percentages).

I think that ICT in educational process contributes most because of ... (%)	... easier availability of the teaching material.	... the teaching material that provides easier understanding of the teaching content.	... the simple teacher/student and student/student communication.
Younger	64	32,3	3,7
Older	82,3	11,8	5,9

Table 2. Attitude towards technology (results in percentages).

We got similar results as in previous open question. This was the major part of the questionnaire where we could identify their beliefs about technology contribution and was specific to the second result of the study from the previous section. As we can see from Table 2, the majority of the students from both groups believe that the main contribution of ICT is to deliver information, teaching material, tools and databases. This is the reality. But why this is the case? Here we have to add that all of the courses at FOI are delivered as blended learning courses. It means that students also have a rich classroom experience and communication with teachers as well as among each other and they are much more looking at LMS for material and supporting tools than to communicate. Therefore, when answering that question they just prioritized their answers and put first what is more obvious to them. By their experience, ICT is mostly used to deliver content, i.e. teachers mostly used ICT to deliver content. One of reasons for this kind of situation is that there are still too many students in lecturing and seminar groups and teachers are rather keen on delivering material through LMS and perhaps spare some time to provide qualitative exercises that Kozma is talking about (which support cognition). On the other hand, teachers have to invest a lot of time in purpose of developing the e-content. To support this claim, we asked students if they have ever encountered, during their study period, materials which helped them in easier understanding of the teaching content. As a result, 82% of younger and 94% of older students answered that they have encounter this kind of material on some courses. So, majority of them agreed that there are some excellent positive examples. We also asked them about the

negative side of the ICT use: whether they have encountered a course which has so large amount of materials (documents, applets, animations, movies...) presented in a non-systematic way that, because of its prolixity, does not contribute to the learning adoption. About half of both groups answered that they have encountered at least one such course. Here we have an example where extensive use of ICT in curriculum delivery does not necessarily contribute to students' learning and achievement of learning goals. What is the ultimate goal the most students have? The highest priority goal is to pass the exam and for most of them this is the only goal. In short: you have pro-technology oriented students, majority of which wants to invest minimal effort required to pass the exams (especially mathematics exam since mathematics is not their core business).

Further, we put them an open question on what kind of more ICT related support they would like to be provided for them (existing or not existing). Some of answers, that describe "dark side" of ICT usage, can be found in Table 3. As we can see, younger students want to have everything on the plate and they put themselves in more passive than active role. For example, a lot of students asked for more "shortened" material for preparing exams and monthly tests. One student even suggested teachers to prepare so called short version of the exercise book with only those examples that are absolutely necessary for written exam (because the exercise book is too extensive). There is a little attention dedicated to leaning on creativity, so teachers have to be careful not to fulfil all students' wishes and in that way imprison their creativity. On the contrary, we have to support creativity and innovation during study period, because today, jobs and careers considerably lean on them. It is encouraging that older students are more aware of employability competences and they have also more sophisticated learning needs.

	Specify some of the most useful materials available through e-learning system (existing or not existing).
Younger (1st year) students	• Shorter electronic presentations with just the contents that come on the exam
	• More examples of solved old midterm exams
	• A script with answered exam questions
	• More exercises with solutions
	• List of presentations and exercises
	• Graphical presentations and animation
Older students	• Self-evaluation tests for students to check their knowledge
	• Links to interesting pages

Table 2. ICT contribution.

Conclusion

The use of technology in teaching process is inevitable, because it has entered into each aspect of everyday life and students are motivated to learn by use of ICT. The survey results show that more engaging technology applications give better learning results. If we are not able to produce this kind of improvement, just

increasing availability of learning materials also makes a positive difference. But we must not forget that pedagogy precedes technology and that setting educational goals comes before looking for appropriate ICT tools or other media. Setting technology in the first place might have an opposite effect and lead to saturation and loss of students' engagement. Therefore we have to be aware of the negative effects of using technology. For example, if it is used for only material delivery and straightforward instructions on how to pass exam, it will have a negative effect on creativity and deeper understanding, as well as on the self-engagement of the students. Because of that teachers have to expose students to different teaching approaches, material and tools that encourage problem solving and creativity. Finally, in the planning process teacher has to start with the list of learning outcomes and potential obstacles that can emerge during learning and teaching process implementation. Upon that, he has to decide on teaching and assessment methods as well as where to implement technology to enhance learning, achieve learning outcomes and personalize learning process. An absolute imperative must be to support student's creativity and innovation.

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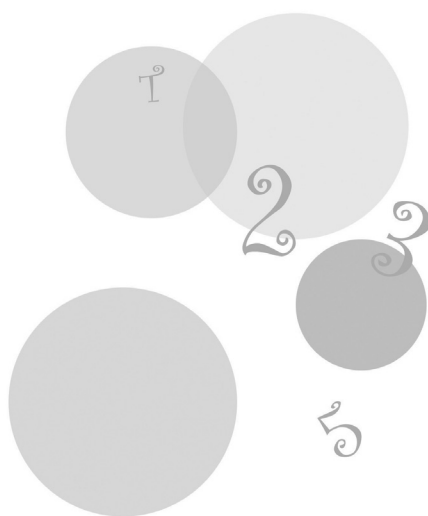
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3.

How to achieve desirable learning outcomes within teacher studies



In the third chapter of the monography in their articles the authors answer the question *How to achieve desirable outcomes of mathematics learning*. Some of the authors are of the opinion that firstly, in school mathematics it is required to replace persistent insisting on formalities, whenever possible, with experimental aspects of mathematics. Other authors see more desirable learning outcomes occurring in the presence of more frequent and structured application of information technologies. They research the attitude of students of teacher studies towards using ICT in the classroom, as well as critically assess relevant issues, criteria and methods for selecting the appropriate on-line materials for teaching mathematics. The third section reports on the efficiency of original approaches and procedures in teaching mathematics, by means of which students are guided towards higher levels of learning outcomes. Modern learning conditions require a competent teacher and pupil/student for long distance learning and application of intelligent systems. It is therefore proposed that additional room be found for achieving such competences within reviewed programmes of teacher studies.

Between axioms and reality

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Abstract. Contemporary teaching and learning of mathematics is supposed to be focused on raising mathematical literacy. This does not mean that we renounce mathematics or its theoretical bases, but it means a lot more: what we learn at mathematics is expected to be promptly understood and logically positioned into the already acquired knowledge, and at the same time we are expected to build mental links necessary for the use of mathematics in every day life. This is the only way we can avoid insufficient functional knowledge of mathematical contents. And at the same time we have to bear in mind that mathematics is in the function of mathematical literacy development important for everyone, not only for those who like mathematics and enjoy in its beauty and elegance.

Mathematics, indeed, is an abstract knowledge; however, it may be brought closer to several pupils by using adequate examples and models. Each pupil can experience it in reality, and more or less clear models guide him/her towards the path of “axioms”. We should bear in mind, that only rare individuals can find their way through the abstract world of mathematics, and that a vast majority of people experience that world only in realistic illustration.

Keywords: teaching and learning of mathematics, mathematical literacy, abstract world, realistic illustration

Introduction

First of all, we have to ask ourselves what objectives and goals give us arguments for mathematical instruction and what are the reasons for it. Do we have adequate curricula? Do we have compelling reasons for mathematical instruction reform and for different approaches to teaching mathematics? Obviously, certain fundamental reasons for mathematical instruction are linked with the development of proper functional capabilities of the learning population, and there is also a reflection underneath on what does the learning population really get within the instruction of mathematics.

The questions on eligibility of teaching mathematics and on the contents that should be taught to certain groups or at certain levels of schooling are not asked very often. Two aspects of teaching are noticeable: one linked with the problem of knowledge testing at the end of schooling at a certain grade (Jensen, Niss, Wedege, 1998), and the other linked with the level of mathematical skills or the need to increase the level of skills within a certain population (OECD, 1995; OECD, 2000).

The Needs of Reality

During the instruction of mathematics we should master the “applied” mathematics or the mathematics which we need in our every day life. First of all, it is a question of our capability of acting in the society, thus the acquisition of certain fundamental minimum knowledge and skills that all pupils should master, except, perhaps a few children with specific disabilities. Each individual should then further develop his/her competences linked with his/her work or job, he/she is doing. The types of competences, knowledge and skills are diverse for different professions; learning population acquires them mostly within specific education. In fact, schools should provide basic understanding and skills, on the basis of which further specific knowledge and skills could be built.

Some people need and want to master specific knowledge at a higher level. This is not a goal that many people follow, however it seems necessary to motivate as many people as possible, in particular younger generations, to set for themselves the goals which are high enough. In any case, that goal should not prevail and twist the mathematical curriculum of Primary Schools.

Within the instruction of mathematics the mathematical problem itself has a significant meaning, i.e., problem solving and problem setting. It is true that solving problems has so far been more emphasised and in most cases routine problems, whereas problem setting has been neglected. In order to understand problems and the recording of problem situations, as well as establishing adequate solving strategies, solving and problem setting them are inseparably connected. With their help we can find “mathematical” links between the seemingly different problems and we can show our creativity for solving them. An individual’s self-confidence is also closely linked with that, as one can prove his/her knowledge of mathematics and show his/her skills in its application, by successfully solving mathematical problems, and thus show confidence in his/her abilities of mastering new knowledge and skills. Through the effective knowledge of mathematics an individual is able to solve more difficult mathematical problems and tackle new challenges.

The knowledge of mathematics acquired by an individual at school should be sufficient for successful acting in the social reality and for improving his/her social status as well as the entire society as a whole. An individual is expected to recognise, interpret, assess and critically judge mathematical aspects of reality, be it in terms of economy, politics or from social point of view.

We should never forget mathematics as a part of culture and history of a certain society in general. This means to understand mathematics in its entire role and not

only as a set of knowledge and skills. Unfortunately, it is both, in schools as well in the individual learning of mathematics, perceived as accumulating this and that kind of formulas and recipes which does not reflect the greatness of mathematical culture, but only bits and pieces extracted from the mathematical mosaic. It is important to know the historical development of mathematics, the social context of mathematical concept beginnings, its symbolic, theory and problems in order to be aware of human culture development, and it is also important for the motivation for learning and deepening mathematical knowledge.

Only after several years of learning mathematics an individual can understand mathematics as a discipline. One becomes aware of qualitative and intuitive understanding of certain concepts as for example symmetry, sample, infinity, structure, paradox, recursion, proof, etc. We can find a number of the deepest, the mightiest and the most exciting ideas mankind has ever developed in mathematics.

Mathematical Education

We have to be fully aware of the fact that “school” mathematics is not unilaterally defined, it depends on social values and cultural environment. Evidently, it is not identical to “pure” mathematics, but it is a selection of adapted mathematical contents. Even the sequence, according to which those contents are mastered, the teaching method and knowledge testing has been selected.

The applicability of “pure” and “school” mathematics has been in the today’s world largely overestimated, and therefore the argument of utility does not really count when justifying its teaching throughout the entire period of obligatory education, although it is generally accepted that mathematics is strongly present with its social applicability in practically in all the segments of modern life, from education, economy to industry etc. Practical mathematics has always been significantly developing next to the “pure” mathematics. Even nowadays mathematical studies in bookkeeping, insurance, management and information technology have been developing within professional institutions with very little contribution of “pure” mathematics (Ernest, 2000).

Modern society and modern lifestyle are deeply mathematised, we are faced with mathematics at every step, a lot of things are organised by integrated complex numerical and algebraic systems be it in the department stores, stock markets, taxation systems, social security systems, education systems or industry etc. Those automated systems are used to perform very complex tasks of data collection and to carry out certain actions, and mathematics helps organising a large proportion of our life with very little intervention into the systems as well as their verification once they have been put in place. Despite that, there is no need for a wider or deeper knowledge of mathematics in order to act successfully in the society, and here we do not mean that we should control those systems. A success of a certain generation at the international competitions form mathematics most certainly does not ensure economic and social boost, however there is a need for a narrow elite supervising the functioning of those systems and servicing them. The objectives

of general mathematical education should therefore not be set according to specific needs of that very small group of people.

It seems that most people do not need a deepened knowledge of mathematics, and the minority which applies their knowledge of mathematics, acquires their knowledge outside school education system. It is a kind of “paradox of significance”, since the society shows at the same time its objective importance and subjective insignificance of mathematics (Niss, 1994). Although society is becoming more and more mathematised, only a handful of people deal with it. It is therefore faulty to emphasise that everyone should get as much mathematical knowledge as possible because of growing mathematisation of the society in general.

In the second half of the 20th Century there was a tendency to establish a better education, the international community started to pay attention to the problem of illiteracy and was trying to find ways to eradicate it. Here also teaching and learning mathematics was underlined to help raise the level of mathematical literacy. There was a well known movement called “mathematics for all”. However, the majority of pupils do not achieve the expected knowledge level and understanding of mathematics, and they do not develop adequate skills in reference to the planned curriculum. Because of this, there are more and more considerations about a differentiated instruction of mathematics, the so called “serious teaching of mathematics” to be limited to a small group of pupils which would deepen their mathematical knowledge and develop mathematical competences with a reasonable input of their efforts and time (Niss, 2002). Of course, this does not mean that mathematical instruction is not needed for a vast majority of pupils, but it means that different approaches are needed for them to master and consolidate their mathematical knowledge as well as raise their adequate competences contributing to the increase of the level of mathematical literacy.

We should also include the elements of understanding and awareness into the mathematical instruction and not only developing capabilities. The language of mathematical instruction is full of orders and commands, pupils are required to do this and that – and we practically cannot find any discussion on the procedures, on reasons for using this path of solving a problem, on understanding of a certain problem, etc. There is an open question on what understanding and awareness of mathematics really means. Ernest (2000) cites elements which are supposed to be part of mathematical understanding and awareness, among them as follows:

- qualitative understanding of certain fundamental mathematical concepts as for example infinity, symmetry, structure, recursion, proof, chaos, randomness etc.,
- understanding the main branches and concepts of mathematics, as well as feeling their interconnections and interdependence, and also unity of mathematics,
- awareness of how mathematical thinking is trickling into every day life and where it has already been present, although we do not speak of mathematics directly,
- critical understanding of application of mathematics in the society: recognition (cognition), interpretation, evaluation and critical assessment of in-

tegration of mathematics into the social and political systems,

- awareness of historic mathematical development, social context of its origins, symbolic, theories and problems,
- feeling mathematics as a central element of culture, arts and life which is today, as it was once, closely linked with science, technology and all the aspect of human being.

We should by no means neglect the gap between the preset objectives of mathematical instruction and the impact that is demonstrated as its result. May the objectives of instruction be as noble as possible, high profile and established with those or other purposes, at the bottom line it is necessary to verify whether they have been achieved or not. Each reflection on mathematical curriculum has to contain three levels, as follows:

- the planned and scheduled curriculum,
- adopted and implemented curriculum and
- “acquired” curriculum, i.e., pupils’ achievements and learning results.

The ascertainment of what objectives of mathematical instruction have been achieved in school practice ought to influence primarily mathematical instruction itself. Since teaching is a planned activity, the expressed objectives should be closely connected with the realisation of teaching goals. If there is no such connection, a certain imbalance and disharmony appears which then reflects in discomfort of teachers and pupils.

How much abstraction?

There is still an overwhelming conviction that mathematical truths are universal, independent from mankind, from its culture and values – supposedly they were discovered and not invented. In fact, we cannot consider this as a cornerstone for mathematics – mathematics was actually built in a social environment and was a product of culture, and was fallible as any other scientific branch. Social constructivism emphasises social and cultural origins of mathematics and positions the eligibility of mathematical knowledge on its quasi empirical grounds. It is necessary to put forward two presumptions of social constructivism:

- the assumption of reality: the existence of solid physical world as we are told by common sense, and
- the assumption of social reality: each discussion presumes the existence of mankind and language.

With the presumptions of the existence of social and physical reality we can set the principles of social constructivism, so that we add to the first two principles, which are considered as the basic principles of constructivism, another four principles:

- knowledge is not acquired in a passive way, but pupils build it actively,
- the cognition function is adaptable and functions in the organisation of the world of experiences, and not in discovering the ontological reality,

- personal theories, which are derived from the organisation of the world of experiences, have to match with limitations based on physical and social reality,
- theories are subordinated to the cycle: theory – prediction – test – failure – adaptation – new theory,
- the commencement of the socially agreed theory of the world, social patterns and the rules on the language use,
- mathematics is the theory of forms and structures which is created within a language (Ernest, 1998).

Mathematical concepts are therefore created through the abstraction of direct experience in a physical world, by generalisation and reflexive abstraction of prior established concepts, through searching for sense in a conversation with others and through different combination of those methods. Mathematics is a scientific domain and human knowledge, closely linked with others. Language ensures formation of theories on social and physical reality, and those are, through dialogue among people and interaction with the physical world, subject to improvement – better adaptation to limits put forward by the physical and social reality. Mathematics is involved in this advancement, and also the conformity of mathematical structures in the areas outside mathematics is constantly verified. Mathematics therefore takes the course of development while solving practical social problems, and the so called unreasonable efficiency of mathematics is not an accidental miracle, but a result of empirical and language origins as well as functions of mathematics (Ernest, 1998).

It is necessary to underline that teaching mathematics depends a lot on teachers, their convictions and their perception of mathematics. The perception of what mathematics really is influences the conception of how it should be presented. The way someone presents mathematics shows his opinion on what is crucial in mathematics. The answer to the question which is the best way of teaching is indeed given in the response to the question what mathematics is all about (Hersh, 1986).

Various literature on teaching and learning mathematics emphasises situational factors providing conditions for both, teachers' and pupils' experiences in the process of education and during mathematical instruction (Cole, 1990). Each individual country disposes of values, beliefs and traditions of their particular education system, which are reflected in the adopted school system as well as in the expectations of pupils, teachers, parents, school leadership and other stakeholders.

When do we know mathematics

As we have already emphasised, the answer to why should we learn mathematics depends on the role that mathematics occupies in the society and on the philosophy of mathematical education itself. Taking into account the educational, social and political aspect we can divide reasons for mathematical education into three groups, i.e., mathematical education as follows:

- it contributes to the technological and socio-economical development of wider social community,

- it contributes to political, ideological and cultural sustainability and development of society,
- it ensures conditions for an individual to better act in various social spheres: education, profession, private and social life and as citizens in general (Niss, 1996).

While analysing the objectives of mathematical education as a reflection of social need, we are faced with troubles in detecting and identifying the actual impact of mathematical education on those needs. Most researches done in mathematical education are based on the presumption that mathematical education contains certain “inner” values, having in mind the significance that is expected to be displayed in a positive value of mathematical education guaranteed by the sole fact that we deal with education in the area of mathematics (Skovsmose, 2006). Skovsmose (2006) emphasises this presumption as a provision that the “mathematical educators” can act as “ambassadors” of mathematics and their action is most certainly directed towards general benefit.

Anyway, the result of mathematical education is supposed to be reflected in certain knowledge of mathematics. What does it really mean that someone knows/masters mathematics? Instead of the concept of mathematical knowledge we more often meet the expression of mathematical competences when we want to holistically present the “output state” of an individual. Ernest (2004) differentiates between “applicability” and “relevance”:

Applicability signifies a narrow conceptual applicability which demonstrates itself immediately or in a short period of time, without taking into account the wider context or long term objectives. Utilitarian education shows mathematics as a narrow technical science, isolated from values and social context.

Relevance may be seen as a relation among three concepts (S, I, G), where S stands for situation, activity or object to which that relevance has been attached, I stands for individual or group of people who attach relevance to the situation S and G stands for goal personalising values of individuals or groups in this case. The object S is thus relevant if it is considered as such by an I at achieving the aim G. Example: “school mathematics” (object S) is following the opinion of several politicians and responsible people for education (group I) relevant because it develops mathematical competences and skills necessary for technological professions (goal G).

In search for the answer to the question what does it mean to know mathematics Ernest (2004) studied several different goals to be achieved by teaching and learning mathematics as follows: utilitarian knowledge, practical knowledge linked with one’s job, expert knowledge at a higher level, understanding mathematics, mathematical self-confidence and understanding mathematics as a part of culture. Those goals are not excluding each other and are not always necessary or needed for all pupils, however, as a whole they represent, together with related capabilities and abilities, “mathematical competencies”.

Someone who mastered all those goals would be mathematically competent; maybe we would then say that someone knows mathematics (Wedegé, 2009). Wedegé (2009) presents utilitarian knowledge and practical knowledge linked with occupation as the ability of “doing something” (know how), thus useful for pupils and students as future employees. Expert knowledge at a higher level may be

useful for some people, for others relevant, whereas understanding mathematics is relevant for all (know that). Mathematical self-confidence and understanding of mathematics itself as a part of culture represents affective and social dimension of mathematical competence.

Mathematics in the real world

Modern teaching and learning of mathematics should be linked more to every day life. One of the key components is supposed to be independent establishment of notions and concepts through solving the problems of the “real” world and thus establishing a “natural” bond with mathematics. By creating models which are entering awareness in terms of mathematised models and at the same time adapted to the real situation we encourage mathematical thinking. This strong bond with the real world ensures the applicability of acquired knowledge, since we discover reality as a source and the area of application. Children apply all their specific capabilities in practice.

The model that was created from the context has a clear didactic significance and means a lot to pupils, since it represents putting in place the mathematical notion or concept. The example of the model of number line which is upgraded into the model of real axis, makes sense if it is mastered by pupils as a representation of natural numbers and integers, and later on as a representation of rational in finally real numbers. Due to the high level of abstractness, although it is a question of quite clear graphical display, pupils probably do not understand it in the initial years of education when they only master the concept of number as a quantity. Teachers have to be careful when using models, since enforcement of illustrations that pupils do not link with the concept to represent it, is senseless or even harmful.

An adequate model is usually the bridge between informal and formal knowledge and encourages the process of vertical mathematisation. Freudenthal (1991) mentions horizontal mathematisation as the connection between everyday world and the world of symbols, whereas the vertical mathematisation is supposed to be “movement” inside the world of symbols. He emphasises that there is no sharp separation line between the two and that both, the horizontal and the vertical mathematisation are important for the development of basic mathematical notions and concepts as well as formal mathematical languages.

The process of learning begins with realistic problem. Through activities within horizontal mathematisation pupils build informal and formal mathematical model. By solving problems, comparing procedures of solving and interpreting them pupils find themselves in the phase of vertical mathematisation which ends with the problem solution. The interpretation of solution and the solving strategy shows the application of the acquired mathematical knowledge.

The model is an important part of pupils’ mathematical development as they can apply the same model in various contexts. Teachers have to be all the time fully aware of the significance the model has and also of the fact that all pupils do not build equal models. Instruction should be regulated so that it suits to all the pupils regardless what model they had built for a particular mathematical notion or concept, only if this model is really in line with mathematical notion or concept.

As only some pupils are capable of abstract thinking only their models can be abstract “enough”. Those pupils are at an adequate level of development, capable of understanding the concept of unknowns, the letter signs for numbers, etc. Pupils who are not capable of abstraction, in fact, never know how to “calculate” with letter signs, and they only reproduce individually trained procedures. Because they cannot find the connection between the mathematics, which they know (master) and the context that they do not understand, they can only memorise rules and procedures that for them have no meaning.

In case of pupils who are capable of abstraction we can also originate on realistic problems which are not directly linked to a material situation, as the abstract world is for them not unfamiliar and have an image about it. A number of other pupils can develop knowledge and capabilities that ensure them a successful professional and personal growth, and of course, for them the mathematical world is always linked with material illustration.

Conclusion

At the end it is necessary to emphasise that qualitative mathematical instruction is one of the most important factors influencing pupils’ achievements and knowledge of mathematics. If we want to have good instruction, we have to train and educate teachers. It is also crucial for professors of mathematics that, in order to successfully develop pupils’ mathematical knowledge and literacy, they urgently have to understand mathematics themselves, and here not only ability of handling mathematical contents is taken into consideration. They have to understand concepts and processes much deeper and explain them logically by using adequate mathematical language and examples.

The capability of a (future) teacher to find several representations of a certain concept or content, and to realise and clearly present the connections between various mathematical presentations or conceptions, is closely linked with teachers’ self-confidence and trust in his own knowledge of mathematics. Positive experiences with mathematics together with adequate trust in one’s own knowledge of mathematics have a favourable influence on successful presentations of mathematical contents at the performances that future mathematical teachers have, as well as on the qualitative teaching of mathematics. Also future professors of mathematics should have, in the course of their studies, an opportunity to understand mathematics, and only afterwards concentrate on mathematical didactics. Many students have deeply rooted preconceptions on mathematics and its teaching and that would make it difficult for them to perform teaching of mathematics successfully.

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Nice tilings, nice mathematics!?! --- ---

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Abstract. The squared papers in our booklets, or the squared (maybe black and white) pavements in the streets arise an amusing problem: *How to deform the side segments of the square pattern, so that the side lines further remain equal (congruent) to each other?* More precisely, we require that each congruent mapping of the new pattern, mapping any deformed side segment onto another one, leaves the whole (infinitely extended) pattern invariant (unchanged).

It turns out, that there are exactly 14 types of such edge transitive (isotoxal) quadrangle tilings, sometimes with two different forms (e.g. black and white) of quadrangles. Such a collection of tilings seems to be very nice, perhaps also useful for decorative pavements in streets, in flats, etc.

I shall sketch the solution of the problem, that leads to fine (and important) mathematical concepts (as barycentric triangulation of a polygonal tiling, adjacency (neighbour) operations, adjacency matrix, symmetry group of a tiling, D-symbol, etc). *All these can be discussed in an enjoyable way*, e.g. in a special mathematical circle of a secondary school, or in more elementary form as visually attractive figures in a primary school as well.

My colleague, István Prok [P] has an attractive computer program on the Euclidean plane crystallographic groups with a nice interactive play (for free download), see our Figures 3–5.

A complete classification of such Euclidean plane tilings (not only with quadrangles) seems to be interesting for university students as well, hopefully also for the Audience. This is why. I give some references, where you find also further ones.

Introduction

After a longer experimental period, when dealing with a lot of squared papers, the school pupils (students) will get good and erroneous constructions in individual and collective work as well. They can conclude (probably in the second or third occasion, step-by-step, under direction of the teacher, as little as possible) to the following really fine idea (being described now in a compact form [M94]):

Our Figure 1 shows the usual square tiling with its 2-dimensional face centres indicated by small triangles Δ , with 1-dimensional edge centres denoted by $\diamond$, and 0-dimensional vertices $\circ$.

The opposite continuous —, dashed - - - - and dotted lines, respectively, are introduced as well, so that we obtain the so-called barycentric subdivision of the square tiling into barycentric triangle tiling denoted by C. That is important in the description of our simple square tiling (moreover of any polygonal tiling, furthermore of any “topological” one; with corresponding changes, of course [LMS94]).

4 barycentric triangles (1, 2, 3, 4 in Figure 1 top left) support to any edge — from both sides. These form the so called *edge domain* $VPWQ$, playing an important role in description of possible edge modifications. Namely, an edge can be chosen just freely between the vertices V and W in $VPWQ$. Then a plane group (as a rule of periodicity) with fundamental domain (repetitive unit) will tile the whole square tiling with the starting edge domain, the modified edge in it (see also Fig. 2: 2.3, 2.4, 2.5, 3.2, 3.3, 3.4).

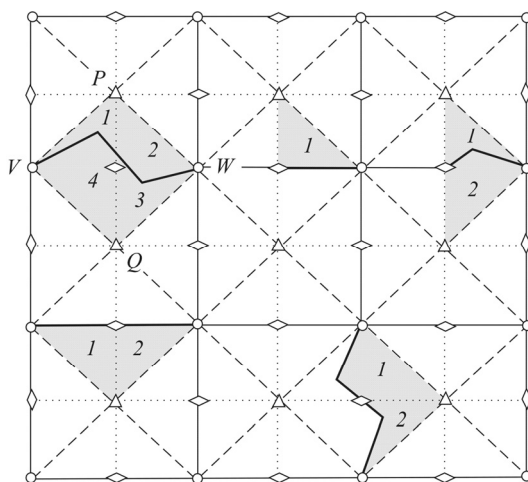
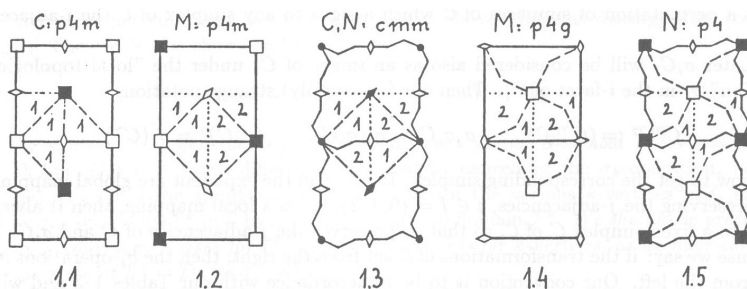


Figure 1. The square tiling and its barycentric subdivision. Edge domain $VQWP$ of 4 barycentric triangles. Symmetries of the edge domain by smaller fundamental domains.



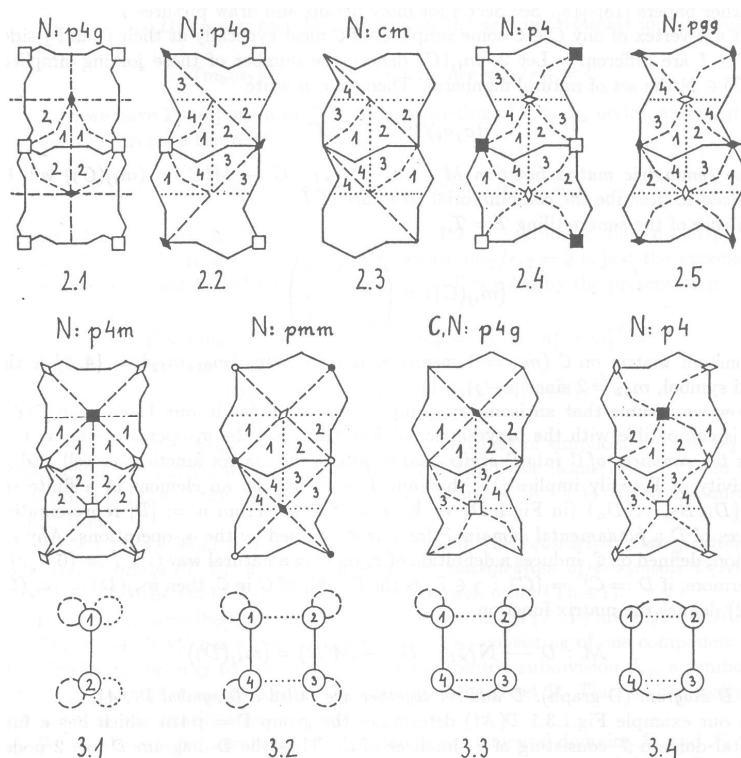


Figure 2. The 14 families of quadrangle tilings with equal edges:
 1. Equal tiles with edge symmetry; 2. Equal tiles with vertex symmetry;
 3. Non-equal (black-and-white) tiles with D-diagrams.

But the modified edge can also be symmetric in its midpoint (right bottom of Figure 1, and in Figure 2: 1.3, 1.5.). Or it is symmetric in the side edge itself (left bottom of Figure 1, and Figure 2: 1.2, 1.4). Here the edge cannot change, but extra label (mark), in the half edge domain, shows the situation. Or it is symmetric in the midline of the side (Figure 1 top right, and Figure 2: 2.1, 3.1). Or it is symmetric in both of the last mirror lines (Figure 1 top middle, and Figure 2: 1.1), this is just the original square tiling.

The remaining task to solve, how to display these edge domains in a regular way (as formulated in the summary), so that they form a 'topological quadrangle tiling' in the plane, and to any two new edges there exists an isometry of the plane that maps the first edge onto the second one and the whole plane tiling will be mapped onto itself.

To this last step **we need** the classification of the 17 plane crystallographic groups, as the program [P] shows it, or **at least to overview those possibilities, how the edge domain can be a fundamental domain (repetitive unit) for a tiling with the edge domains above?** This task need only finitely many side pairings of this quadrangle edge domain and its parts above (Figure 1).

That means, *this problem provides also a theoretic benefit to overview some plane crystallographic groups* (7 from among of 17 ones). This could be an attractive experiment for an ambitious teacher in the special mathematical circle of a secondary school, or also of a primary one.

I collected the complete list of our solution: the 14 types of our “*edge transitive*” (*isotoxal*) *quadrangle tilings* as illustration from [M94], where *the more general scientific problem on spatial cube tilings has also been solved*. The Reader finds also the plane generalization in [LMS94].

The solution

Our Figure 2 shows the 14 types of edge transitive square tilings by two typical tiles with the edge domains of numbered barycentric triangles. We divided the list into 3 classes: 1 – 3.

Class 1 contains five members 1.1 – 1.5 where the symmetry group of the edge (i.e. the stabilizer group of the edge centre) maps the two adjacent tiles to each other.

Class 2 also consists of five tilings 2.1 – 2.5 where we have also equal tiles but not by the edge stabilizer.

Class 3 has four surprising tilings 3.1 – 3.4 with two types (e.g. black and white) of tiles not mapped to each other, in spite of that all the edges are mapped onto each other.

We have indicated to each tiling, whether convex (*C*), non-convex (*N*) realization (sometimes both ones, think of!) is (are) possible. Moreover, you find the crystallographic plane group to each tiling (7 groups of the 17 ones).

On D-symbols. Illustration with computer pictures

You find also the very important adjacency diagram (adjacency graph) to *the so-called D-symbol* (D, M) (for initiative of B. N. Delone, M. S. Delaney, A. W. M. Dress) at class 3 (find the other ones to classes 1 – 2, please!).

This would be the next level of abstraction, mainly for university students or to audience of a scientific conference. Of course, this is only to indicate an important method to research in a scientific level for other tilings in the plane; or – much more important and with new scientific results (e.g. in [M94]) – in a homogeneous space (e.g. that of constant curvature), ***where many problems are open yet.***

To each D-diagram D in Figure 2: 3.1 - 3.4 we consider the barycentric triangle subdivision C (Figure 1) of the square tiling. Any such triangle of the edge domain (1, 2, 3, 4 at most) will be indicated by a vertex of the diagram D . The dotted lines, now also as σ_0 g operation of C and of D , the dashed - - - - lines as

σ_1 operation of C and of D , the continuous — lines as σ_2 operation of C and of D appears as adjacencies of the diagram D (labelled or coloured edges of graph D) just by the common sides of the barycentric triangles in C .

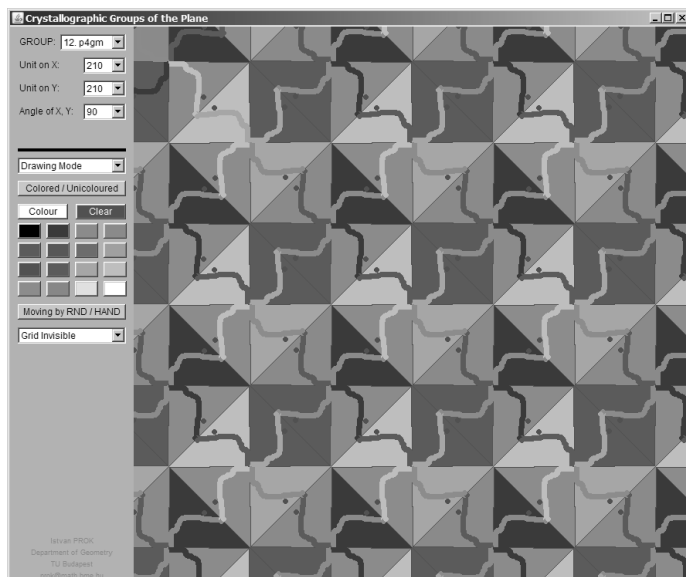


Figure 3. I. Prok's program provides a drawing mode with various colours or black and white pictures. <http://www.math.bme.hu/~prok>

Please observe that the images of the edge domains or smaller fundamental domains in Figure 1 will also be induced by this adjacency operations.

That means that **the whole symmetry group denoted by Γ of the tiling** will be described if we give the (symmetric) so called adjacency matrix function $M = m_{ij}$ on diagram D which describes how many barycentric triangles of side i and j (the half of them) surround a k -vertex of C $\{i, j, k\} = \{0, 1, 2\}$, according to the corresponding ij -path in D . That means that each element (vertex) of diagram D describes the corresponding triangle class (orbit) of C by the symmetry group Γ , so that Γ preserves the adjacencies of C .

Now the adjacency matrix function M is constant:
$$\begin{pmatrix} 1 & 4 & 2 \\ 4 & 1 & 4 \\ 2 & 4 & 1 \end{pmatrix}$$

for each element of D and of C , because of the square tiling or quadrangle tiling, where 4 quadrangles meet at each vertex.

Consider e.g. Figure 2:3.3 with its picture and its diagram, where the adjacency operations can be given by involutive (involutory) permutations (in general) as now follows

$\sigma_0 : (12)(34)$; $\sigma_1 : (1)(2)(34)$ as the loops show it at vertex 1 and 2; $\sigma_2 : (14)(23)$.

Now $(\sigma_1\sigma_0)$ at the tile centres (2-centres) of triangles 1, 2 in C shows the dihedral corner for Γ , $(\sigma_1\sigma_0)$ at tile centres of triangles 3, 4 in C shows the 4-rotation in Γ by matrix entries $m_{01} = m_{10} = 4$ (please check them carefully!). The $(\sigma_2\sigma_1)$ composition (always in opposite order by our convention now) provides the line reflection at the common triangle vertices (0-centres) of triangles 1, 2, 3, 4 in C , again by matrix entries $m_{12} = m_{21} = 4$ (check it, please!). The $(\sigma_2\sigma_0)$ composition provides the trivial stabilizer in Γ at the edge centre (1-centres, 1-vertices) of triangles 1, 2, 3, 4 in C by matrix entries $m_{02} = m_{20} = 2$. Thus the edge domain serves as a *fundamental domain for the plane group $\mathbf{p4g}$* , indeed. This procedure can be algorithmized and computerized as well (e.g. as in our paper [DHM93]).

Another edge transitive tiling to $\Gamma = \mathbf{p4g}$ is pictured by computer [P] in Figure 3 (also by

Figure 2: 2.1) where we have drawn an edge motif into a fundamental domain of the coloured picture can be very attractive and amusing.

This method is general, we can draw all the 14 types by this strategy, and also the other Euclidean isotaxal tilings as well [LMS94].

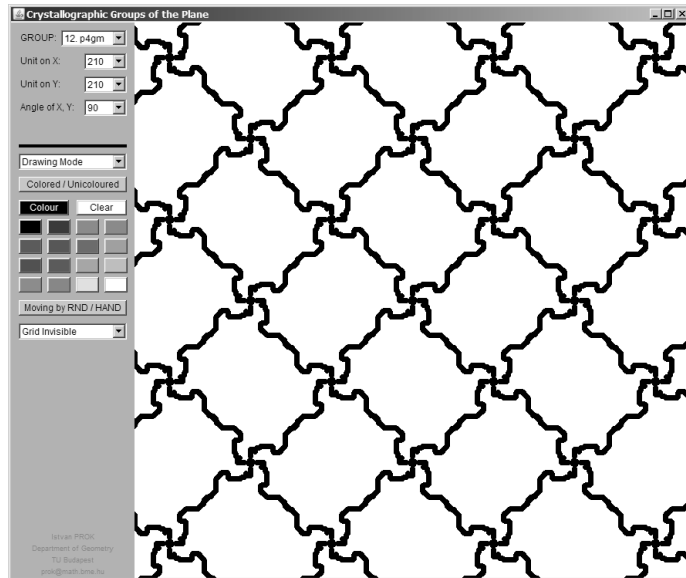


Figure 4. Black and white pattern by the former plane group $\mathbf{p4gm}$ no. 12.

Finally, you see two (not too nice) black and white computer illustrations by the author

(Figure 4-5). The first picture is drawn just by the former pattern by the group $\mathbf{p4g}$ (Figure 4).

The second one is by the group $\mathbf{pgg}$ (find it also in Figure 2 please!).

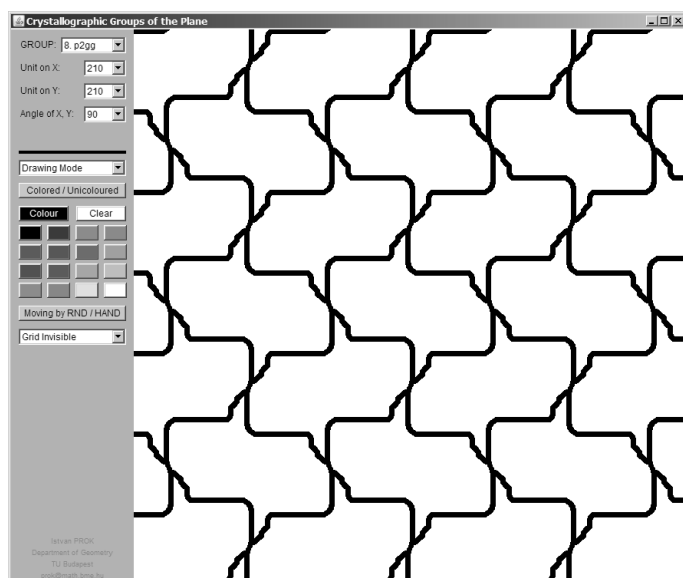


Figure 5. Pattern by plane group **p2gg** no. 8.

You can continue drawing!!!

I thank my colleague Dr. Jenő Szirmai for his help in preparing the manuscript.

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Use of technology for enhancement of teaching mathematics: perspective, problems, and criteria and selection method

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Abstract. Already in the eighties of the last century it has been recognized that “we are on the threshold of a revolution in mathematics — how we think about it, how we practice it, and how we learn it. The revolution centers on the computer as a mathematical tool. As in every revolution, people are taking sides — some would like to see a complete exploration of the computer’s role in human mathematics, while others feel that using a computer in math is cheating” [12]. “Despite the availability of hardware and software, mathematics staff in a technology-rich secondary school rarely used computers in their teaching” [15]. We are presenting some of reasons for this phenomenon. We are also aware that “mathematical developments lay at the heart of recent advances in engineering, biomedical science, commerce, and information technology. Students in these areas need mathematical education but their backgrounds, abilities and attitudes vary widely” [8]. We are introducing some resources that are available on the Internet and could enhance teaching mathematics. “Since the World Wide Web is becoming wider at an increasing rate, it is virtually impossible to take any kind of accurate snapshot of the state of its development” [7]. We are presenting methodology for selection by defining criteria and selection method to choose the best appropriate. Those criteria ensure better accomplishment of learning objectives and avoiding risks of giving up from using resources because they are not appropriate for teachers by being either too complicated or time consuming to use.

Keywords: teaching mathematics, teaching with technology, decision making, mathematics

Introduction

Internet and technology have offered new teaching materials, software and other resources that can be implemented in schools to enhance students’ learning

and performance. It has been debated if technology, in general, is enhancing educational process in sense of ensuring better accomplishment of learning objectives. We are presenting just some of argument for and against using the technology. Some of arguments for usage are better engagement of student in learning, stimulated cognitive grow and performance, better identification of student's weaknesses in content area, computers and internet allow teaching at the moment with many information than adhere prepared lesson. Arguments against usage are need of change of teaching style to fit some technological device and open question about methodological approach that can insure improved students achievement.

The role of technology can be summarized National Research Council's statement "The process of using technology to improve learning is never solely a technical matter, concerned only with the properties of educational hardware and software. Like a text book or any other cultural object, technology resources for education function in a social environment, mediated by learning conversations with peers and teachers" [2].

In professional literature has been recognized that use of technology doesn't increase success of teaching process by itself [1]. Preparation with using technology consumes much more effort and time then classical preparation. Benefits of involving such effort are not obvious, and rarely adequately recognized in working or wider community. Mostly, motive to give such effort is teachers' inner motivation driven by enthusiasm of preparing for better future or just looking for self-fulfillment.

Educational science and practice, e.g. [9],[11],[12],[15] recognized great technological change of the world. With that change of world they recognize urgent need of change of methodological approach in teaching mathematics. Educational process should be more focused on acquiring permanent and useful knowledge, skills and abilities. Involving technology can make process of teaching mathematics modern. Using technology can save time on operational part of teaching process. Saved time can be spent on explaining basic ideas in mathematics (basic knowledge), discussions and creative mathematical thinking (reflective knowledge).

It has also been recognized that technology doesn't necessary mean better education. Some possible negative effect on students, could be fall of persistence, patience, accuracy and concentration - what teachers usually believe that mathematics give to their students. Technology and on-line teaching applications can also have great problem: be very interesting and time consuming but with small educational benefit.

By being aware and giving relatively small effort probability that those problems happen can be significantly reduced. There is something new that arise with use of technology. It can't be called problem, maybe possibility or question: "What is main question for our mathematical subjects: "why" or "what if". Answering this question can give us should we or maybe must we use technology in education.

In our paper we present method for selection software in course Selected chapters of mathematics. We present decision criteria and selection method.

Decision theory

Decision about using technology can have large influence on teaching and learning process. Using new technology, software, hardware or just available on-line materials introduce threats and opportunities, has their own strengths and weaknesses. In this part we are introducing method for recognizing those aspects in software selection process.

Teaching process is dynamic process. Before taking teaching actions, teacher should think if action will produce desired goal. In the decision process he or she should take new information if available and be open for feedback. Teachers decide about those actions generally based on their creativity, experience or intuition. Some actions influence achieving course objectives and can be called strategic. Decisions about those actions shouldn't be made only on those principles. Systematic analytical approach should also be used. There are different approaches and systematic methods of mathematical models that characterize the problem and argue the decision. In the systematic approach lies the difference between good and bad decisions. In fact, a good decision will be one that uses a quantitative approach, based on logic, taking into account all the available input data and possible alternatives. The omission of some alternatives, which may even be very insignificant, can lead to the wrong choice. It is known that sometimes a good decision can result in unexpected outcomes but in general analytical approach in decision making is better than one done just on intuition. One of strategic decisions is decision on need of use teaching software and selection the best appropriate if needed.

Decision theory studies decision-making processes in a systematic and analytical way. The environment in which decision can be taken may be more or less deterministic and uncertain, and therefore the decisions we make in terms of certainty, or conditions of risk or uncertainty. Although, we can't be totally sure what will be results of our actions our decision making problem can be characterized as one in terms of certainty. Decision maker should also be aware that there are times when a decision is an isolated one-time decision, but rather as the first in a series of sequential decisions that are interconnected in some future time intervals.

In this paper we decided to use the Analytic Hierarchy Process (AHP) technique for multi criteria decision making. The procedure for using the AHP can be summarized as:

1. Model the problem as a hierarchy containing the decision goal, the alternatives for reaching it, and the criteria for evaluating the alternatives.
2. Establish priorities among the elements of the hierarchy by making a series of judgments based on pairwise comparisons of the elements.
3. Synthesize these judgments to yield a set of overall priorities for the hierarchy.
4. Check the consistency of the judgments.
5. Come to a final decision based on the results of this process [16].

There are different commercially available or free software tools that were developed for the analysis of multi criteria decision making using pairwise comparisons such as Expert Choice, Decision Lab, D-Sight, ERGO, MakeItRational and various other tools. Our calculations were made in Expert Choice.

Educational software

Educational software is software whose primary purpose is in teaching and learning. Four different software packages were in final consideration for use in course Selected Chapters of Mathematics. We describe some of properties of the software.

GeoGebra is teaching and learning tool that integrates geometry, algebra and calculus. It is a cross-platform application written in Java and can serve for development of instructional materials in mathematics in many different forms, types and styles, and for all levels of mathematical education. *GeoGebra* has a built-in Cartesian coordinate system and can accept geometric commands (drawing points, lines, vectors, perpendicular line, angle bisector, . . .) and algebraic commands (pairs of coordinates, equation of a curve, function, . . .). This double representation, the geometric and the algebraic, is one of the greatest advantages of *GeoGebra*. Moving the objects in the Geometry window changes the expressions in the Algebra window accordingly and vice-versa, editing the expressions in the Algebra window results in the respective change in the Geometry window. Moreover, with a Spreadsheet window, which is also dynamically connected with Geometry and Algebra window, *GeoGebra* is ready for statistics commands and charts [10].

Mathematica is most widely used, complete and currently the most powerful advanced computer algebra system. *Mathematica* computations can be divided into three main classes: numerical, symbolic and graphical. Unlike the usual programming languages such as C and C++, it is not restricted to a small number of data types. It uses symbolic expressions to provide a very general representation of mathematical and other structures. *Mathematica* has a large number of built-in functions, but also includes own powerful programming language which supports several programming styles including:

- Procedural programming with block structure, conditionals, iterations and recursion
- Functional programming with pure functions and functional operators
- Rule-based programming with pattern matching and object orientation

The biggest disadvantage of *Mathematica* is price because it is too expensive for average users [18].

Maxima is a powerful computer algebra system written in Lisp programming language which combines symbolic, numerical and graphical abilities. It is a free and open source program which is being continuously improved by a team of volunteers. Compared with *Mathematica*, *Maxima* has more basic and simpler interface, but has big advantage in price. Currently, two most popular interfaces for *Maxima* are *wxMaxima* and *XMaxima*. For new users is the best to start with *wxMaxima* interface because it has convenient icons which help locate *Maxima* functions for common tasks. Through menus and buttons in *wxMaxima* new users gradually learn basic *Maxima* syntax. Also, *wxMaxima* implements its own math display engine to nicely display *Maxima* output. *XMaxima* is more lightweight interface with very small number of menu commands than *wxMaxima*, but it is more stable. Therefore, experienced users rather use *XMaxima* interface because they already know *Maxima* syntax and it is faster to them just type the name of command than

to search particular *Maxima* command in menus (if it is in menu at all). *XMaxima* is also a faster environment for testing and playing with code ideas than *wxMaxima* [13], [14].

SAGE is an open source mathematics computing environment written in very powerful and popular Python programming language. Most mathematics computing environments contain some kind of mathematics- oriented high-level programming language. Some of these mathematics-oriented programming languages were created specifically for the environment they work in while *SAGE* is built around an existing Python programming language. This means that expert Python programmers are also expert *SAGE* users. Beginners must first learn Python to being able solving problems with *SAGE* because *SAGE* is just powerful mathematics extension of Python. While most mathematics computing environments are self-contained entities, *SAGE* takes the umbrella-like approach of providing some algorithms itself and some by wrapping around other mathematics computing environments (*Maxima*, *GAP*, *BLAS*, *LAPACK*, *mathematics Python packages*, etc.). *SAGE* is built out of nearly 100 open source programs and can be used to study elementary and advanced, pure and applied mathematics like basic and advanced algebra, calculus, elementary and advanced number theory, cryptography, numerical computation, group theory, combinatorics, graph theory, linear algebra, etc. *SAGE* interface is a notebook in a web browser or the command line. With notebook interface, *SAGE* can connect to locally installation of *SAGE* on hard disc or to a *SAGE* server on the network. *SAGE* server on network is great tool for windows users because *SAGE* native port is *Linux*. Windows users must install virtual machine if they want run *Linux* and *SAGE* locally on Windows. *SAGE* currently works best with the Firefox web browser [17].

Case study: selection of educational software

In this part we present our decision making process of selection educational software.

A. Decision goal, the alternatives and the criteria

Course Selected chapters of mathematics is course in second year of pregraduate study of informatics. Using technology is expected for all courses so there is now need of decision if it's needed or not. Decision goal is to select best appropriate software to accomplish course objectives. In section Educational software we presented four alternatives that are considerable for use. Criteria for selection are Mastering Software by Teacher, Mastering Software by Students, Price, Covering Syllabus, Available Materials and User Help.

B. Establishing priorities among the elements

Figure 1. Hierarchical structure of decision problem shows hierarchical structure of our problem.

Figure 2. shows pairwise comparisons of importance of each of criteria. Pairwise comparisons are done by 1-9 scale recommended by AHP methodology. Final

rating of our criteria are: Price (34,8%), Covering Syllabus (32,5%), Mastering Software by Teacher (12,9%), Mastering Software by Students (9,0%), Available Materials (6,5%), User Help (4,2%).

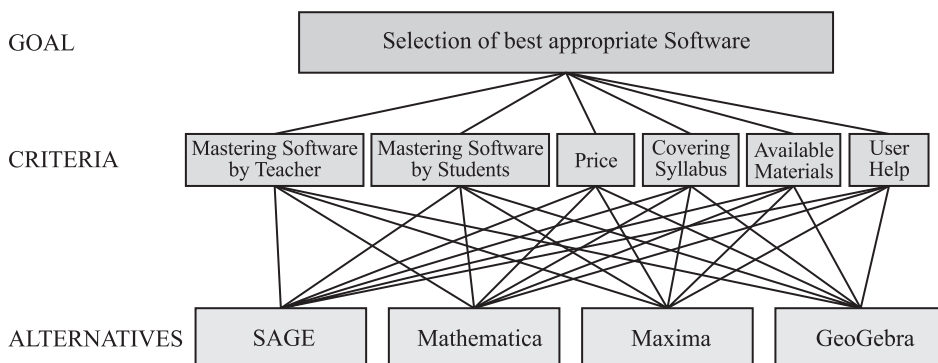


Figure 1. Hierarchical structure of decision problem.

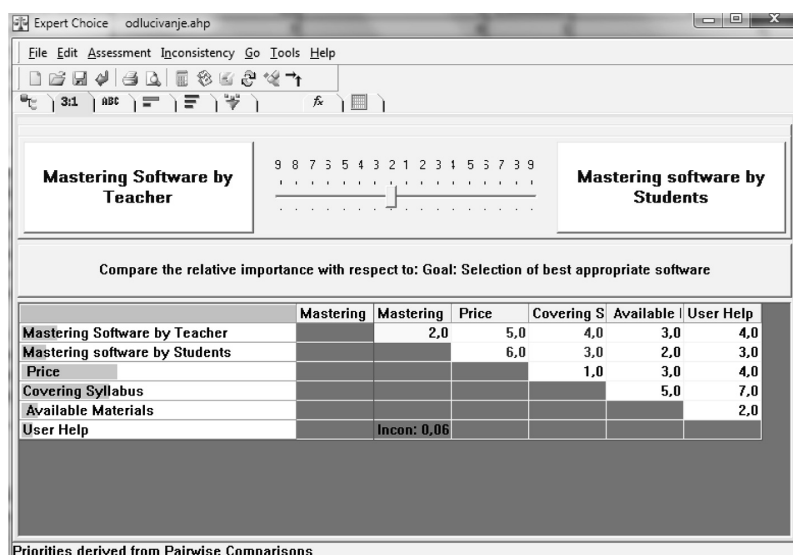


Figure 2. Pairwise comparisons of criteria .

Similar method was used to pairwise comparison of alternatives according to every criteria.

C. Overall priorities and consistency

Gathering all comparisons we can conclude that *SAGE* is best appropriate software for use in course Selected chapters of mathematics. In Table 1. Overall priorities in decision process is shown rating of every alternative according to every criteria. All comparisons were checked on consistency. Consistency ration on every comparison table were less then 0,10 what is acceptable by AHP methodology.

	Mastering Software by Teacher	Mastering Software by Students	Price	Covering Syllabus	Available Materials	User Help	Overall points
Weights	12,9%	9,0%	34,8%	32,5%	6,5%	4,2%	
<i>SAGE</i>	0,124	0,218	0,320	0,293	0,362	0,236	0,275
<i>Mathematica</i>	0,319	0,078	0,040	0,477	0,375	0,472	0,265
<i>Maxima</i>	0,074	0,149	0,320	0,186	0,165	0,106	0,207
<i>GeoGebra</i>	0,484	0,555	0,320	0,044	0,098	0,186	0,253

Tablica 1. Overall priorities in decision process.

D. Final decision

Using AHP methodology for multi criteria decision making we can conclude that *SAGE* is the most appropriate software for use in course Selected Chapters of Mathematics. Some of top advantages of *SAGE* are price, available materials and coverage of syllabus. Although, comparing to other software *SAGE* is just best in price, and in every other criteria there is one or even two alternatives better. For example, *Mathematica* is better in Covering syllabus, Available Materials, User Help but it is commercial software relatively difficult to learn for students. Teachers are more familiar with *Mathematica* while they graduated mathematics, but that isn't reason good enough to make it obligatory for students of informatics. *GeoGebra*, even free and easy to learn is not covering course program good enough to be the best choice.

Conslusion

Technology has enlarged opportunities to enhance mathematics teaching. There are some positive and negative aspects of using technology. Preparation for use of technology in teaching consumes more time. This effort doesn't produce tangible results in short time period so it's not recognized in community as it should be. Usage of technology opens new perspective on mathematics teaching methodology. Because of change of world, in some aspects, it becomes necessary to accomplish learning objectives. In our paper we have shown application of AHP, multi criteria decision methodology, as possibility for best appropriate software selection methodology. In decision process we used six criteria and four alternatives.

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What is the future of the integration of ICT in teaching mathematics

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Abstract. In recent years the need for introducing information and communication technology into the teaching process has posed one of the unavoidable changes in the educational system. Present generation of students are so proficient in usage of the information and communication technologies in their daily lives, that this change in the educational system can not be viewed as an investment in a better future, but as a necessity in order to keep pace with technology and students. Considering the integration of ICT in teaching mathematics, it is clear that the replacement of board and chalk with digital presentation material does not cover all the aspects that technology and mathematics can improve when working hand in hand.

One of the important prerequisites for quality integration of ICT in teaching mathematics is the teacher's personality, i.e. his knowledge, willingness and desire to improve his lessons bringing mathematics closer to the present generations of pupils.

The aim of this paper was to investigate the readiness of the future mathematics teachers at the elementary and high school levels to integrate ICT into their teaching of mathematics. Factors influencing described readiness that are considered in this paper are teacher's university education and initiative with regard to personal digital competence and infrastructure.

We conducted the survey research on the samples of individuals from the population of students enrolled in final years of a five-year Master of Arts in Teaching Primary Education programme at the Faculty of Teacher Education in Osijek, Croatia ($n = 196$), and Master of Arts in Teaching Mathematics and Computer Science programme at the Department of Mathematics in Osijek, Croatia ($n = 36$). The obtained results indicate that identified aspects have impact on considered readiness and that teacher's university education causes the differences in the attitudes towards his digital competencies necessary for quality integration of ICT in mathematics lessons.

Keywords: teaching mathematics, ICT integration, teacher's university education, teacher's initiative, digital competences

Introduction

Information and Communication Technology (abbreviation: ICT) has changed our daily activities in many ways. Since these changes are evident amongst younger members of our society, they are evident on primary and secondary schools' students. Considering that ICT plays an increasingly important role in society, especially if we take into account social, economic and cultural role of computers and the Internet, it is clear that the time has come for the actual entry of ICT in the field of education. The combination of ICT and the Internet certainly opens many opportunities for creativity and innovation, but also for approaching the teaching material to current generation of students.

Since some authors even in year 1980 predicted that till year 2000 the main methods of teaching will include the application of computers at all levels and in all areas (Bork, 1980), we can conclude that even 30 years ago the application of ICT in teaching was the subject of study and research.

Visualisation of teaching materials facilitates understanding in mathematics and the use of ICT tools facilitates appointed visualisation process. Therefore this paper deals with the introduction of ICT in the teaching mathematics.

The future of the ICT integration into the teaching mathematics definitely depends on future generations of mathematics teachers who are: (i) teaching primary education students who will teach mathematics in the lower grades of primary education, (ii) teaching mathematics students who will teach mathematics in the upper grades of primary education and in all grades of secondary education. For that reason this survey research is conducted on the sample of individuals from these two populations of students.

We assumed that university education is causing the difference in attitudes towards self-initiatives focused on digital competences and infrastructure. Accordingly, in this study the direction of that influence is investigated. Furthermore, we investigated how a university education, and described self-initiatives affect the willingness of teachers to use ICT in teaching mathematics.

The term competence involves the knowledge, skills and attitudes required for performing a job. Special types of competences are digital competences that comprise safe and critical use of ICT in work, leisure and communication (Marčetić, 2010). Core digital competences include using the multimedia technology to locate, access, storage, produce, present and exchange of information and also to communicate and participate in the Internet network (Ala-Mutka, 2008). As a final conclusion about digital competences, we can say that digital competences include reliable and critical use of ICT for employment, learning, self-development and participation in society (Marčetić, 2010).

Study framework

Way that young teachers can contribute their knowledge about different forms of ICT use, the Swedish associate professor at Linköping University, Sven Andersson, explored. The survey was conducted in Swedish elementary schools, and as

a final conclusion regarding to the application of ICT in their work, teachers have found that new technologies improve their attitude toward finding the knowledge to develop their own competences, finding teaching materials and methodological ideas and the relationship between student and colleagues. Improving their digital competences gave them the idea and inspiration for the development of teaching methods, their own knowledge of computers or, a starting point for the upcoming ICT activities in schools (Andresson, 2006).

A study conducted by Sang, Valcke, van Braak and Tondeur shows the influence of gender, constructivism beliefs, self-efficacy in teaching, self-efficacy in the use of computers and attitude about computers. The study was conducted on 727 students of a Chinese university. The results showed that the potential integration of ICT is in correlation with all these variables, except gender (Sang, 2010).

The paper by Drent and Mellissen discusses the influences that stimulate or limit the innovative use of ICT by teachers in the Netherlands. Some of the factors that were studied are: pedagogical approach, ICT competences and personal entrepreneurs. The results showed that all these factors have an impact on innovation in the use of ICT. The authors conclude that the ICT competences are requirement for the use of ICT in teaching, but that the innovative use of ICT is affected by other factors (Drent, 2008).

In the area of Flanders (Belgium), unlike Great Britain and Canada, ICT competences are not included in the national curriculum, just the guidelines for schools to focus innovative educational processes in the process of integration of ICT into teaching are given. This study explores how, the school in general and teachers personally, conduct the new expectations arising from these guidelines. Specifically, it examines the ICT competences that teachers currently adopted (for actual use in teaching), and which competences they intend to adopt in the future (prefer using them). Research has shown that the majority of teachers is familiar with the concept of ICT, but still 2.6% of respondents has never use a computer, either in class or preparing for teaching. Of the total number of hours per week spent at the computer, most of them is related to professional help, then leisure and finally at teaching. The main result of research shows that teachers in the primary education tend to increase and expand their ICT competences (Tondeur, 2007).

Model, sample, data

The inception of this survey research is marked with the discussion about aspects of some issues influencing teachers' readiness to integrate ICT in teaching mathematics. It resulted in establishing model of situation (Figure 1) and designing survey for collecting data about the populations of interest.

Teacher's self-initiatives taken with purpose of quality integration of ICT in teaching mathematics were analyzed with respect to personal digital competence and infrastructure of teacher himself. Digital competences were deliberated in terms of acquiring, holding, developing and updating, and categorized as core and special. Holding core digital competences were analyzed through possessing knowledge in following: fundamental components of computer, basic Internet terms,

using electronic mail, creating digital textual and presentational materials, creating spreadsheets, drawing and image processing, creating and publishing web-sites, and using Internet as additional source of information necessary for creating teaching materials. Further, holding special digital competences were analyzed through possessing knowledge and skills in using some specialized ICT tools appropriate for integrating in mathematics lessons, such as IT board and educational mathematics software. Specific attention was assigned to the existence of awareness about necessity of constant developing and updating of digital competences. Teacher's personal ICT equipment is modelled by model variable digital infrastructure.

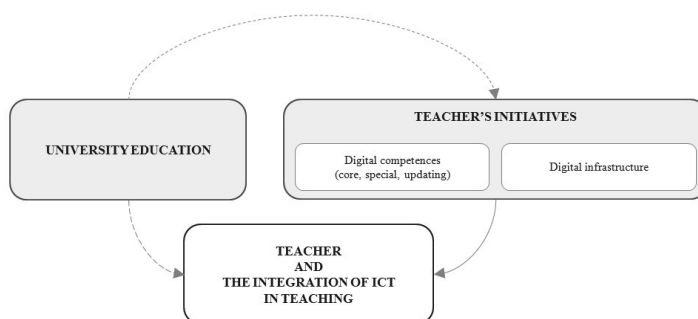


Figure 1. Konceptualni model

Method for data collecting employed in this research is an online survey. Data are obtained from the samples of individuals from the population of students enrolled in final years of a five-year Master of Arts in Teaching Primary Education programme at the Faculty of Teacher Education in Osijek, Croatia ($n = 196$), and Master of Arts in Teaching Mathematics and Computer Science programme at the Department of Mathematics in Osijek, Croatia ($n = 36$). 232 responses were recorded from November 2010 till January 2011 from the sample of individuals from the populations of interest. Sample structure is depicted in table below (Table 1).

Feature	Category	Frequency	Relative frequency (%)
STUDY PROGRAMME	Teaching primary education	196	84,48
	Teaching mathematics	36	15,52

Table 1. Sample structure

Responds collected from the sample of individuals are discrete quantitative data that were recorded on meaningful integer numerical scale from 1 to 5. They label a grade given to certain statement, where 1 stands for "I totally disagree" and 5 stands for "I completely agree".

In table below (Table 2) statistical distribution of variables of interest, that model aspects of teacher's self-initiatives with respect to personal digital competences and infrastructure, is described.

Considered feature	Relative frequencies of evaluation					Descriptive statistics of sample	
	1	2	3	4	5	Mean	Standard deviation
Core digital competences	0,006705	0,021073	0,068966	0,144636	0,758621	4,627395	0,760839
Special digital competences	0,281609	0,100575	0,21408	0,217672	0,186063	2,926006	1,478185
Constant updating of digital competences	0,018103	0,064655	0,244828	0,326724	0,34569	3,917241	1,003039
ICT infrastructure	0,006466	0,036638	0,090517	0,19181	0,674569	4,491379	0,854369

Table 2. Statistical distribution of variables of interest

Methodology

After obtaining data about variables of interest from the sample of individuals, we performed statistical analysis directed to estimation of population mean and proportion as well as to testing of hypothesis formulated on the basis of research hypotheses and with purpose of making inferences about considered populations.

We estimated population mean and proportion based on a single sample from the population of interest using large sample confidence intervals. Confidence level applied for the purpose of this paper is 95%.

Tests of hypothesis are carried out by implementing the following steps (McClave et al., 2001): (1) introduction of null hypothesis representing status quo, (2) introduction of alternative hypothesis, (3) selection of appropriate test statistic, (4) determining the rejecting region referring to the values of the test statistic considering level of significance α , (5) collecting data from the sample, (6) computing test statistic, (7) making decision whether to reject null hypothesis, (8) making inference about population. Level of significance (α -value) of the tests conducted for the purpose of this paper is 0,05. In order to make inferences about difference between two population means and proportions we performed large-sample tests of hypothesis utilizing z -statistics.

Outcomes

This study investigates in what manner teacher's university education influences the assessment of self-initiatives directed to acquiring, holding, developing and updating own digital competences and infrastructure.

From the data obtained from the sample of students we computed sample numerical descriptive measures and estimated population mean utilizing 95% confidence interval for previously mentioned aspects of self-initiatives. Results are depicted in table (Table 3, Table 4) and they indicate that teaching mathematics

students evaluated all considered aspects of self-initiatives with greater grades than teaching primary education students.

Considered feature	Descriptive statistics of sample		Confidence interval (95%) for mean estimation	
	Mean	Standard deviation	Lower bound	Upper bound
Core digital competences	4,619615	0,770176	4,583649	4,655580
Special digital competences	2,876701	1,468281	2,792696	2,960705
Constant updating of digital competences	3,892857	1,002423	3,830019	3,955695
ICT infrastructure	4,461735	0,878015	4,374547	4,548922

Table 3. Descriptive statistics and confidence intervals for estimating mean for the sample of teaching primary education students.

Considered feature	Descriptive statistics of sample		Confidence interval (95%) for mean estimation	
	Mean	Standard deviation	Lower bound	Upper bound
Core digital competences	4,669753	0,707465	4,592430	4,747076
Special digital competences	3,194444	1,506317	2,992427	3,396462
Constant updating of digital competences	4,050000	0,998742	3,903103	4,196897
ICT infrastructure	4,652778	0,695250	4,489402	4,816154

Table 4. Descriptive statistics and confidence intervals for estimating mean for the sample of teaching mathematics students.

Quantitative data obtained from the samples of students are described by graphical method utilizing numerical measures *pth* percentile and range (Figure 2) with respect to identified categories of students.

Furthermore, we conducted large-sample one-tailed test for comparing two population means, and thus compared respondents' judgements of analyzed self-initiatives directed to digital competences and infrastructure with respect to faculty education. The alternative hypothesis represents the existence of a difference between the means of judgements of analyzed self-initiatives in favor of teaching mathematics students. This hypothesis is designed on the basis of previous discussion. From results depicted in table (Table 5) at $\alpha = 0,05$ we conclude: (i) the samples do not provide sufficient evidence for us to conclude that there is a statistically significant difference between considered means, (ii) there is a statistically significant difference in means of judgements of last three aspects of analyzed self-initiatives in favor of teaching mathematics students.

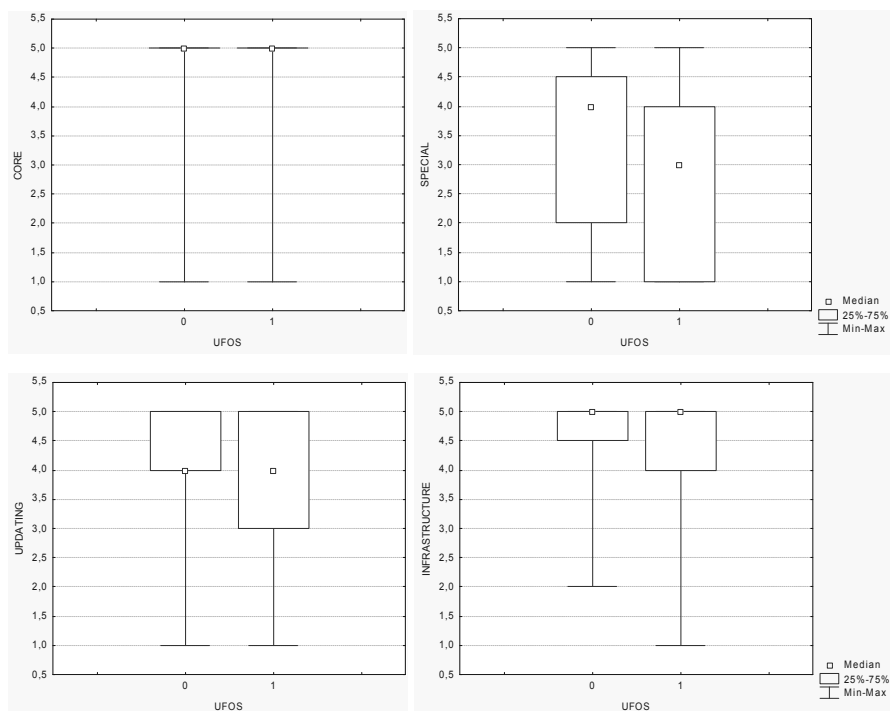


Figure 2. Categorized box plot for aspects of self-initiatives.

Considered feature	p -value	$\alpha = 0,05$	z	$z_{\alpha}=1,644854$	H_0
Core digital competences	0,12383	$p > \alpha$	1,15604	$z < z_{\alpha}$	not reject
Special digital competences	0,00211	$p < \alpha$	2,86061	$z > z_{\alpha}$	reject
Constant updating of digital competences	0,026241	$p < \alpha$	1,939156	$z > z_{\alpha}$	reject
ICT infrastructure	0,020156	$p < \alpha$	2,050542	$z > z_{\alpha}$	reject

Table 5. Results obtained from the one-tailed test for comparing two population means of judgements of analyzed self-initiatives with respect to faculty education.

In addition we analyzed closely each of previously introduced aspects of self-initiatives by computing sample numerical descriptive measures and estimating population mean utilizing 95% confidence interval (Table 6, Table 7).

Considered feature	Descriptive statistics of sample		Confidence interval (95%) for mean estimation	
	Mean	Standard deviation	Lower bound	Upper bound
Having knowledge in fundamental components of computer	4,571429	0,771279	4,462777	4,680080

Having knowledge in basic Internet terms	4,760204	0,504999	4,689064	4,831344
Having knowledge in using electronic mail	4,877551	0,372526	4,825073	4,930029
Having knowledge in creating digital textual document	4,903061	0,359188	4,852462	4,953661
Having knowledge in creating digital presentational materials	4,877551	0,411758	4,819546	4,935556
Having knowledge in creating spreadsheets	4,397959	0,919752	4,268392	4,527526
Having knowledge in drawing and image processing	4,448980	0,854820	4,328560	4,569400
Having knowledge in creating and publishing web-sites	3,658163	1,185591	3,491147	3,825179
Having knowledge in using Internet as additional source of information necessary for creating teaching materials	4,816327	0,482450	4,748363	4,884290
Necessity of having knowledge and skills in utilizing IT board	3,790816	1,053501	3,642408	3,939225
Necessity of having skills in applying some educational mathematics software	4,112245	0,904492	3,984828	4,239662
Personal ability of working in <i>Geometer's Sketchpad-u</i>	3,448980	1,087198	3,295824	3,602135
Personal ability of working in <i>GeoGebra</i>	2,316327	1,293748	2,134074	2,498579
Personal ability of working in <i>Wolfram Mathematica</i>	1,301020	0,720540	1,199517	1,402524
Personal ability of working with IT board	1,923469	1,163223	1,759604	2,087335
Updating of digital competence of mathematics teachers is necessary for proper application of ICT tools in teaching.	3,811224	0,877103	3,687666	3,934783
In order to update own ICT skills and to integrate ICT tools in appropriate way in teaching, the mathematics teachers should regularly read the relevant IT publications.	3,678571	1,054295	3,530051	3,827092
In order to update own ICT skills and to integrate ICT tools in appropriate way in teaching, the mathematics teachers should regularly monitor the relevant web portals.	3,750000	1,019678	3,606356	3,893644
In order to update own ICT skills and to integrate ICT tools in appropriate way in teaching, the mathematics teachers should attend computer science seminary.	4,112245	0,975417	3,974836	4,249654

In order to update own ICT skills and to integrate ICT tools in appropriate way in teaching, the mathematics teachers should self-initiative and independently study ICT tools.	4,112245	1,001360	3,971182	4,253308
The prerequisite for quality preparing of mathematics lessons is owning a personal computer (at home).	4,469388	0,879441	4,345499	4,593276
The prerequisite for quality preparing of mathematics lessons is having Internet access (at home).	4,454082	0,878772	4,330288	4,577876

Tablica 6. Descriptive statistics and confidence intervals for estimating mean for the sample of teaching primary education students.

Considered feature	Descriptive statistics of sample		Confidence interval (95%) for mean estimation	
	Mean	Standard deviation	Lower bound	Upper bound
Having knowledge in fundamental components of computer	4,583333	4,349266	4,817401	0,691789
Having knowledge in basic Internet terms	4,805556	4,647486	4,963625	0,467177
Having knowledge in using electronic mail	4,972222	4,915830	5,028614	0,166667
Having knowledge in creating digital textual document	4,972222	4,915830	5,028614	0,166667
Having knowledge in creating digital presentational materials	4,972222	4,915830	5,028614	0,166667
Having knowledge in creating spreadsheets	4,888889	4,781047	4,996731	0,318728
Having knowledge in drawing and image processing	4,722222	4,548541	4,895903	0,513315
Having knowledge in creating and publishing web-sites	4,611111	4,392914	4,829308	0,644882
Having knowledge in using Internet as additional source of information necessary for creating teaching materials	4,944444	4,831661	5,057228	0,333333
Necessity of having knowledge and skills in utilizing IT board	4,138889	3,845558	4,432220	0,866941
Necessity of having skills in applying some educational mathematics software	4,666667	4,485810	4,847523	0,534522
Personal ability of working in <i>Geometer's Sketchpad-u</i>	4,083333	3,849266	4,317401	0,691789
Personal ability of working in <i>GeoGebra</i>	3,361111	2,861523	3,860699	1,476536
Personal ability of working in <i>Wolfram Mathematica</i>	3,166667	2,783012	3,550321	1,133893

Personal ability of working with IT board	1,750000	1,412859	2,087141	0,996422
Updating of digital competence of mathematics teachers is necessary for proper application of ICT tools in teaching.	3,972222	3,736607	4,207837	0,696362
In order to update own ICT skills and to integrate ICT tools in appropriate way in teaching, the mathematics teachers should regularly read the relevant IT publications.	3,777778	3,406153	4,149402	1,098339
In order to update own ICT skills and to integrate ICT tools in appropriate way in teaching, the mathematics teachers should regularly monitor the relevant web portals.	3,861111	3,463615	4,258607	1,174802
In order to update own ICT skills and to integrate ICT tools in appropriate way in teaching, the mathematics teachers should attend computer science seminary.	4,083333	3,727317	4,439349	1,052209
In order to update own ICT skills and to integrate ICT tools in appropriate way in teaching, the mathematics teachers should self-initiative and independently study ICT tools.	4,555556	4,306992	4,804119	0,734631
The prerequisite for quality preparing of mathematics lessons is owning a personal computer (at home).	4,666667	4,424022	4,909311	0,717137
The prerequisite for quality preparing of mathematics lessons is having Internet access (at home).	4,638889	4,407948	4,869830	0,682549

Tablica 7. Descriptive statistics and confidence intervals for estimating mean for the sample of teaching mathematics students.

In addition for each of the previously introduced aspects of self-initiatives we performed large-sample test for comparing two population means of judgements of analyzed self-initiatives. The alternative hypothesis represents the existence of a difference between the means in favor of teaching mathematics students. At $\alpha = 0,05$ we revealed:

- The samples do not provide sufficient evidence for us to conclude that there is a statistically significant difference between means of grades for necessity of having knowledge in fundamental components of computer and basic Internet terms ($p=0,46289$; $p=0,29858$). The necessity of acquiring and/or holding all other considered core digital competences is evaluated statistically significant higher by teaching mathematics students than by teaching primary education students ($p < \alpha$).
- The necessity of acquiring and/or holding all by this paper covered special digital competences is evaluated statistically significant higher by teaching mathematics students than by teaching primary education students ($p < \alpha$).

- Both categories of students equally recognized the necessity for constant updating of digital competences by reading ICT publication and attending ICT educations ($p > \alpha$).
- Teaching mathematics students evaluated statistically significant higher the necessity of self-initiative and individual studying of ICT tools with purpose of updating ICT skills and quality integration of current ICT tools in teaching mathematics ($p = 0,000885$).
- The samples do not provide sufficient evidence for us to conclude that there is a statistically significant difference between means of grades for necessity of owning a personal computer and having Internet access ($p = 0,072$; $p = 0,07746$) in order to prepare mathematics lessons.

Afterwards, respondents evaluated if their self-initiatives, that they are taking in order to update their knowledge, skills and digital infrastructure, made them ready to utilize ICT in teaching mathematics. Using large-sample 95% confidence interval we estimated population mean for described readiness. In that manner we obtained intervals $[3,776023; 4,335088]$ and $[3,501265; 3,784449]$ for teaching mathematics and teaching primary education students, respectively. Besides we conducted large-sample test for comparing two population means of previously mentioned evaluation. The alternative hypothesis represents the existence of a difference between the means of that evaluation in favor of teaching mathematics students. This hypothesis is designed considering previous analysis. Gained result ($p = 0,003934$) at $\alpha = 0,05$ reveals that there is a statistically significant difference in means of evaluation of analyzed readiness influenced by taken self-initiatives in favor of teaching mathematics students.

Ultimately, by utilizing large-sample 95% confidence interval for a population proportion we estimated the proportions of students who are ready to integrate ICT in teaching mathematics, i.e. those students whose grade for identified readiness is at least 4. Thus we gained intervals $[0,54397; 0,84492]$ and $[0,48656; 0,62568]$ for teaching mathematics and for teaching primary education students, respectively. Utilizing the same method we estimated the proportions of students who are not ready to integrate ICT in teaching mathematics, i.e. those students whose grade for identified readiness is at most 2. Thus we gained interval $[0,07229; 0,162404]$ for teaching primary education students, while in the sample of teaching mathematics students there is no such student. Furthermore we conducted large-sample test of hypothesis for comparing two population proportions of the students who are and are not ready to integrate ICT in teaching mathematics. The alternative hypothesis represents the existence of a difference between mentioned proportions in favor of teaching mathematics students and teaching primary education students, respectively. This hypothesis is designed considering previous analysis. Gained results ($p = 0,054888486$; $p = 0$) at $\alpha = 0,05$ reveal the following: (i) samples provide insufficient evidence to detect the difference between the proportions of the students who are ready to integrate ICT in teaching mathematics, (ii) proportion of the students who are not ready to integrate ICT in teaching mathematics is statistically significant greater in the population of teaching primary education students than in the population of teaching mathematics students.

Conclusion

The results obtained in this study indicate that faculty education causes differences in attitudes towards self-initiative focused on digital competences and infrastructure. We have shown that higher levels of computer education causes more positive attitudes about the application of considered self-initiative, as we expected. The teaching program of primary education study covers less computer science areas and covers them at a lower level than a program of mathematics teacher study, it is expected that teaching mathematics students will better evaluate the self-initiative focused on digital competences and infrastructure, which was shown in this research.

However, we must emphasize that the teaching primary education students and teaching mathematics students recognized the need of constant actualization of digital competences by following the appropriate computer science publications and participation in education. This finding gives us the right to claim that the teaching primary education students are directed towards in terms of IT training, because the awareness of the necessity of the update knowledge and skills is extremely important in working with ICT tools.

We will endeavour to follow the difference in this work considered phenomena between these two populations of interest. Considering the fact that ICT increasing its part in the teaching process, students themselves should understand that their digital competences are bond that binds them to the creative and innovative application of ICT in teaching. This research is the starting point of a larger study which is planned to cover different populations of future and current teachers of mathematics at the elementary and high school level.

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The effect of students' learning style on the selection of elective modules

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Abstract. This paper investigates whether there is a connection between students' learning style and the enrolled elective module within the framework of a study program, and is it possible to predict a student's success on the basis of its learning style and elective module with the use of a machine learning classification method. The study involved students of the fifth year of graduate studies whose learning style was examined with the use of the Felder-Soloman Index of Learning Styles. The correlations between the dimensions of learning style detected with the use of the Felder-Soloman Index of Learning Styles are also investigated as well as relationship between the performance of students, selected elective modules and the dimensions of learning style. In data processing stage, ANOVA method and machine learning classification methods were used. Results showed that students with a dominant active, visual, sensory or intuitive dimension choose all elective modules, while students with a strong sequential dimension tend to choose the module with the developmental direction. Students with a strong global dimension prefer the module with the focus in computer science courses. The paper emphasizes the need to adapt teaching styles within individual elective modules to the learning styles of students who entered that module. The results obtained in this work can serve as guidelines for designing teaching materials and in planning purposes of teaching, with the goal of improving students' learning outcomes.

Keywords: learning style, ANOVA, machine learning methods, elective courses, student success

Introduction

In recommendations of the Bologna Declaration¹, special attention was given to eligibility, ie. opportunity that students themselves create the content of their studies. Therefore, in 2005. the higher education institutions create new programs

¹ Tuning, 2005., EUA tuning educational structures in Europe, available at: <http://www.let.rug.nl/TuningProject/index.htm> (accessed at March, 4th 2005.).

under which the contents of a certain percentage of time is left to students' choice according to their interests. In the same time, the process of restructuring the High schools of education into the Faculties of Teacher Education was conducted in Croatia. The previous practice of intensified program of study in a particular subject in which students of teacher studies trained for two fields of work: a) teachers in primary education and b) the subject teachers of children aged up to 14 years is abolished. However, the creators of the program of teacher studies continue to insist on the need to give the students the opportunity to achieve additional competencies in accordance with their interests within the framework of teacher studies, with the aim of better education of children in primary education. This was in line with the Bologna recommendations. Therefore, the creation of teacher training programs was approached at three levels: compulsory courses, elective courses of modules and free elective courses. The elective module in teacher study at that time was defined as a set of courses whose contents narrowly guide students to additional competencies. In the program of teacher studies that was created at the Faculty of Teacher Education in Osijek, which is positively reviewed² by the National Council for Higher Education in 2005, courses of elective modules and free elective courses cover nearly 30% of the total ECTS student workload.

After the first semester, students of integrated undergraduate and graduate studies of Faculty of Teacher Education in Osijek prescribe for one of the optional modules which qualifies them to be competent in a specific area of education of children of junior school age. Three modules are offered to them whereat the module with the development orientations (Module A) with chosen subjects of pedagogy and psychology trains them to a wider understanding of the specific issues of child education, then the IT (information technology) module (Module B) trains them thoroughly to use information technology in education process and for the IT education of children in primary education, while the foreign language module (module C) further trains them for the early teaching of children of junior school age to foreign language.

Considering the results of previous studies by other authors (Pask, 1988 in Hativa, Birenbaum, 2000; Naimie and dr.2010) which show that compliance of learning styles and teaching styles affect the performance of students, the aim of this study was to examine whether there is a connection between students' learning style and the enrolled election module, in order to adjust the teaching style with the learning style, all for the purpose of increasing students' success. For the purposes of this study, a survey was carried out in 2010./11. academic year among students of the fifth year of the Teachers University in Osijek. For testing the learning styles of students the Felder-Soloman Index of Learning Styles was used, and data on student performance were collected. The correlation between the dimensions of learning styles detected with the Felder-Soloman Index of Learning Styles and the elective modules was investigated, as well as the differences in learning styles depending on the elective module by using the one-way analysis of variance. For the

² Minister of Science, Education and Sports, signed on 16 June 2005. accreditation to the Faculty of Teacher Education in Osijek for the performance of an integrated five-year undergraduate and graduate university teacher studies.

purposes of classifying students according to their success, the k-nearest neighbor method of machine learning was used.

About the index of learning style

Felder and Silverman (1988) proposed a model of learning styles which classified participants according to preferences to one of the four dimensions: sensory-intuitive dimension, visual-verbal dimension, active-reflective dimension, and sequential-global dimension. Each of these dimensions distinguishes two basic types. In their paper (Felder, Silverman, 1988) the basic properties of each dimension were stated, and the differences between each type of these dimensions were highlighted. Thus, they stated that respondents who prefer sensory learning like to learn from data, facts and experiments, while those who prefer an intuitive learning like to learn from principles and theories. Those who belong to the visual type most easily remember information if they were presented by illustrations, graphs, films and demonstrations, while it is easier for the verbal type to remember what is said or written. Respondents belonging to the active type learn the easiest if they try out things, and they like team work, while those with prominent reflective type prefer independent work. Sequential participants learn new material in increasing increments, while global participants learn new material in "larger jumps".

As a continuation of that research, Felder and Soloman (1991) developed the index of learning style as instrument (questionnaire) that examines the tendency of respondents to the dimensions of learning style of Felder-Silverman model. It consists of 44 questions where each dimension is set to 11 questions on which it is answered back with the answers *a* or *b*. The number of *b* is subtracted from the number of *a* responses for each group of questions so that for each dimension the result is a number between -11 and 11. If the absolute value of the obtained result lies in the interval [1,3] then we are talking about a balanced relationship within the dimensions. If the score is in the interval [5,7] then there is a moderate preference for one dimension, and if the score is in the interval [9, 11] then the preference for one dimension is distinct.

Review of previous research

Studies have shown that in cases where the teaching style is similar to the learning style of students, the students acquire the material more easily and are more efficient in learning than those students whose learning style does not match the teaching style of their teacher (Pask, 1988, in Hativa, Birenbaum, 2000). Naimie, Siraj et al (2010) explored the impact of teaching style and learning styles of students and the impact of their match or mismatch on the performance of students. In a sample of 310 students using the Felder-Soloman Index of Learning Styles, observation and interviews to collect data, they showed that matching the style of teaching and learning can increase student success. Mismatch occurs when a student's preferred method of processing information is not in accordance with the preferred style of teaching of their teacher which can cause poor performance

of students because of their lack of motivation, or a student may be bored and may not be interested, write exams worse, become discouraged and in some cases change the course (Felder, Spurlin, 2005). Amir and Jelas (2010) also investigated the congruence of teaching styles and learning styles of students. They used the translated version of Grasha-Riechman Teaching and Learning Style Inventories on the sample of 545 students and 120 courses in order to investigate which of the learning styles are prevalent at the University Kebangsaan, Malaysia.

Methodology

The study was carried out in 2010/2011 academic year at the Faculty of Teachers Education at the University in Osijek. The sample included 109 fifth-year students. The instrument used to examine students' learning styles was Felder-Soloman Index of Learning Styles translated into Croatian language, which is available as a free web application. Data processing and analyses were performed by StatSoft Statistica 8 software. In order to examine whether there is a difference between the learning styles of students and their preferences in selecting module, the statistical method one-way analysis of variance (ANOVA) was used. Since the ANOVA showed a certain dependence between variables, it was assumed that it is possible to create a model of machine learning that will be able to assess in which elective course a student should be classified on the basis of the information about students' learning style and selected module as input variables. For this purpose the k -nearest neighbors algorithm was tested as one of the machine learning methods.

ANOVA

Analysis of variance (ANOVA) is a statistical method used to test for significant differences between means of two or more groups, under the assumption that the sampled populations are normally distributed (NIST/SEMATECH, 2010). The basic formula for calculating the variance s^2 of n measurements is (McClave, Benson, 1998):

$$s^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n - 1} \quad (1)$$

where $\bar{y}$ is the mean of the n measurements. In ANOVA, the variation in the dependent variable is partitioned into components that correspond to different values of the independent i.e. group variable.

The goal of ANOVA is to split the total variation in the data into a portion due to random error and portions due to changes in the values of the independent variable(s).

In our experiments, each of the learning style type separately is used as the dependent variable, while the selected study module was used as the independent variable (valued as module A, B, and C).

The descriptive statistics showed that the data of learning style variables are normally distributed, therefore the assumption for ANOVA was satisfied. The test statistic, used in testing the equality of treatment means is the F test. The critical value is the tabular value of the F distribution, based on the chosen p-level and the degrees of freedom.

***k*-nearest neighbor methodology**

The ***k*-nearest neighbors algorithm** is an instance-based learning method used for classifying objects based on closest training examples in the feature space, although it can also be used in regression type of problems. It is one of the most frequently used machine learning methods, successfully applied in many areas, such as data mining, statistical pattern recognition, image processing and many others (Teknomo, 2011). Govindarajan and RM. Chandrasekaran (2010) showed that it is efficient in text mining for direct marketing. The advantage of this method is in its simplicity to use and interpret, while the weakness is in a high computational cost in the search process which is proportional to the size of the training samples. (Kudo et al, 2003).

The method operates in a way that an input vector is classified by a majority vote of its neighbors. The neighbors denote the input vectors for which the correct classification is known. The classification is performed such that a new input vector is assigned to the class most common amongst its *k* nearest neighbors, where *k* is a positive number, typically set to a small value. If *k* = 1, then the input vector is simply assigned to the class of its nearest neighbor. In our experiments, the size of *k* is determined in the *n*-fold cross-validation procedure (due to a small sample size, the value of *n* = 3 was used) which showed that the *k* = 2 is the best choice for the observed data). In order to measure the distance between an input vector and its neighbors, the Euclidean distance is used where the distance between points **p** and **q** is the length of the line segment connecting them.

In *n*-dimensional space, if **p** = (*p*₁, *p*₂, ..., *p*_{*n*}) and **q** = (*q*₁, *q*₂, ..., *q*_{*n*}) are two points, then the Euclidean distance from **p** to **q**, or from **q** to **p** is given by (Teknomo, 2011):

$$\begin{aligned} d(p, q) = d(q, p) &= \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2 + \dots + (q_n - p_n)^2} \\ &= \sqrt{\sum_{i=1}^n (q_i - p_i)^2} \end{aligned} \quad (2)$$

This method is used in our experiments for the purpose of classifying students according to their study successfulness, measured by the average grade of study. In the preprocessing stage, the output variable (i.e. the average grade) is transformed into a binary format, such that value of 0 indicates success below 4, while the value of 1 indicates success equal or higher than 4 (grades were expressed in the range from 1 to 5). The input variables were eight dimensions of learning style

and a selected module. In order to perform the k -nearest neighbor method, the data sample is randomly divided into a train set (75% of the data), and a test set (25% of the data). The model is estimated on a train sample, while the test sample is used for evaluating its accuracy. A confusion matrix which shows correct and incorrect classification is presented for the obtained model, and correct classification rate (i.e. the hit rate) is computed separately for category 0 and category 1. The final assessment of the model successfulness is made upon the average classification rate of both categories of the output variable.

Results

The study first analyzed the distribution of preferences of respondents according to each dimension of Felder-Soloman index of learning style. Table 1 shows the classification of the tendency of respondents to significant, moderate affinity for the dimensions of the Felder-Soloman index of learning style and on a balanced ratio between dimensions.

Faculty of Teacher Education ($N = 109$)

Preference	Learning style							
	A	R	SE	I	VI	VE	SEK	GL
significant	11	0	3	0	3	0	18	0
moderate	49	2	37	2	14	3	50	1
balanced relation	39	8	52	15	57	32	30	10

A – active, R – reflective, SE – sensory, I – intuitive, VI – vizual, VE – verbal, SEK – sequential, GL – global

Table 1. Distribution of learning styles.

It is recommended (Felder, Spurlin, 2005.) to consider only subjects with moderate or significant tendency towards a dimension, so for the purpose of this study, these respondents are classified into one category as shown in Table 2. For the category of significant/moderate tendency values of 5-11 for each dimension of the Felder-Soloman Index of Learning Styles were used, and for a balanced ratio between the dimensions, values of 3 to –3 were considered. The results showed that 55.05% of respondents have a preference for the active dimension, 15.60% toward the visual, and 0.92% toward the global dimension.

Active – reflective			Senzory– intuitive		
UM _A /I _A	U _{A-R}	UM _R /I _R	UM _{SE} /I _{SE}	U _{SE-I}	UM _{SE} /I _I
55,05%	43,12%	1,83%	36,70%	61,47%	1,83%

Vizual – verbal			Sequential – global		
UM _{VI} /I _{VI}	U _{VI-VE}	UM _{VE} /I _{VE}	UM _{SEK} /I _{SEK}	U _{SEK-GL}	UM _{GL} /I _{GL}
15,60%	81,65%	2,75%	62,38%	36,70%	0,92%

*UM – moderate preference, I – significant preference, U – balanced relation

Table 2. Classification according to the intensity of preferences.

Before the analysis of variance, a correlation analysis was conducted to investigate the existence of linear relationships between selected module as the dependent variable and the dimensions of learning style as independent variables. The results showed that the sequential-global dimension is in correlation with the selected module, but the types of sequential-global dimensions are not in correlation with the active, reflexive, visual and verbal learning style type (level of significance $p < 0.05$).

The results of the ANOVA showed a statistically significant relationship ($p < 0.05$) between categories of modules and sequential learning style ($F = 4.124$, $p = 0.019$) and between categories of modules and global learning style ($F = 4.124$, $p = 0.019$), i.e. students who have chosen different modules also differ according to their learning style. We also noted that the active, visual, reflective, sensory and intuitive types fit into all modules, sequential types predominantly elect developmental module (Module A), and global types frequently use IT module (Module B). Detailed results of the ANOVA test are shown in Table 3.

Learning style	Degrees of freedom	F	p
active	106	0,478765	0,620881
reflective	106	0,478765	0,620881
sensory	106	0,197495	0,821086
intuitive	106	0,197495	0,821086
vizual	106	0,161496	0,851079
verbal	106	0,161496	0,851079
sequential	106	4,124447	0,018839
global	106	4,124447	0,018839

Table 3. ANOVA test results.

In order to investigate a possibility of predicting whether a student's success will be above average if the information about the chosen module and the information about his/her learning style is known, a model for classifying students with respect to performance is created by using the k -nearest neighbors method. Results in the sense of hit rates for each category of students are shown in Table 4.

Algorithm for calculating the distance	Hit rate for category 0 (students with grade below 4)	Hit rate for category 1 (students with grade higher than 4)	Average hit rate
Euclidean, $k = 2$	75,00%	62.00%	68.5%

Table 4. The results of k -nearest neighbors method in the classification of students according to their study success obtained on the test sample.

Table 4 shows that the k -nearest neighbor method successfully classifies 75% of students who belong to the category of 0 (those who have an average student success under 4), while the model correctly identifies 62% of students who have an average student success above or equal to 4 (category 1). Thus, less successful students are easier to identify than very successful ones according to the applied method. The average rate of accuracy of the model for both categories of students was 68.5%, which is better than the random choice, so the model can therefore be considered as successful in classifying students according to their study success.

Conclusion

The aim of his study was to determine the students' dominant learning style with the use of Felder-Soloman Index of Learning Styles, to investigate whether there is a connection between the dominant learning style of students and selected elective module using one-way analysis of variance, and finally to create a model for predicting student success with regard to elective courses and learning style by using the k -nearest neighbors method of machine learning.

Research has shown that most of our respondents had the least preference towards reflexive, intuitive and global type of learning style, and most had significant or moderate affinity towards the sequential type. The results of the ANOVA showed a statistically significant relationship ($p < 0.05$) between the sequential-global dimension of learning style and selected elective modules. Also, it was shown that the sequential and global types of learning style are in correlation with the selected module that students attend. The method of k -nearest neighbors, based on the input variables describing students' learning style and elective module, produced the average hit rate of 68.5% in classifying students, therefore demonstrated that it is possible to create a model of machine learning that will be able to predict whether the students will be extraordinary successful or not. Considering that this is preliminary research, the results obtained should be restricted to the observed sample, and the future research work should be focused on increasing the accuracy of the model by testing other methods.

Since this work showed that students who choose a particular module have different learning styles, recommendation for further research is to more investigate the adjustment of teaching styles with appropriate learning styles in order to facilitate efficient learning to as many students as possible, and to enhance student success. In that way, depending on the dominance of students' learning styles, teachers could contribute in improving the learning outcomes by adapting their teaching materials and methods.

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Self-efficacy of the first year ICT students

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Abstract. The purpose of this study is to research and validate items of Bandura's social cognitive theory of self-efficacy among ICT students of the first year at Faculty of organization and informatics at the University of Zagreb. Our research was conducted in 2009/2010 year in summer semester in mathematics on the sample of 202 students. After confirmatory factor analysis we have got five factors which can influence students' self-efficacy: mastery experience with social persuasion, psychological states, vicarious experience, grade and family support. We have found that there are differences in self-efficacy for those students who passed the mathematics exam and those who failed. There are three common factors which influence self-efficacy of both successful and less successful students – mastery experience with social persuasion, psychological states, and vicarious experience. Two remaining factors are grade factor influences successful students' self-efficacy, and family support factor influences self-efficacy of less successful students. There are many reasons why is self-efficacy important for school and university students and one of the most important is that academic self-efficacy is the good predictor of academic outcomes.

Keywords: mathematics, self-efficacy, mastery experience, vicarious experience, social persuasion, psychological states

Introduction

Self-efficacy may be defined as the level of confidence individuals hold in their abilities to accomplish certain courses of action or achieve specific outcomes (Bandura, 1977).

Self-efficacy has been thoroughly researched in the past few decades. I has been shown that there are different kinds of relationship between self-efficacy and *mathematical anxiety* (feelings of tension and anxiety that interfere with the manipulation of numbers and solving mathematical problems), *test anxiety*, *self-esteem* (general evaluation of yourself), *self-regulation* (implies conscious awareness and coordination of thinking processes and strategies, and involves selecting and developing appropriate strategies in order to achieve explicit or implicit learning

goals (Duncan & McKeachie, 2005)), *efficacy in problem solving, mathematical achievement*,...

For example, Jain and Dowson (2009) have shown that all “self” (regulation and efficacy) factors are positively and statistically significantly related to each other, but negatively related to mathematics anxiety. Hoffman (2010) has shown that self-efficacy was significant predictor of *problem solving efficiency* for less complex problems as evidenced by greater accuracy and reduced problem-solving time. These results also yielded new information concerning the role of mathematical anxiety and self-efficacy in relation to problem-solving time. Students with higher degrees of confidence in task success are presumed more accurate in part because they do not divert time-consuming resources to manage stress and anxiety and are able to effectively calibrate effort (Hoffman, 2010).

Bandura's self-efficacy theory

Bandura (1997) assumes that self-efficacy beliefs are key factor in human capabilities system. It is shown that different people with similar skills, or same person in different circumstances can perform an action badly, well or perfectly depending on the changes in self-efficacy beliefs. Bandura's theory assumes that there are four factors which influence self-efficacy – mastery experience (ME), vicarious experience (VE), social persuasion (SP), psychological states (PH). According to this theory self-efficacy is strongly affected by one's previous performance, e.g. mastery experience prove particularly powerful when individuals conquer obstacles or succeed on demanding task. Successful performance in the discipline can have permanent effects on one's self-efficacy. Examples of vicarious experience are: observing others, watching peers, teachers, and adults succeed at a task. Social persuasion means encouragement from parents, teachers, or peers whom students trust, and that can improve students' confidence in their academic capabilities. Examples of psychological states are high anxiety, stress, mood, tiredness – all of that can weaken self-efficacy. Self-efficacy theory suggests that behavior may be changed in response to self-efficacy expectations which, in turn, can strengthened (weakened) by different types of feedback given to individuals (Bandura 1977).

Research

Our research was conducted in 2009/2010 year in summer semester in mathematics on the sample of 202 students. The research was conducted on-line in system for e-learning Moodle and students did it at the end of the second semester. Doing of a questionnaire was on voluntary basis but students were motivated by getting one point (out of 100 possible to get on different course activities).

Research questions were:

1. Which factors affect the self-efficacy at studying mathematics for *the whole group* of ICT students?
2. Are there any differences in self-efficacy between *successful* and *less successful* students, e.g. those students who passed mathematics via continuous

monitoring through the semester or via exam at the end of semester, and those who didn't pass?

We used original items from "Sources of self-efficacy in mathematics: A validation study" (Usher & Pajares, 2009). The items were translated into Croatian and modified to suit for given students' group (freshmen of ICT on the mathematics course). There is one item added. All changes are shown in the following list.

Questionnaire items:

1. (ME1) I got excellent grades on *math mid-term tests and tests*. (*Modified.*)
2. (ME5) I have always been successful with math.
3. (ME10) Even when I study very hard, I do poorly in math.
4. (ME15) I got good grades in math on my last *mid-term test*. (*Modified.*)
5. (ME20) I do well on math assignments.
6. (ME25) I do well on even the most difficult math assignments.
7. (VA2) Seeing adults do well in math pushes me to do better.
8. (VP6) When I see how my math teacher solves a problem, I can picture myself solving the problem in the same way.
9. (VP11) Seeing kids do better than me in math pushes me to do better.
10. (VP16) When I see how another student solves a math problem, I can see myself solving the problem in the same way.
11. (VS21) I imagine myself working through challenging math problems successfully.
12. (VS9) I compete with myself in math.
13. (P3) My math teachers have told me that I am good at learning math.
14. (P7) People have told me that I have a talent for math.
15. (P12) Adults in my family praised me when I was good *at school math*. (*Modified.*)
16. (P17) *Adults in my family still praise me when I have good result on math mid-term tests and tests.* (*Added.*)
17. (P22) I have been praised for my ability in math.
18. (P14) Other students have told me that I'm good at learning math.
19. (P4) My classmates like to work with me in math because they think I'm good at it.
20. (PH8) Just being in math class makes feel stressed and nervous.
21. (PH13) Doing math work takes all of my energy.
22. (PH18) I start to feel stressed-out as soon as I begin my math work.
23. (PH23) My mind goes blank and I am unable to think clearly when doing math work.
24. (PH19) I get depressed when I think about learning math.
25. (PH24) My whole body becomes tense when I have to do math.

We have modified items ME1, ME15, P12 and added item P17. The modification was necessary because the original questionnaire had been made for high school students so we have added phrase "*on mid-term tests*" in items ME1 and

ME15. We have divided the item “*Adults in my family have told me what a good math student I am*” in items P12 and P17 because we wanted to separate two different types of family praise, the one related to high school students and another related to university students.

Kaiser-Meyer-Olkin’s (KMO) measure for the whole group of students is 0,916 which confirms the fact that the collected data are appropriate for implementation of confirmatory factor analysis.

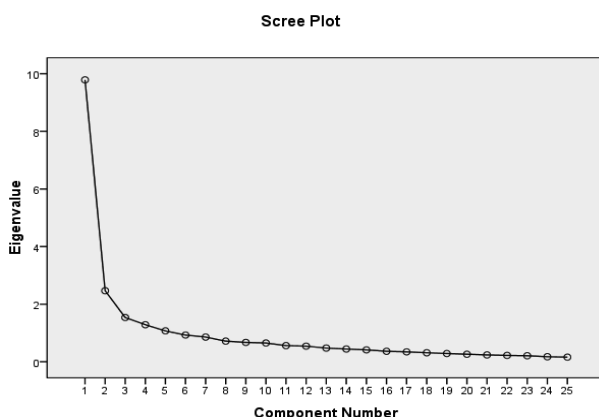


Figure 1. The diagram for 25 variables. x-axes represents variables, and y-axes eigenvalues.

Component	Initial Eigenvalues		
	Total	% of Variance	Cumulative %
1	9,789	39,156	39,156
2	2,467	9,867	49,022
3	1,538	6,151	55,174
4	1,287	5,148	60,322
5	1,074	4,297	64,619
6	,928	3,714	68,333
7	,858	3,432	71,764
8	,717	2,867	74,632
9	,673	2,692	77,323
10	,653	2,611	79,934
11	,562	2,247	82,181
12	,544	2,177	84,358
13	,476	1,903	86,262
14	,443	1,774	88,035
15	,412	1,649	89,685
16	,365	1,460	91,145
17	,343	1,373	92,518
18	,314	1,257	93,775

19	,287	1,147	94,922
20	,263	1,052	95,974
21	,238	,952	96,926
22	,222	,888	97,815
23	,210	,840	98,655
24	,176	,704	99,359
25	,160	,641	100,000

Table 1. Eigenvalues, individual and cumulative percentages of variable variance for the whole group of students.

Results

After confirmatory factor analysis on *the whole group of students* we have got five factors (Picture 1, Table 2) which can influence students' self-efficacy: mastery experience together with social persuasion, psychological states, vicarious experience, grades and family support. The percentage of variance explained by those five factors is 64,619% (Table 1).

	Factors				
	Mastery experience and social persuasion	Psychological state	Vicarious experience	Grades	Family support
ME1	,320	-,265	,149	,801	,116
VA2	,039	,027	,785	-,047	,147
P3	,433	-,088	,368	,405	-,105
P4	,710	-,135	,301	,277	,121
ME5	,755	-,403	-,033	,048	,124
VP6	,374	-,380	,529	-,012	-,181
P7	,827	-,233	,119	-,012	,035
PH8	-,123	,762	-,197	-,268	-,007
ME10	-,345	,491	,092	-,260	,273
VP11	-,013	,031	,798	,112	,171
P12	,445	-,174	,159	-,068	,630
PH13	-,106	,715	,107	-,031	-,082
ME15	,240	-,178	,065	,852	,002
VP16	,173	-,182	,671	,092	-,010
P17	,002	-,043	,248	,106	,824
PH18	,400	-,591	,196	,185	,094
ME20	,677	-,329	,107	,344	,088
VS21	,698	-,223	,079	,170	-,029
P22	,403	-,238	,301	,240	,144

PH23	-, 402	,631	-, 054	-, 094	, 030
ME25	,686	-, 368	, 050	, 282	, 146
VS9	, 170	-, 091	,537	, 158	, 251
P14	,737	-, 185	, 208	, 332	, 058
PH19	-, 341	,731	-, 222	-, 156	-, 147
PH24	-, 261	,795	-, 147	-, 057	-, 084

Table 2. Rotated component matrix for the whole group of students.

We have divided the whole group of students in two subgroups: *successful* (students who passed mathematics via continuous monitoring through the semester or via exam at the end of semester) and *less successful* (those who didn't pass mathematics).

KMO is 0, 868 for the group of successful students and 0, 821 for the less successful students which confirms that the data are appropriate for implementation of two confirmatory factor analyses.

We have found that there are differences in self-efficacy for those students who passed the mathematics exam and those who failed. There are three common factors which influence self-efficacy of both successful and less successful students – mastery experience with social persuasion, psychological states, and vicarious experience. Two remaining factors are grade factor which influences successful students' self-efficacy and family support factor which influences self-efficacy of less successful students.

The percentage of the variance explained by the four factors which affect successful students' self-efficacy is 59, 458%, while the percentage of the variance explained by four factors which influence less successful students' self-efficacy is 56, 423%. If we assume that the factor loading of variable by factor less than 0, 5 doesn't have to be considered, and if we take out variables P3 and VS9 of factor analysis in the group of successful students, and variables ME1, ME10 and ME15 in the group of less successful students it will be shown that the percentage of variance explained by four factors is 62, 47% (and KMO 0, 870) for the successful students and 59, 310% (and KMO 0, 826) for less successful students.

Considering the meaning of the variable P3 (*"My math teachers have told me that I am good at learning math."*) we can conclude that a teacher's praise doesn't affect successful students' self-efficacy. That is not surprising because a teacher, who works with a large group of students, doesn't have an opportunity to know each and every particular student and to establish communication such as it is in small groups.

The variable VS9 (*"I compete with myself in math."*) doesn't affect successful students' self-efficacy because the majority of good students wants to collect only a minimum of points needed to pass the exam. That confirms the fact that there is a small percentage of A, B and C grades at the end of the semester.

Further, it can be concluded that the variables connected with good grades and points (ME1 and ME15) don't affect less successful students' self-efficacy because they didn't get grades which they perceive as good until the moment of doing the questionnaire. That is also confirmed by the variable ME10 (*"I got good grades in math on my last mid-term test."*) which ended up taken out together with other two variables.

Conclusion and discussion

Two Bandura's factors have merged on the whole sample as well as on other two subgroups, what does not surprise, because students who perceive their past performance in mathematics as successful are more likely to receive frequent praise on those very performances. Conversely, students who interpret their effort in math as futile are more likely to receive (or perceive) messages from others that they are not capable. (Usher & Pajares, 2009; Bandura, 1997)

The grade factor was sorted out – items ME1 (*"I got excellent grades on math mid-term tests and tests."*) and ME 15(*"I got good grades in math on my last mid-term test."*) – in the group of successful students which still confirms that there is a strong positive relationship between mathematics self-efficacy and achievement in mathematics (Ayotola, Adedeji, 2009).

The family support factor has grouped – items P12 (*"Adults in my family praised me when I was good at school math."*) and P17 (*"Adults in my family still praise me when I have good result on math mid-term tests and tests."*) – with item ME15 (*"I got good grades in math on my last mid-term test."*) which has a negative correlation coefficient, in the group of less successful students. This proves that family support and praise is not important only to primary and secondary school students but it is also important to university students, and that less successful students have more pronounced fear of disappointing their parents. Further, the family support factor (items P12 and P17) hasn't sorted out in the group of successful students, which doesn't mean that family support is unimportant to successful students, but their self-efficacy hasn't been affected by the fear of disappointing their parents. Živčić-Bećirević (2003) says that successful students use encouraging thoughts more often, have more pronounced learning motivation and interest for the materials, and less pronounced fear of disappointing parents.

This research gives us useful information on the factors which build students' self-efficacy and ensure us on how important is students' environment at home as well at university, if there is aim to raise the achievement in learning mathematics. The information on self-efficacy can help teachers to tailor teaching strategies and communication methods so that they can give the best possible feedback to students. In the end, it is important to emphasize the role of praise in raising self-efficacy regardless high school students/university students' age and well-known fact that "nothing succeeds as well as success".

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Exploratory learning of mathematics in primary school: advantages and examples (poster)

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Abstract. A well known quote by Confucius states: “I hear and I forget. I see and I remember. I do and I understand.” Explorative learning is a well known form of teaching, but is relatively rarely used in mathematics. In the last decades the approach in which pupils discover laws and relationships has been more and more developed for teaching sciences. In this approach the necessary terms and other theoretical background is explained after some conclusions have been obtained e.g. on the basis of an experiment. Mathematics on the other side is better known as a theoretical, and less as an experimental scientific discipline, and thus it seems that it is harder to develop lesson plans appropriate for exploratory learning. On the other side, mathematical formality is not a subject in its own right, although it seems so to many children during their schooling and they often keep this opinion when they grow up. By exploratory learning (as well as, for example, by using projects and open problems) formulas can be introduced gradually, at the end, resulting from the need to describe a method concisely.

Many papers emphasize the advantages of learning through exploration, and particularly for learning geometry there are many elaborate examples, see e.g. Varošanec, S., (2006.): Učenje otkrivanjem (<http://web.math.hr/nastava/mnm1/ucenje.doc>). The purpose of this presentation is to enlarge the pool of ideas for developing the explorative approach in teaching mathematics, particularly for non-geometric topics, and thus to encourage an increase in the use of this teaching form in the teaching of mathematics in primary school. Furthermore, an emphasis shall be put on the fact that the often heard argument “we do not have time to separate from the classical lessons for exploratory learning/learning through projects” in fact is no argument at all, because this other approach covers the same subject-matters, but increases the understanding and also the interest of pupils for mathematics.

Keywords: exploratory learning, classroom projects, mathematical experiments, constructivist information-processing approach to teaching and learning, interactive constructivist approach to teaching and learning

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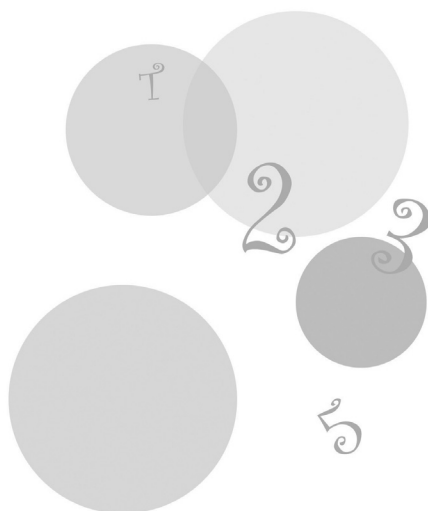
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4.

Teacher and students in teaching practice



In the fourth chapter the authors speak of the results of research and/or success stories of strategies in mathematics teaching which have resulted in desirable outcomes of learning among children and students. Teachers are in favour of active methods of learning and teaching, and, regarding children of younger school age, professionally structured play. Some of the authors are of the opinion that firstly, in school mathematics it is required to replace persistent insisting on formalities, whenever possible, with experimental aspects of mathematics. In their opinion, students of teacher studies need to master the interactive constructivist approach to learning and teaching mathematics and be ready to apply the popular approach in the mathematics classroom, whenever opportunity arises. The authors report general awareness among parents, teachers and society of the importance of early mathematical education. Likewise, a growing number of parents of children with special needs are interested in integrating their children into regular public instruction, which is also suggested by the national development strategy of the Republic of Croatia. Teachers warn of insufficient knowledge and skills in this segment imparted on students during their university studies and indicate the need for the development of curricula which would enable students to recognise, educate and support pupils with special needs.

Mathematical experiments

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Abstract. Mathematics is known as a theoretical science. Because of its formalism it is one of the least popular school subjects and sciences. The current trend to popularisation of mathematics, developing after 2000 intensively and systematicall on an international level, still hasn't shown larger effects on changing the attitudes of pupils towards mathematics. Independently of all broader oriented activities, the role of the mathematics teacher as a popularisator of mathematics is exceptional, because by his approach he directly contributes to the formation of pupils opinions that are later hard to change.

One of the main obstacles to success in popularisation of mathematics is that mathematics is a theoretical science. In comparison to other sciences it seems that it is hard and time-consuming to develop and prepare activities connected to school curricula that are at the same time such that pupils actively, not only by thinking and writing, discover mathematical laws and the beauty of mathematics. Almost nobody, not even math researchers and teachers, experiences mathematics as an experimental science. The purpose of this presentation is to show that, particularly on the primary and secondary school level, mathematics also has experimental aspects and to show concrete examples of activities adequate for children that emphasize this aspect of mathematics.

Keywords: popularisation of mathematics, learning through discovery, interactive mathematics, constructivist information-processing approach to teaching and learning, interactive constructivist approach to teaching and learning

What is good education? Systematically giving
opportunity to the student to discover things by himself.

(George Polya, *How to solve it?*, 1945.)

Introduction

Mathematics is one of the least popular school subjects and sciences. The current trend to popularization of mathematics, developing after World Year of Mathematics 2000 intensively and systematically on an international level, as yet

has no significant effects on changing the attitude of students towards mathematics. The role of the mathematics teacher as a popularisator of mathematics is exceptional, as the opinions on mathematics (and other sciences) developed during school are hard to change later. One of the main obstacles to success in popularisation of mathematics is that mathematics is a theoretical science. Even worse, often its formal aspects stand in the foreground, while the real content (and methods) they represent are not easy to perceive. Almost nobody, not even math researchers and teachers, experiences mathematics as an experimental science. In a very strict sense, this might be true, but on the other side, mathematical experimenting – with numbers, shapes, ideas – can lead to discovery of patterns and properties, formulation of conjectures, even to first ideas of proofs.

In this paper we restrict ourselves to a narrower meaning of a mathematical experiment: we consider activities that not only aim to discover a mathematical fact by pure reasoning, but have a strong hands-on aspect. As it is well known that learning by doing has a greater impact on true understanding of facts, such mathematical experiments can help to improve the understanding of mathematics, and also the children's attitude towards mathematics. While higher mathematics lacks a big number of topics that can suitably be converted into lesson-plans with a strong experimental and discovery component, primary and secondary school curricula show an abundance of topics that can be treated in an experimental and hands-on way. It is fairly obvious that almost all topics in school geometry can be introduced and explained by discovery learning, and most explorations can be formulated as hands-on experiments. But, it is less obvious that also most of the arithmetical and algebraic topics can be treated in the same way.

The purpose of this paper is to describe a few examples for an experimental approach to some of the standard curricula topics. While it may seem that it is hard and time-consuming to develop and prepare mathematical activities connected to school curricula that are at the same time such that pupils actively, not only by thinking and writing, discover mathematical laws and the beauty of mathematics, these few examples are intended to show that it is not hard to find ideas for converting a standard lesson-plan on most mathematical topics to a inquiry based one.

Experiment 1: Discovering the meaning of digits by a magic trick

A classical mathematical magical trick [1] is the following one. The magician gives chooses a spectator and instructs him (or her) to

- (1) choose a three-digit number whose first and last digit are different;
- (2) reverse its order of digits;
- (3) subtract the smaller from the larger of the two numbers from steps (1) and (2);
- (4) reverse the order of digits of the result obtained in (3), with the remark that if (3) resulted in a two-digit number, it should be considered a three-digit number by placing a zero in front of it;
- (5) add the numbers obtained in steps (3) and (4).

Then the magician surprises the audience by guessing the final result. Namely, the result will always be 1089.

The performance of this trick is one of the easiest and most fun ways of starting an experimental investigation leading to the understanding of meanings of digits and their positions. The experimental part consists in trying out various starting numbers and discovering patterns. This should lead not to formulating the conjecture “The result is always 1089.”. But, there is much more to it – some children will have the idea to experiment with three-digit numbers with first and last digit equal, and consequently arrive at a more precise formulation of the conjecture (“The result is 1089 if and only if the starting three-digit number has unequal first and last digits”). The experimental calculations also offer first insights into the proof (depending on the age a more or less formal one), which is basically the answer to the obvious question: “Why is the conjecture true?”. Many subquestions can be posed in order to help the students to discover the proof, or at least to get a feeling of the steps to prove (e.g. “Why is it important to start with a number having different first and last digits?”). There is also room for generalizations (possibly even some children will suggest them on their own): What happens if the starting number has two digits? Four or more digits? At a higher level, one can also generalize the problem to other bases. Several related questions, each of which can be analyzed experimentally, can be found in [2].

This “1089-trick” wonderfully distinguishes the experimental discovery approach to the classical “problem-solution” approach. Even if choosing the same property would be used to illustrate the meaning of digits in base 10, a “problem-solution” approach in which the students would be told to prove the statement: “When starting from a three-digit number with first and last digit different and performing the steps (1)–(4) one always obtains 1089.” would not only probably be too hard for most students, but would hide much of the real mathematics in the problem. Here, under real mathematics, we mean: the ability of formulating and testing conjectures, finding examples and counterexamples, and discovering possible generalizations (or limitations) of a mathematical statement. And all of that leads to the discovery of the need of a mathematical proof and finding of the proof itself.

Experiment 2: Discovering arithmetical progressions by another magic trick

A number of magic tricks [3], [4] use calendar leafs and properties of arithmetic progressions form their mathematical background. One example is the following simple trick: The magician asks the spectator to circle 4 numbers in a column of a (monthly) calendar leaf and add the four numbers. Upon being told the sum, the magician knows which four numbers were circled. Namely, the four numbers form an arithmetic progression with difference 7, so their sum is 42 plus four times the first number, so all the magician has to do is subtract 42 and divide by 4. Where does the experimental part come in? First, children can try various months and columns, and in this way they can discover the rule about summing members of an arithmetical progression. Furthermore, there are several possible additional exploratory questions: What if one takes three or five numbers in a column? How about rows? And does the trick work with other rectangular tables of numbers?

Can you construct such a table that does not contain the numbers 1 to 31? Can you formulate a general rule? To answer such questions children have to try out different possibilities: they have to experiment.

Experiment 3: Discovering addition by magic squares

Magic squares [5], [6] are fascinating mathematical objects, and obviously they can be used to improve the proficiency in addition. But, they can also be used in the very beginnings of learning addition. Namely, if numbers are represented as dots (like on dominoes), the “magic” property can be discovered even by small children not yet able to do addition, simply by counting the dots. The experimental part can consist in constructions of various magic squares. Note that for maximal benefit it is best to work with a 3×3 grid, but not only in the classical setting of numbers 1 to 9, but also letting children try other numbers and also with varying conditions of being magic (e.g. with or without requiring that diagonals sum up to the same sum as rows and columns). As they construct – in the worst case simply by counting the dots – with or without help from the teacher the children notice that, say, consecutive squares with 8, 1 and 6 dots result in 15 dots altogether. Additionally, guided by suitable questions, they will also notice the commutativity (it is irrelevant in which order they count the dots, the sum stays the same).

The experiment can be extended to classic notation with numbers instead of dots, to training of subtraction (filling in of missing numbers, knowing the magic sum), to finding multiplicative magic squares to gain proficiency in multiplication, and of course to various sizes, other grids or even to three-dimensions (“magic cubes”). Note that constructions of magic squares with fractions, decimal numbers, general real (or even complex) numbers can serve to help children gain proficiency in arithmetic with the mentioned sorts of numbers. In all cases there is an additional benefit – the (quite probable) possibility of children noticing various properties of magic squares (e.g. that up to rotation and reflection there is only one magic square with numbers 1 to 9).

Experiment 4: Discovering fractions by tilings

It is well known [7] that children (and even adults) often have problems with understanding of fractions and performing arithmetic operations with them. There are many ways (some suggested in [7]) to deal with this problem, and most have a hands-on aspect, e.g. when suggesting coloring parts (fractions) of a picture in different colors. A variation on this theme can be achieved by using tilings with regular polygons (isosceles triangles, squares and regular hexagons), with or without decoration. The tiles can easily be made of thick cardboard paper. The main advantage to the version when one uses a given picture divided into fragments, that are then used to lead the pupils to the idea of fractions, is that by using tilings one can easily vary the denominator (number of tiles), and also use them for illustration of arithmetic operations (particularly easy: division by a positive integer). The experimental version can consist in asking children to construct examples of specific fractions, which can also lead to discovery of canceling (2 tiles of 8 are example of one fourth just as 5 of 20 are). Also, the children can experiment with various shapes and discover the independence of the number (fraction) of the object

they describe, noticing that e.g. two squares of a tessellation with five squares and four hexagons of a tessellation with ten hexagons both deserve the same name – the fraction “two fifth” that is describing the ratio. There are many other facts and properties of fractions that can easily be discovered by experimenting with tiles, e.g. by grouping into “Egyptian fractions” [8] and thus improving the skills in addition and subtraction. Other experimental approaches to fractions include using origami [9], cooking [7], music [7] and obviously measuring of various physical properties (time, length, area, volume, . . .).

Experiment 5: Discovering areas using a tangram puzzle

Although the tangram is well-known as a didactical tool, it is unfortunately rarely used in the math classroom. This is quite a pity, since it can be used for many mathematical topics for various ages. One can use it to learn the basic terminology (triangle, square, parallelogram, etc.), develop the concept of area and perimeter, angles, congruency and similarity, convex polygons, symmetry, fractions and irrational numbers (the square root of 2), even coordinate geometry and combinatorics [10], [11] as well as some more advanced topics [12]. Additional benefits are the development of reasoning skills and the discovery that a (mathematical) problem can have more than one solution, or be insoluble. There are many sources for ideas [13], [14], so let us give us just one example of an experiment with tangrams. Say each child is equipped with a complete tangram (e.g. because they had to make their own tangrams from cardboard paper for homework) and one wants to teach the class the concept of area using a unit square. The smallest square can be used as the unit and the children can explore (by experimenting: using the tangram pieces and tracing them in their notebooks) the areas of the tangram pieces and various triangles and quadrilaterals (and other polygonal shapes) that can be made from the pieces. Note that is quite probable that not all children will obtain the areas of the same shapes, since there are many possible shapes¹ that can be made from the tangram pieces, so there is much place for communication and exchange of ideas. The exploration can be extended using other tangram-like dissection puzzles, be it some that are already known [15], [16] or own dissection puzzles.

Discussion

It is the opinion of the author – in fact, her firm belief – that it would be possible to develop lesson-plans for mathematics (based on any standard curricula) that are thoroughly inquiry-based, particularly for the first eight years of schooling.

The five previous examples were meant as a source of inspiration. In particular, they serve as a “proof” that it is not hard to develop experimental and exploratory concepts for use in the mathematical classroom. Generally, it is quite easy to find or develop ideas for experiments in geometry, and with a little creativity it is not much harder to formulate lesson plans based on experiments and inquiry for

¹ Thirteen convex polygons are possible using all seven pieces: one triangle, six quadrilaterals, three pentagons and three hexagons, as the Chinese mathematicians Fu Tsiang Wang and Chuan-Chih Hsiung have proven in 1942.

all, or almost all, topics covered in standard mathematical curricula for primary (and also secondary) school. Also, one should not forget the obvious methods of mathematical experimentation using software (spreadsheets, dynamical geometry, etc.), although they have a backdrop with respect to the hands-on activities: one has to put faith into the computer (e.g. that the computer “knows” what a quadratic function is and that it correctly plots its graph).

N. B. The experimental approach does *not* eliminate the teaching of a mathematical proof. Although there is a possibility that children discover mathematical properties and become convinced of its truth without seeing the need for a proof, it is one of the tasks of the teacher (besides being a coordinator and monitor of the experimental activities) to show that ideas obtained through experience are not yet proven and lead them to the conclusion that a proof is necessary. But, even for discovering the necessity of a mathematical proof the experimental approach is often profitable, because in many cases the children will discover the need – and even the ideas – of a more efficient approach than trying out all the possible cases (even if there are only finitely many of them) and thus arrive naturally to the basic ideas of a proof.

Possible issues

Some teachers – and probably not only them – will now say that, while the discovery approach is a nice idea, it is expensive, it requires too much effort and/or is incompatible with the curriculum and they do not have time or requisites to change to such a teaching style, except perhaps once in a while. Let me give some answers to these objections.

The first objection – expenses – is easily put aside. Namely, all hands-on mathematics activities can be prepared with the minimum amount of money [17]. For most of them all one needs is paper, cardboard, scissors, glue, pencils, small items like pebbles, dice, playing cards, daily newspapers and similar requisites – all of them are cheap and easy to obtain, aren't they?

Secondly, while it is true that discovery oriented teaching of mathematics requires more creativity from the teacher than, say, for chemistry, where it is (or should be) much more obvious how to set up and use experiments, there are enough good and adaptable existing ideas to be found in books [18], [19], [20] and on the internet [21] (like the five ones described in this paper). The effort for the search for ideas and their adaptation for use in the classroom is far less than the obtained effect (after all, once developed, the lesson-plans can be used over and over again). Also, the overworked teacher may even find that suddenly mathematics is fun. Yes, even for the teacher. And one should not underestimate the effect of a teacher who enjoys what he or she is teaching. As Jerry P. King says² in the book *The Art*

² Speaking of the math classroom, King says: *The third group in that chamber had only one member: the teacher. This person had three characteristics which we found were more or less common to the secondary school mathematics teachers that were to follow: he did not like mathematics, he did not understand mathematics, he did not believe mathematics important.* Further,

of *Mathematics* [24]: one of the important reasons why pupils do not like mathematics is that many mathematics teachers seem not to like mathematics. Still, one should be cautious: not every “fun” mathematical problem is suitable for the use in a classroom, and many ideas have to be carefully adapted for the use in the classroom.

The third point is a touchy, but often heard one. Most of the ones objecting to the inquiry-based concept of teaching say that the usage of the discovery method, this is a good, but time-consuming illustration of a topic and it will take away the time needed to “explain the theory”. Note that inquiry-based teaching is not an addition, but a replacement of the classical *method* – it is not a change in the content. In this approach, “theory” is just a conclusion, a clear statement of facts discovered by the pupils themselves. Many [25] (not only constructivists) agree that a child learns better – in the sense of understanding, and not just memorizing of facts and learning of formal manipulations – if it is actively involved in the learning process. This is particularly true for younger children, as it is a commonly known fact that children are concrete learners until around age of twelve. Another advantage is that with this style of teaching is much easier to respect the differences in learning style and talents. Also, a general tendency [26] in mathematics and science teaching is to develop curricula and lessons that develop the problem-solving skills, and discovery-oriented teaching clearly supports the development of these.

Instead of a conclusion...

... a question for teachers:

If you have the choice between a teaching method that requires

- 1) little effort, since you and everybody uses it since ages, although it has proven to be deficient and uninspiring both for the teacher and the student and
- 2) some effort, mostly because it is new and there are not yet many prepared teaching materials for this one, but promises better learning outcomes as well as more fun for both the teacher and the student,

which method will you choose?

We end this paper as we begun – with a quote from the famous Hungarian mathematician George Pólya (1887–1985):

Mathematics presented in the Euclidean way appears as a systematic, deductive science; but mathematics in the making appears as an experimental, inductive science. Both aspects are as old as the science of mathematics itself. But the second aspect is new in one respect; mathematics “in statu nascendi”, in the process of being invented, has never before been presented in quite this manner to the student, or to the teacher himself, or to the general public. (George Pólya, *How to solve it?*, 1945).

describing how the pupils came to this conclusion, King says: *He taught mathematics on weekdays with less enthusiasm than he showed on Saturday when he mowed his lawn.*

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Natural learning of mathematical terms in preschool

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Abstract. Based on the theory of social constructivism, on the importance of the environment quality and on children's self-motivation for learning by doing, the paper shows the simultaneous development of the preschool children's understanding of the following terms: amount, number, weight and their relations as mathematical categories. In the preschool "Dječji svijet" in Varaždin, data on children's natural, practical learning of basic mathematical terms in social interactions with other children and preschool educators was collected and interpreted during the period of two years by means of the ethnographic method. This was done by means of exploring the functioning of scales and by solving practical problems that occurred spontaneously during the process of exploration. Subsequent continuous joint reflection on ethnographic data by educators and a pedagogue enticed professional dialogue which enabled setting up and applying changes in the material offer for the children's exploration and problem solving, as well as changes in the socio-pedagogical atmosphere of educational groups. The assumption that in a high-quality learning environment the existing children's experiences will be amplified and new experiences, ones which would help children better understand the basic mathematical terms, will be gained and thus make a holistic impact on all aspects of children's development, was confirmed. The observed results show direct connection between the quality of (social and physical) environment and the quality of children's learning and understanding of mathematical terms. The materialization of children's thinking and understanding of mathematical terms in action, as described and interpreted in this paper by means of a whole range of practical examples of children's learning, shows that the environment has a positive effect on the learning process.

Keywords: preschool child's learning (process), basic mathematical terms, learning environment, understanding mathematics in action

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Supporting development of mathematical skills of pre-school children through projects

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Abstract. In developmentally appropriate programs, mathematics has an important place. Value of mathematics can be seen in its presence in everyday life. Mathematical concepts such as numbers, measurements, spatial relations etc. are close and accessible for pre-school children. Considerately, mathematical activities can be realized during pre-school period, so activities for supporting development of mathematical skills have been designed within children project named "Space". The project, which lasted for three months, has been an opportunity for supporting children's development, including mathematical skills, as it has showed. The aim of this paper is to present activities, complementary element of project, which supported development of children's mathematical skills. Children have been observed and narratives about their engagement in activities have been made, and mathematical tasks according to their developmental abilities have been realized. Children gained mathematical concepts through games, different activities and tasks, and autonomous research.

Keywords: mathematics, mathematical skills, pre-school children, children's project

Introduction

Mathematics is an essential part of everyday life. In appropriate development programmes it has an important place. Slunjski (2006) states that development of initial mathematical concepts with children needs to be in accordance with development features of an individual child and features of the learning process of a pre-school child. Mathematical concepts such as numbers, measuring, spatial relations etc. are close and accessible to pre-school children. In practice this means that teaching mathematics starts from child's experience with application of concrete materials that children will get to know (firstly tactile and visual), play with them and then compare, count and classify. . . Čudina-Obradović (2002), Čudina-Obradović and Brajković (2009) state that child's activity is very important

in a project. In the course of the project children make decisions, discuss, explain and justify their ideas to their peers, draw conclusions from data, make notes, make suggestions and remarks and take responsibility for their work and achieved results. Since mathematical activities can be carried out in the course of the pre-school period, the activities have been created that motivate mathematical skills within the framework of the project "Space". In the course of three months children acquired mathematical concepts through games and independently do research tasks.

4. Beginning and aim of the project "Space"

The project started in an older educational group when a boy received a telescope as a gift from his parents. Every day he was talking about stars, the Moon, the Sun with great interest and enthusiasm. He also brought various encyclopaedias on space in the group and other children participated in their thorough research. In no time we visited the library, borrowed some picture books and books, introduced parents with posters on Solar system and stars. The children's living room turned into the "space". By following the children's interest the project "Space" was created with the main aim to meet the children's interest in space, planets of the Solar system, space ships, astronauts and to motivate them to put questions, research, think independently, draw conclusions, widen their knowledge of space. The paper deals also with mathematical skills and abilities that children achieved in the course of the project. Kirsten et al. (2004) state that mathematics is the basis of technological progress taking place in the world today, starting from the industrial revolution over space research to the era of telecommunication and Internet. This children have to be offered mathematical literacy. Kindergarten teachers should help children become aware of the function of mathematical concepts and develop mathematical competence in direct interaction with the world around them. Kirsten et al. (2004) state that early experience in mathematical concepts would fortify the basis of mathematical thinking and good basis would enable children to apply mathematical knowledge in practice and successfully fit into technologically complex global community.

5. Material gathering and initial activities

Children's curiosity about space grew together with the development of the project. Besides books they searched for answers through autonomous tasks such as observing phases of the moon comparing them in colour, size, distance from the Sun and the Earth. During the project a scene play emerged under the title "Sun's family" and shadow theatre (for which it was necessary to make puppets-the planets and obtain a plexiglass plate and a spotlight). The scene play "Sun's Family" was performed at an international meeting of kindergarten theatre groups in Podgorica, Montenegro, with the help of a father a visit to Planetarium and Technical museum in Zagreb was organised. Gradually the families joined the project and one of the

grandfathers helped us build a rocket. Children would bring into the rocket different items such as telephones, calculators, different boxes to make control board and write letters and numbers. The boxes were used to make robots, different space ships and the scale model of solar system. During these activities children learned about spatial relations, developed and spotted directions in the space and the concept of distance, connected items and notions and cause-effect relations between them, developed principles of counting and geometrical shapes, researched, independently solved problem tasks and acquired knowledge.

6. The course of the project

As the project developed it was possible to group the activities into the research-cognitive group, activities of speech and literacy development, artistic-creative activities, musical, mathematical, physical, manipulative and social games. It was very often difficult to group the activity to only one area since they often entwined. As for mathematics the following activities were included: counting, comparing numbers, introducing spatial relations, comparing and classifying objects.

I. Counting:

1. Mechanical counting
 - a) Recognizing numbers and their sequence
2. Counting by grouping
 - a) Understanding the sense of counting
(Game example: *Who will gather more stars?* — Fluorescent stars are put in different places in the room. The room is darkened. Children sit in a circle and every child throws a big die and gathers as many stars as the die shows. At the end when all stars are gathered every child counts their stars. The child with the most stars wins the game.)
3. Recognizing numbers and grouping according to the quantity
 - a) Numbers as an amount of items
(Activity and game example: *Stars* — making cookies, which includes measuring, scaling, mixing, shaping. Children are offered a recipe, ingredients, digital scale, a bowl, roll and star moulds. One child reads a recipe, the second measures, the third mixes all ingredients and they together make a pastry and cut with a mould stars, take them to the kitchen for baking and later on eat them and offer to their friends.
Measuring robot — measuring activity, different size robot toys are offered from small to big, Lego blocks for measuring, paper, colours, pencils. Next to the robot the children put together Lego blocks and then count them together to check how tall the robot is. The child writes the number of blocks next to the robot drawing.
How many stars are there in the sky? — work sheets, attaching numbers, spatial relations. Groups with different numbers of stars are drawn. The child should write the appropriate number next to the group.

Draw as many stars as it says! — colour pencils, pencils are offered and the child should draw the exact number of stars in the bubble.)

II. Comparing numbers

a) Equal, one more, one less

Comparing the size of the group, children notice the differences.

(Game example: *Space-domino* — coloured picture cards presenting a rocket or two planets or three stars or one sun. It is played as domino.

Memory — stars of different colours and with a different number of points; child should find pairs, it is played in pair.)

b) Comparing numbers “in head”

Without counting fingers and items a child should say which number is smaller or bigger.

All these skills and knowledge children acquired in the course of the project by moving around the space, handling the objects, comparing them and discovering their features and comparing quantities. Parents took part in children's research and offered new contents, activities, their help and support. The aim of the prepared work sheets was to focus and keep children's attention, understand the relations among items, comprehend drawings and numbers as a substitute for real items and relations in the environment, understanding and memorizing numbers and their connection to quantity, gradual adjustment to independence and persistence. Work sheets motivated children's discussion in solving the tasks. Their work and endeavour, persistence in solving the tasks were regularly praised.

III. Looking into the spatial relations

During the projects the children learned about spatial relations, relations among items, their features.

(Activity and game example: *Planets are above us* — observation by telescope, experience in Planetarium. Planets closer to and further from the Sun and the Earth — observing the distance in picture books, books, encyclopedias, posters, finger measuring, ruler and paper tape measuring, children think of the ways to determine the planet distance, they also notice that some of them glow more than the others, their colours change: from yellow to red- asking whether the red planets are warmer. Planet orbit — planets are rotated on the black circle around the Sun and their orbit is observed, the scale model made of collage paper and carton with a nail in the centre is made.

IV. Comparing items

(Activity and game example : *Planets* — cut pictures of planets, planets made of styrofoam ball, comparing their sizes and put in order according to their size from smaller to bigger and vice versa, styrofoam is painted in appropriate colours of the planets and paper planets are stuck on paper, they make scale models, put them in order according to their distance and sequence.

Day and Night — motion game, one child gives instructions and other children listen and stop moving if it is day or squat when night.

In posters and books children notice features of the planet : warm or cold, big or small, red or blue, with a ring or without it.

Solar system-oppositions — motivating children to list opposite features in solar system (small/big, near/far, warm/cold, left/right, up/down, high/low, day/night...)

Stars of different size — children make ascending and descending sequences according to minimum differences for features required. Stars are made of paper and plastic.)

V. Classification of items

a) Categorizing and sorting

Children find common features of different items and ways to categorize and sort them (planets, rockets, aliens) and they also sorted geometric figures.

(Activity example: *Rocket* — work sheet with the task to colour triangles yellow, circles blue, rectangle green, the rocket is drawn out of geometric figures parts.

Stars — find the pair, stars are of different colours and with different number of points, children find pairs, play in pairs and the pair that collects the most wins.

b) Pairing and joining

(Activity and game example — *Electric circuit* — joining items of equal and pertaining features (astronauts with rockets, stars and sky, astronomer with telescope, space concepts with letters — child connects pictures with words, the light glows or connects the picture of the number with the star or planet number)

Conclusions

Kirsten et al. (2004) state that mathematics imbues the whole human experience. In every culture mathematical concepts are used in everyday life. Mathematics is the basis of stunning technological progress taking place in the world, which makes it necessary for our children to offer them mathematical literacy and good foundations for mathematical thinking, good foundation for applying mathematical knowledge in practice and fit into the technologically complex global community. Main assumption of the project is that children observe, question, research and take specific activities connected to the topic and think independently, draw conclusions and widen their knowledge activating thereby their potential and developing numerous abilities. Working on the project on space I learned and researched together with children. I was often surprised by their questions and ways they arrived at conclusions and how focused they were in research and the problem. After having systematized the activities I became aware of how many areas of child development one project can include and of the mathematical activity significance in child development.

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Real Objects and Problem Solving in Mathematics Education

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Abstract. The manipulative tools are very important because they make abstract mathematical concepts possible to understand through concrete activities.

First Bruner and Dienes tried on the children of their university colleagues the application of such tools with which one can discover algebraic connections. Due to their success and research the application of manipulative tools became generally accepted.

Hypothesis:

There are children, also very talented ones, for whom mathematical plays and mathematical contents within the plays are inaccessible because of their basic deficiencies.

We examined our hypothesis with qualitative methods.

We disclosed the deficiencies during developing classes and we constructed didactic methods to cease them.

1. Non-mathematical plays to acquire experience relating to values of dice and cards
2. Mathematically relevant tasks to be done by objects

The application of tools made to help learning Mathematics (Dienes set, coloured rods, logics set) really does not require background knowledge of Mathematics from the students, but quite a high level of symbolic thinking is needed. This is usually acquired during family- and kindergarden games by children, however, there are exemptions, too, first of all socially disadvantaged children. We carried out our observations among children from 8 to 10 years old.

Manipulative tools and mathematics education

Using manipulative tools has a long history in mathematics education. Now we can use virtual manipulative tools. For example, see http://www.ct4me.net/math_manipulatives.htm. Physical manipulatives, however, have not lost their importance.

How does one use manipulative tools to help better understand mathematical concepts and to develop problem-solving skills? Piaget (1962) noted the role of objects in mathematical learning; Vygotsky (1962) stressed the relationship of learning to society. Cole (no date) demonstrated how, in practice, these views are not in conflict: they can be combined to an acceptable degree.

Bruner's *Learning Theory* (Bruner, 1966) was enriched by important new ideas in mathematics, both being didactic – that new knowledge is arranged in two ways (Bruner, 1991). The first is the narrative mode, which addresses the history of knowledge acquisition; the other one is the logical approach, which is based on a logical categorization. Equally important, it can be said, is the theory of modes of representation (1966). Bruner proposed three modes of representation: enactive representation (action-based), iconic representation (image-based) and symbolic representation (language-based). The parallel processes of learning formulated by Paivio and Clark (1991) correlate a dual coding theory. Visual and verbal information complement each other.

Bruner's theory was refined by Nakahara (2008):

- S2. Symbolic representation
Representations used in mathematical notation, such as numbers, letters, and symbols
- S1. Linguistic representation
Representations that use everyday languages such as Japanese or English
- Illustrative representation
Representations that use illustrations, figures, graphs and so on
- E2. Manipulative representation
Representations such as teaching aids that work by adding the dynamic operation of objects that have been artificially fabricated or modeled
- E1. Realistic representation
Representations based on actual states and objects

These levels, having been determined by Bruner and Nakahara, show the nature of representative modes, but not at the level of problem-solving. One can also present difficult problems for students in the frame of realistic representation. Bishop (whose research-interest involves the social and cultural aspects of mathematics education) paid more attention to models; he said the real world has a mathematical model, and every mathematical concept has a corresponding physical correlate (Bishop, 1988).

Hypothesis

There are children, often quite talented, for whom mathematical plays and mathematical contents within plays are inaccessible because of their basic knowledge deficiencies. For them, realistic representation is a great aid, as it is independent of any intellectual aptitude.

Experiences in multigrade schools in Hungary and in Kenya

We examined our stated hypothesis with qualitative methods. Photographs of the work, educational resources, and teachers' and students' written reflections are part of our research; they can be viewed at the conference website.

Our goal was to help disadvantaged students play these math games because we wanted to offer the same experience for unprivileged students than were offered to students from middle-class families. Studies were carried out for students 8-9 years of age.

Method: Real objects as manipulative tools

- a. We wanted to show the symbolic value of certain objects. We used, dice, cards, etc. We also at times utilized drawing and writing as symbolic activity.
- b. Many of the learning tools were used to help in the understanding of mathematical concepts and relationships. We did not use standard educational tools, although they are important. We believed tools that are designed to promote the abstraction of mathematical concepts require a relatively high level of symbolic thinking, yet we wanted to build on foundations. Our tools were familiar to the children: for example, certain objects are found in the kitchen.

The common plan of the experimental lessons:

1. Presentation by computer
2. Problem solving and hands-on activity through real objects
3. Story telling: *What happened at the lesson?*

Examples

Our tested presentations

1. Glasses, and about measuring
2. Excursion, and practicing spatial orientation
3. In the past: of Egyptian number writing and first steps to proofs
4. Traveling, and about data handling
5. Polyhedrons

Glasses, and about measuring

Aim: To learn how to measure and compare objects while involving concepts of capacity. Our goal is to teach children how to connect their everyday life experiences to mathematics. Everyone feels, e.g., one needs a longer time to fill a container by a small glass than by a bigger one. In this unit, we translate our experiences into the language of numbers. Necessary tools: Water or sand, bowls, pots, small and larger glasses. Requirements: To get qualitative and quantitative experiences. Evaluation: The beginner level is to know that a longer time is needed to fill a container by a smaller glass than by a larger one. The advanced level is to understand how to solve word problems on the topic of measuring.

Excursion, and practicing spatial orientation

Aim: To move spatially in three directions. Our aim is to use words to describe the directions. Necessary tools: Clay and sticks, chairs (as cars), and a little space to draw a simple junction for 'cars'. Requirements: Describing direction. Evaluation: The beginner level is to know which arm is the left and which is the right. The advanced level is to use directions properly even when in motion.

In the past: of Egyptian number writing and first steps to proofs

Our aim is to offer experiences for using new symbols and to help children develop a deeper understanding for number writing. Necessary tools: Paper and pencils (or clay or sand) for drawing. Requirements: Using simple symbols, stating "Why?" questions. Evaluation: The beginner level is to read the numbers written in the Egyptian number system. The advanced level is to multiply numbers by duplication.

Traveling, and about data handling

Our aim is to show simple examples of applied mathematics by collecting data (or producing data by rolls of the dice). Also, we intended to help students understand how to better solve word problems. Necessary tools: Timetables and price lists for traveling (or an expert who can give such information), dice. Requirements: Using numbers to solve everyday problems. Evaluation: The beginner level is to put data in the right place in the tables. The advanced level is to pose questions and estimate fees; and to calculate everything correctly on the basis of collected data.

Polyhedrons

Our aim is to help students understand the concept of polyhedrons. Necessary tools: Clay and sticks, and boxes, bricks, and stones. Requirements: Describing direction through the physical objects of cube and brick. Evaluation: The beginner level is to count the edges, the peaks and faces of a shape. The advanced level is to solve more difficult problems specific to polyhedrons.

Conclusion

Students were happy to solve their tasks. For us, it was surprising that some were not previously aware of the dice and hadn't played board games. The activities gave students a lot of experience in using symbols. We started to play further games using math teaching tools, and the educational experience for the students was enhanced.

Each student progressed in terms of their math skills; among them, the more gifted students were able to solve more difficult problems.

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Adaptation of teaching units in mathematics for pupils with disabilities in the regular school system

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Abstract. Children's rights and inclusive education is the topic that has the ability to increase the significance, power and rights in education. (Hart, Š., 2001)

Modern governments take the responsibility for the education of all children. Special attention is given to the abilities and needs of children through individualized approaches, adapted or specialized programs together with the indispensable role of the parents of all children and the active participation of class mates at school in order to promote the right to curriculum¹, which is focused on the pupil (Garner, Dietz, 1996).

For the last thirty years, adapted programs and individualized practice in Croatian schools has been created and applied in different ways.

“Adapted program is a program appropriate for basic characteristics of the child's disability, and usually involves reduction in the intensity and extensity when choosing teaching contents that are enriched with specific methods, means and aids. Adapted program is prepared by a teacher in cooperation with the defectologist of appropriate specialization with permanent or occasional involvement in rehabilitation programs of specialized organizations” (Regulations on elementary education of students with disabilities, NN, 23/1991).

Every adjustment which serves its purpose, and that is to give opportunity and possibility to a child in an inclusive class with its peers, through individual approach, to develop its knowledge and skills, and to make him/her competent for the next level of education or employment, is a good adjustment.

¹ Curriculum is organized arrangement of the learning process and content according to the specific goals and objectives (Knoll, J. accPastuović, 1999.) The curriculum is the developmental pedagogic-educational cycle, which has its own laws and governs concrete learning activities answering the questions: What, Who, the what, how, in what time, how effective, how effective for the children.

In this paper, nine ways of adjustment will be described (Deschenes, Ebeling and Sprague, 1994). Also, types and examples of adjustments that can be implemented in mathematics will be explained.

Planning in the educational system has not yet reached a satisfactory level so changes in planning of work for children with special educational needs are necessary.

Keywords: nine ways of adjustment, mathematics, students with disabilities

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Play – a way towards mathematical competence

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Abstract. The methodic of initial maths instruction is a scientific discipline which studies mathematical upbringing and education in the first four grades of primary school so in it learners acquire the initial mathematical literacy. It is characterised by content of high level of abstraction and most commonly learners can't realize it through experience. At this age child is capable of logical thinking, but only if the thought is based on the action with concrete objects. Thus, in our schools we find ways of thoughtful and efficient unification of learning and mathematical play because games attract learners, motivate them and develop their abstract thinking and logical deduction. By learning maths through play a child acquires abilities and knowledge of mathematical content and has fun at the same time. In didactically mathematical games diverse operations from multiple areas are united although reflective operations should be given priority so learner's wit, acumen, combinatorics and creativity are seen. The content of play is extracted from the maths curriculum so we can conclude that its effect is a sign of successful realization of the same curriculum in the initial maths instruction. In the design and realization of didactical play in the maths classes role of the class teacher is significant and in that process his or her creativity, systematicity and inovativeness comes forth.

In this work didactic games which are most often used in the maths classes are presented and also teachers' attitudes towards application of didactical play in the maths classes.

Keywords: initial maths instruction, role of didactic play, class teacher

Role of didactic play in the initial maths instruction

Methodic of initial maths instruction is a discipline which studies mathematical education and upbringing in junior grades (the first four) of primary school and it possesses features which differentiate it from the following levels of mathematical schooling. In the initial maths instruction educational contents are taught

through planning and organisation, often without immediate contact with reality, but, although a child at this age is able to think logically, that thought still has to be based on the activities with concrete objects. In the initial maths instruction educational achievements are related to spotting processes, initial understanding, differentiation and nomination of fundamental concepts which form the contents of maths as a subject and on which maths instruction on higher levels is grounded. Maths instruction is mostly directed towards the abilities of solving mathematical operations and in doing so it neglects “the crucial abilities such as abstract thinking, logical reasoning, solving problem tasks, the ability to interpret graphic displays” (Đuranović, Klasnić, 2009). By learning maths on concrete objects and solving real problems pupils will acquire knowledge and skills which allow them to successfully implement maths in daily life and, by doing so, appreciate the importance of maths in their own lives. Mathematical competence relates to pupil’s qualification for developing and applying mathematical thinking in problem solving in a number of everyday situations. Thus, they will become active participants in the learning process and also prepared for learning throughout their lives. In Croatian contemporary school we hope to take care of mathematical competences development because research from 2008. (Baranović and others, 2008) has shown that the biggest accent in maths instruction is on solving mathematical exercises and development of logical thinking, less so on the use of maths in everyday life and the development of critical thinking about mathematical conceptions and actions.

According to Glasser’s *Quality school* successful school work meets four basic human needs – knowledge, power, love and fun (Glasser, 2005). Fun is pleasant, joyful and appealing for pupils and it is easily implemented in teaching through play because it creates a positive emotional experience. Marzollo and Lloyd (1972, acc. to Sharma, 2001, p. 124) think that through play pupils learn how to “concentrate their attention, practice visualization, practice adult behaviour and develop the sense of control over its own world”. As a fun activity each game has its own content by which it’s usually named. Since in teaching we mostly study reality, games can be divided according to subjects, so we differentiate between language, music, technical, historical, geographical and mathematical games. Mathematical games instigate mathematical thinking and mental activity. “Although maths is not a game, many games are related to maths and many mathematical topics can be described and explained in a fun way, often through play” (Bruckler, 2010, p. 57).

A game can be created according to the activity areas – sensory, manual, expressive or reflective. According to Jurašić (2010), some mathematical games encourage logical-mathematical intelligence development (numbers) and some the development of visual-spatial intelligence (geometry). Didactic play unites several operations from many areas, but the advantage should be given to contemplative operations in order to express pupils’ ingenuity, acumen, combinatorics and creativity. In play “pupil creates models which he/she understands and likes, ones he/she can later efficiently use to understand abstract mathematical ideas” (Sharma, 2001, p. 124). Exactly that holds the specificity of play’s didactic significance. Considering the fact that didactic play derives from teaching content, its task and goal are submitted to aims and goals of a concrete curriculum – acquisition of certain knowledge and development of certain abilities. Some didacticians advise the play

to be incorporated into harder parts of the teaching process. In which part of the teaching process will teacher apply play depends on the teacher's judgement itself, his/her creativity, systematicness and innovativeness. Before playing the game itself, teacher presents the content of the game, explains its tasks, nominates and describes the used tools, demonstrates the execution of the game as a whole and in detail, explains the regularity of individual procedures and warns about the game's rules. Because they are interesting and competitive, didactic games will motivate pupils to verbalize ideas, devise appropriate solutions and strategies in the game and in such a way spend a lot more time ('willingly') with maths and mathematical concepts.

Every game, didactic as well, is executed by rules established in advance. In such a way the relation with the object of the game and other players is regulated and adjusted – correctness, tolerance, fairness, equity. Following the rules of the game contributes to quality socialization in communication. Pupils have to "combine and mentally strain themselves in order to play the game to the end, they have to control themselves, learn how to win a game, but lose as well" (Đurović and Đurović, 1984, p. 25).

The final effect of didactic play is acquisition of abilities and knowledge of the game's content, with the additional acknowledgement, praise and also a high mark for the best. Because the game's content is derived from the curriculum, its effect is the indicator of the curriculum realisation efficiency.

Teacher's role

Play application is a great challenge and requirement for pupils as well as for the teacher. Pupils are expected to be careful and patient, memorise the rules of the game and solve problems. In the use of the game the teacher's role is also great because didactic game should have its justification in the teaching process – for exercising and application of taught mathematical contents and automatising of certain mathematical operations. Didactic play is not introduced into initial maths instruction because of playing, but because of more successful, more permanent and also easier learning. Nevertheless, according to Sharma (2001), most teachers introduce play into their teaching solely as additional activity which serves greater pupils' motivation and relaxation. Apart from the game planning and knowing the moment and the aim for which it will be applied, teacher has to prepare necessary material and plan the duration of the game well.

By using play in maths instruction, through cinestetical-mathematical activity, teacher provides pupils with a break from solving exercises. Games are often competitive, which additionally refreshes the class and motivates pupils into logical thinking and fast reasoning. In planning and preparing didactic play for maths instruction teacher should use appropriate teaching methods, forms of work and teaching tools which encourage pupils to work creatively and cooperatively. The results of research (Arambašić and others, 2005) have shown that teachers should be provided with backup in order to develop their positive attitudes and belief about

maths and they should be directed towards the selection of the most appropriate teaching methods for their students.

According to Sharma (2001), all types of didactic games can be divided into three big categories: manipulative, representative and structured games with rules. In a manipulative game child develops its logical-mathematical experience which becomes the foundation of meaningful learning and intensifies its cognitive growth. In representative play pupil uses the imagination which is an important component of abstract thinking development at older age because he/she “devises games, composes models, and creates a necessary starting point for transition towards the abstract by using touch and visual perception” (Đuranović, Klasnić, 2009). In structured games pupil “learns how to follow a sequence of instructions, directly develops mathematical skills, mathematical and spacial thinking, strengthens the taught mathematical conceptions, actions and facts” (Sharma, 2001, p. 124).

In constructing quality didactic play teacher uses the available literature, various media (for preparation and execution), informative centres, but mostly shows (alongside methodical and didactic knowledge) ingenuity, creativity and experience.

Teachers' opinions about the implementation of didactic play in maths instruction

94 junior grades teachers participated in the November 2010 research implemented in the professional gathering of the County's professional council in Sisak. Most of the teachers who participated in the survey were highly qualified (56 teachers – 60%).

In the first figure it's visible that all the participants in the survey apply play in maths instruction and 70 of them (74%) apply it once a week.

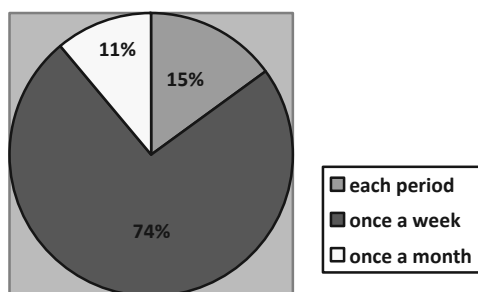


Figure 1. How often do you implement play in maths instruction?

Teachers apply didactic games in equal percentage in the introductory and finishing part of the class, during revision or practise. None of the teachers use didactic play in the main part of the period. 65% of the teachers use mathematical

games without aids and materials more often and 35% state that they use a cube and dice, cards, dominos or other social games.

Games most often applied in maths instruction, according to the frequency of appearance in the survey, are: dice games, napping, dominos, memory, pulling numbers out of a hat, darts, raffle, bingo and jigsaw.

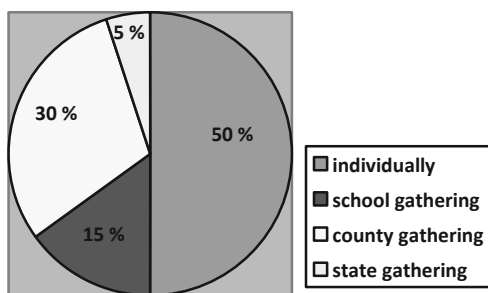


Figure 2. Teachers' opinion about the level of professional training which they consider will strengthen their mathematical competencies

According to Figure 2, teachers consider that individual professional training will help them in strengthening mathematical competencies and application of didactic play in teaching the most (47 teachers). Professional training on the county level follows (28 teachers).

In the questionnaire teachers had the possibility to write what they think of the implementation of didactic play in the initial maths instruction. Here are some of their opinions:

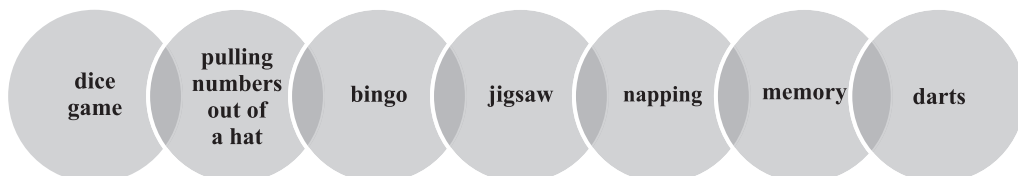
- ✓ Mathematical games are useful because they develop pupils' creativity, games help pupils memorise things faster and easier.
- ✓ YES for games because it's been proven long ago that play gives the most quality learning and long-lasting memory.
- ✓ Interesting games motivate pupils for (new and old) teaching material, give self-confidence and the possibility of active participation for each pupil.
- ✓ Didactic mathematical games aren't recommendable – they are NECESSARY!
- ✓ Mathematical games refresh the teaching and connect maths to real life.
- ✓ Didactic games help the student accept maths as something real and useful.

Examples of chosen didactic games

Some of the most often used didactic games in maths instruction by the junior grades teachers in Sisak are described here (Figure 3):

1. Dice game
2. Pulling numbers out of a hat

3. Bingo
4. Jigsaw
5. Napping
6. Memory
7. Darts



Slika 3. Didactic games

1. Dice game

Pupils throw a dice and call out the number of dots. If they have two dice they can practise addition and subtraction, and if they have three or more dice they practice addition or multiplication with two dice.

2. Pulling numbers out of a hat

Pupils call out the pulled numbers. If the numbers are three-dimensional, first they recognize them with their eyes closed (sense of touch). Then they put the numbers on the blackboard. In combination with the numbers that follow, mathematical exercise is created (numeric phrase or mathematical story).

3. Bingo

Bingo is a game similar to lotto. It's mostly implemented in the first or second grade because of the numeric sequence to 20 and later with numbers to 50 or 100. It can be played individually or in a group.

Individually – material: notebook, pencil, number cards

Course of the game: each pupil draws a table with six fields and writes a number in each field. Numbers mustn't be repeated twice in the same table. Teacher pulls out number cards and calls out the number and pupils, who have it in their table, cross them out. The pupil who crosses out all 6 numbers calls out "BINGO" and he/she is the winner.

In a group – material: blackboard, chalk, number cards

Course of the game: class is divided into two groups. On the board they draw two tables with 6 fields. Pupils from both groups say the numbers which teacher writes on the board. Pupils from each group pull out the numbers alternately and the pulled numbers are crossed out in both tables on the board. The group which crosses out all the numbers first is the winner.

4. Jigsaw

There is an exercise in each square. Exercises are supposed to be calculated, put on the solutions carton with the result facing the carton. On the back of each card there is a picture. If all the exercises are properly calculated, the correct drawing is composed in the end.

5. Napping

Pupils sit with their heads on the desks ('napping'). Teacher walks and pronounces the numeric phrase. Pupil that is touched by the teacher has to repeat the phrase and say the result. If it is correct he/she lifts up their head (wakes up).

6. Memory

Exercises are written on cartons in two different colours – on one side the exercise, on the other the solution. Cartons are placed facing down. Pupils flip the different colour cards. If he/she flips the cards with both the exercise and the solution, he/she takes them and then flips a new pair. If he/she doesn't flip the right combination, he/she returns the cards where they were. The game is played in groups of 3-4 players.

7. Darts

Five concentric circles are drawn on the board or on the placard. (Circles can be divided by vertical lines so they contain more different fields) In each circle (field) one number is written. Pupils stand in two single files in front of the circles at the same distance and throw chalk at the target. They add/multiply the numbers they hit. File that has the most points wins.

Conclusion

Fundamental goal of maths instruction is to create a mathematically literate, competent individual who is able to apply maths in everyday life. Considering the fact that teaching contents are abstract from the very beginning, quality methodical processing and constant connection between mathematical contents and life's reality is important. Because it connects learning and fun, motivates pupils, develops abstract thinking and logical reasoning, mathematical play is often used in teaching for acquisition of mathematical content. In that, teacher's role in devising and conduction of the game in the initial maths instruction is great because didactic play should have its justification in the teaching process. Learning enrichment and motivating pupils to use mathematical games develop their mathematical abilities. In maths instruction games with rules are used more often because they require pupils to memorise both the content and the rules of the game, but also many social skills tied to winning and losing a game.

The research conducted with the junior grades teachers in the County professional councils of Sisak has shown that teachers apply didactic play in maths

instruction at least once a week. They implement it for revision and practising in the introductory or final part of the period. None of the teachers implement the play in processing during the main part of the class, so it is necessary to motivate teachers to find, devise and use didactic games which will help pupils in acquisition of new mathematical content and also in developing mathematical competencies. Of all the games teachers most often implement in teaching, the authors of the paper selected and described several: dice game, pulling numbers out of a hat, bingo, jigsaw, napping, memory and darts.

Teachers who participated in the pole mostly consider that individual training will help them in strengthening mathematical competencies and application of didactic play in the initial maths instruction. In that, their creativity, experience and ingenuity will come forth.

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Didactic games in mathematics teaching

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Abstract. Game is basic child's activity and need which affects the overall development of a child. Didactic games are group of games which are used in educational process with the aim of achieving previously determined educational tasks. They have a positive effect on teaching results, permanence of knowledge, pupil's motivation, attention and activity, and make learning more interesting. Modern school which uses didactic games as a method of teaching is focused on needs and nature of child. Didactic games in mathematics teaching are used for introducing, practising and revising mathematical content.

Keywords: game, didactic game, mathematics, mathematics teaching

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Discovery learning in mathematics by using dynamic geometry software *GeoGebra* – action research

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Abstract. It is necessary to get involved actively in the process of acquiring knowledge to learn mathematical contents successfully. The theory of constructivism emphasizes student's individual activity in which he/she creates his/her own knowledge through his/her own experience, while the teacher prepares and shapes the environment for learning and encourages and guides students in their independent learning. The aim of this approach is for students to learn how to learn, and Polya's problem-solving model provides an answer how to perform in practice – in 4 steps: understanding the problem, devising a plan, carrying out the plan and reflection.

In teaching practice, this can be achieved by using discovery learning or inquiry based learning. In mathematics, dynamic geometry software can be of great help because of its possibilities to display the dynamism and interactivity of mathematical objects so it provides a dynamic virtual environment for research. Among such programs *GeoGebra* is distinguished because it's free and open source, translated into Croatian, intuitive and easy-to-use tool, combining elements of geometry, algebra, analysis and statistics. Moreover, the community of its users on the Internet is large and very active so there are more and more completed examples that can be used in everyday teaching.

There were various problems the author encountered in her teaching practice when trying to implement the ideas described above, so she opted for action research which seeks to respond to the problem: how to help students to discovery learn new mathematical terms, concepts and ideas individually by using computer and *GeoGebra*. This article describes the research and presents a report on the achieved action research with second grade students in High school Čazma in Čazma.

Keywords: action research, *GeoGebra*, Pólya's problem-solving model, constructivism, discovery learning, dynamic virtual learning environment

Introduction

One of the aims of teaching mathematics is to teach students how to think and train them for solving problems and making decisions in their future life. Kurnik (2008, p. 53) stresses that “*the basic guidelines for the modernization of teaching are inducing and starting students’ thinking and ensuring that a great deal of new knowledge is acquired by their own strengths and abilities.*” (free translation by the author) The theory of constructivism emphasizes this approach, it is based on the fact that knowledge is constructed by pupils’ activity and therefore the role of the teacher as a source of information decreases significantly in relation to the role of the teacher who will lead and direct the students in their learning. Knowledge cannot be transmitted from the teacher to a student in a way that information from the teacher is copied and pasted into in student’s head. The student must be involved actively in learning so he/she could reach his/her own insights by individual work. The way of learning is unique for each student because he/she constructs his/her knowledge on his/her own activity based on personal experience. A teacher’s role is to choose appropriate teaching forms, teaching methods, sources of knowledge and thus to shape the environment for learning and to encourage and guide students in their independent discovery of new terms, concepts and principles in the contents they are learning.

Teaching models which match up the requirements above are discovery learning and inquiry based learning (Bognar, Matijević, 2002). **Discovery learning** is an experiential learning that takes place in reality or uses a simulation. The simulation by dynamic geometry software is being used in teaching mathematics increasingly because of its possibilities to display the dynamism and interactivity of mathematical objects so it provides a dynamic virtual environment for the research. Among such programs *GeoGebra* is distinguished because it is free and open source, translated into Croatian, intuitive and easy-to-use tool, combining elements of geometry, algebra, analysis and statistics. Moreover, the community of its users on the Internet is large and very active so there are more and more completed examples that can be used in everyday teaching.

Main feature of **inquiry based learning** is to encourage students to come to certain findings by individual exploring with the appropriate assistance of teacher. It allows a high level of differentiation in teaching because teacher will help each student as much as he/she needs. The student will discover the meaning and relevance of information individually which will lead him/her to a conclusion or reflection on the newly attained knowledge. The aim of this approach is for students to learn how to learn, and Polya’s problem-solving model provides an answer to how to perform in practice – in 4 steps:

1. **Understanding the problem** – read the task with understanding, student can try to answer the following questions: *What is the unknown? What are the data? What is the condition?*
2. **Devising a plan** – perceive pattern (rule, formula, theorem) that connects the known and unknown variables, more complex task divides into several easier.
3. **Carrying out the plan** – perform each step carefully.

4. **Reflection** – checking the results, answering questions *Can you derive the result differently? Can you see it at a glance? Can you use the result or the method for some other problem?* (Pólya, 2004).

More about how Pólya's model of problem solving is installed in *GeoGebra* dynamic learning environment and within students' exploring and experimental learning can be read at Bjelanović Dijanić (2010) and Karadag and McDougall (2009). The following is an excerpt from the report of action research that the author carried out with her students as they learned just to the ideas described above.

Context of research

Action research participants were second grade students in High school Čazma (there are 20 in the class 2c) and me, Željka Bjelanović Dijanić as the researcher and their Math and IT teacher. I teach Math in two classes of general high school and I teach IT subject in all grades in our school so my IT classroom is always available for us. There is one main computer (server) that serves 16 clients, LCD projector and screen in the classroom and Internet access is available to all 16 clients. Sometimes I organize students to learn Math individually or in pairs with interactive digital teaching materials.

The course of action research was followed by three critical friends:

1. PhD. Branko Bognar – assistant professor at Faculty of Philosophy in Osijek, dealing with themes related to action research¹, also a mentor for the methodological aspect of this research;
2. Šime Šuljić – Math teacher at Gymnasium and vocational school Jurja Dobrile Pazin, engaged intensely in using computers in Math teaching, developing interactive digital materials and regularly following the development of cognitive tools in the world;²
3. Željko Kralj – the principal of High School Čazma and Math teacher with a 30-years experience showing great interest in my effort to introduce changes in Math teaching.

The problem and research plan

The action research is based on several educational values: **independence** in learning, the **activity** of all students, **collaboration** between students, **motivation** for learning mathematics and **computer-assisted learning**. The full report of action research goes beyond this article so here only a part of the research is going to be presented related to the independence, the activity and computer-assisted learning.

¹ Doctoral thesis: *Possibilities of enabling teachers to realize action research enquiries by using electronic learning processes* (mentor prof. dr. sc. Ana Sekulić-Majurec, Faculty of Philosophy in Zagreb, 2008.)

² Šime Šuljić is one of the translators of *GeoGebra* into Croatian, the author of dozens of professional articles and lectures about using computers in math teaching.

Aims		
1. Training students for individual discovery learning using dynamic geometry software <i>GeoGebra</i> . 2. Development of new and adaptation of existing digital materials needed for individual discovery learning using computers.		
Criteria		
• Students show more independence in learning new mathematical terms, concepts and ideas. • Students come to conclusions individually. • Almost all students are active in the class. • Students visit Web sites with digital materials at home on their own initiative. • Students are more satisfied on Math lessons because they work on computers more often.		
Activities		
For the first aim it is predicted that students discovery learn lecture units listed below independently (two school hours for each unit) in the IT classroom through three cycles:		
1. cycle	Lecture unit	Transformation of quadratic function $f(x)=ax^2$
	Digital material	http://www.normala.hr/interaktivna_matematika/kvadratna.htm
	Data collection	• the questionnaire 2 (scale, open-ended questions) • students' papers – a worksheet and a short test • photos of the class • teacher's research diary • critical friends' notes, e-mail messages
2. cycle	Lecture unit	Roots of quadratic polynomial and its graph
	Digital material	create a new digital material with the help of critical friend Šime Šuljić
	Data collection	• video recording and photos of the class • notes on systematic observation (videotape) • students' papers – written knowledge exam • group interview • teacher's research diary • critical friends' notes, e-mail messages
3. cycle	Lecture unit	Definitions of trigonometric functions
	Digital material	translate and adapt material with permission by A. Meier http://www.realmath.de/Neues/10zwo/trigo/sinuseinf.html
	Data collection	• video recording and photos of the class • notes on systematic observation (videotape) • the questionnaire 3 (Likert scale) • teacher's research diary • critical friends' notes, e-mail messages
Duration: in November and December 2010.		
For the second aim an additional activity is predicted that teacher does independently and continuously: studying references concerning the development of interactive digital learning materials that correspond to the specifics of teaching mathematics. (Bjelanović Dijanić, 2009, Hohenwarter, Preiner, 2007)		

Table 1. Research plan.

Almost all of these values are disrupted in traditional classes where the activity of teacher in frontal teaching dominates. Students are too reliant on the teacher and most of them are not able to solve the task at home if a similar one is not

resolved in the class. Only interested students are active in the class while others are waiting to copy from the board. Computers and the Internet are the media that surround students every day so it is logical to offer a computer-assisted learning as an alternative.

However, the computer is not going to improve the current situation by itself. I even tried to realize discovery learning within computer-assisted learning in previous generations of students but we encountered some difficulties and the results were not improved essentially. That is why I decided to introduce changes in my educational practice and carry out the action research in the class 2c. All of the 20 students said they wanted to participate and their parents signed an agreement statement. I expect that we are going to identify why students have difficulties in individual discovery learning using computer and *GeoGebra* based on systematic observation and critical questioning, and that we all together are going to overcome the difficulties in learning mathematics content with new teaching methods.

Therefore, we stated the problem of action research with the question:

How to help students to discovery learn new mathematical terms, concepts and ideas individually by using computer and dynamic geometry software *GeoGebra*?

A confirmation for the validity of decision to explore previously stated problem using action research, I found in the book “Research Methods in Education” (Cohen, Manion and Morrison, 2007, p. 226) where it is stated that action research can be used to explore methods of teaching – replacing traditional method with discovery method and in developing new methods of learning. According to the classification Zuber-Skerritt (see Cohen et al, 2007, p. 232) this research is emancipatory because the participants (teacher and students) feel responsible for solving the problem presented and will attempt to resolve it through a cyclical process: plan – action – observation – reflection. For being as much successful as we can be it was planned to implement the activities through three cycles as seen in Table 1.

The process of conducting action research

1. cycle: Transformation of quadratic function

The teaching materials that we were used is on the Web of association Normala *Interactive mathematics*³ and have been used with the previous two generations of students. The author of material is Šime Šuljić. Students themselves formed pairs according to who they like learning with. I noticed that each pair consists of students of equal ability and prior knowledge and they progressed at a pace that suits their abilities. They rarely called for a help, the instructions were clear to them. However, in the third exercise most students had difficulties: “*Read the instruction but called me because they did not know what to do (better students). Some understood how to choose the vertex, but did not understand what about the other points.*” (research diary, 15/11/2010) Critical friend Željko Kralj was present at

³ <http://www.normala.hr/interaktivna-matematika/kvadratna.htm>

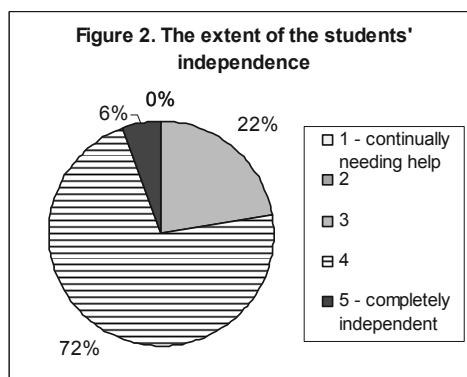
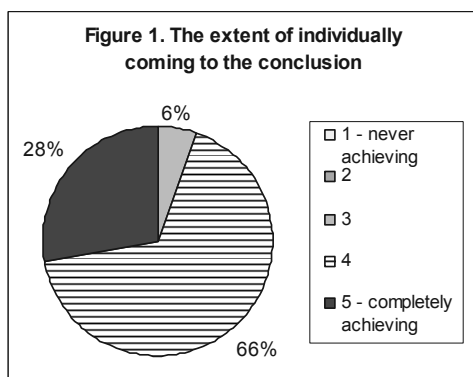
the class and he carefully observed what was happening. After the class he said this way of teaching seems very demanding for the teacher because it requires walking around the students all the time but they don't progress at the same speed at the same time so the teacher should know all the lessons through which they are going through very well. He also noticed some students really did well while another did not understand what to do (research diary, 15/11/2010).



Picture 1. i 2. The computer in the class – frontally or individually (in pairs).

Picture 1. shows how we use a computer and a projector in frontal teaching and Picture 2. shows how students learn in pairs using digital materials. In the e-mail sent on 14/12/2010 Branko Bogar gave his review of these two photos: *“There is possible to see the whole class where the teacher is dominant at the first photo, while at another one students are active. On the faces of students (second photo) it is possible to observe the interest and concentration on the activities they are engaged.”*

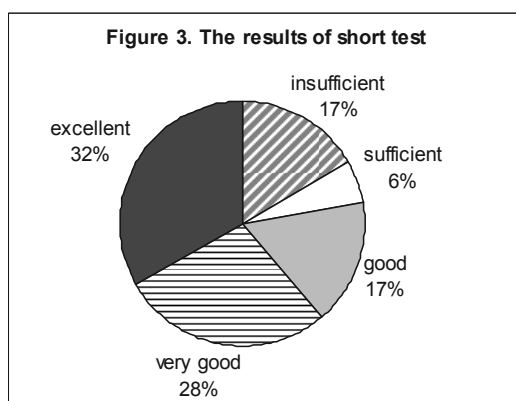
In questionnaire 2. I wanted to determine the extent of the students' independence in learning and reaching conclusions, and if they were visiting the Web site at home.



The students knew the digital material is available on the Internet, but in the class I did not emphasize it would be good to visit these Web pages at home a few more times. Still, 11 of 18 students visited the Web site at home.

Related to the worksheet, all students solved the tasks requiring short answers, and about a half of the students solved the task describing the effect of parameters on the graph. Most students had problems with the task 19 (determining the equation of quadratic function from the sketch) and the task 20 (plotting the graphs of quadratic function) so these answers were analyzed and we solved an example on the board. During the conversation several students noticed the problem: they learned using computer but were examined on a paper (research diary, 23/11/2010).

The analysis of short test containing two tasks of plotting (as in task 20) and three tasks of reading the equations from the sketches (as in task 19) showed very good results and the progress for most students:



After the first cycle reflection I submitted the results to critical friends. Željko Kralj was in contact with me and the class very often, he reviewed pupils' papers and had no remarks except some students should be neater when plotting graphs. Branko Bogonar suggested to try to collect more qualitative data.

2. cycle: Roots of quadratic polynomial and its graph

I tried to make a digital teaching material for this unit myself (two dynamic applets and a worksheet). However, students had difficulties understanding the instructions at the first class in the IT classroom (23/11/2010) so Šime Šuljić helped me redesigning the material. Next time (25/11/2010) they began with the new material⁴ and I recorded the situation in the class with a video recorder. After reviewing the video recording I wrote: *“Teacher walks around the class trying to be discreet, looking behind the students what they are doing and intervenes if notices students do not understand well. The students are working, they rarely call for help, rather turn to neighbors if necessary.”* (research diary, 30/11/2011) We took a few photos:

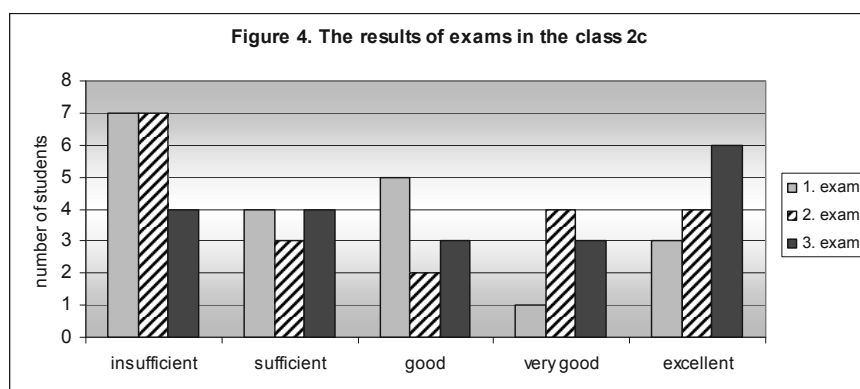
⁴ <http://free-bj.t-com.hr/zbjelanovic/apleti/nultocke.html>



Picture 3. i 4. The students work with the digital material and the worksheet.

Šime Šuljić gave his review of these photos: “*The first photo shows the working material – the applet and the work of students in pairs. Obviously, they discuss their ideas. Schoolgirl right opposite is focused on the problem. On the second photo the students fill in the worksheet relying to the interactive content on the computer. It seems they occasionally check the result on the calculator. The photo tells me the working temperature is high.*”

We did the remaining lecture units in the classroom, several of them frontally with *GeoGebra* on the LCD projector. We should also prepare for the written exam of this lecture topic. The results of the third exam can be compared with the results of the previous two:



In both previous exams pass rate was 65% while in the third even 80%. Besides decreasing the number of negative grades, we can notice the increase of the number of excellent grades. Željko Kralj reviewed students' papers and talked to students. He was very satisfied with the results (highlighted the pass rate of 80%), but cautioned the students who wrote and drew with unsharpened pencil and drew a coordinate system without ruler.

To complete the reflection of the second cycle and to obtain students' reviews of the realization of critical-reflexive practice, we organized a group semi-structured interview.

	Students' answers in the group semi-structured interview
Advantages	• interesting • it's easier to learn because of lots of examples • individually coming to the conclusion is encouraging • it stays in our memory longer • we can visualize what we do, visually remembering more • we can learn at own pace • we can repeatedly pass the same thing to clarify what is not clear to us • learning material on the Web, we can do it at home
Disadvantages	• it is not explained how you explain • the problem with understanding instructions • computer plots automatically, but we need to do it gradually • we should write the most important thing in a notebook
Suggestions for improvement	• it's better to work in pairs, we can discuss with our partner • to have a worksheet so we could write on it • this is better for practice than learning, to practice on a computer after understanding • the sliders should be more precise, not skipping, so we can check our homework • feedback, when we gave the wrong answer to show the correct

Table 2. Group interview (taken from the transcripts and categorized).

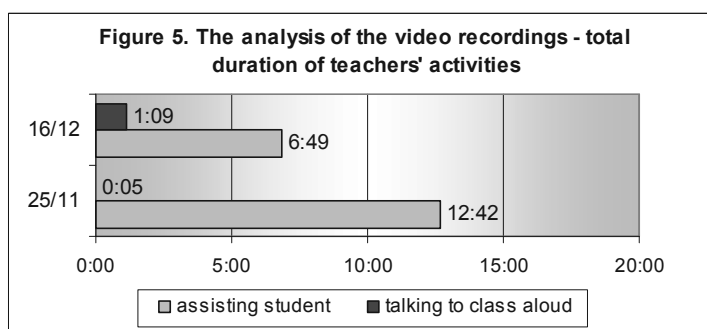
3. cycle: Definitions of trigonometric functions of acute angle

Šime Šuljić and I together adjusted the German material by Andreas Meier⁵ for this lesson. I wrote to him to give us the permission for using and adopting the material in Croatian but he did not respond so we decided to proceed under the Creative Common license (share alike). We adjusted the material⁶ and stated who the original author was. Meier had prepared a lot of interesting interactive exercises but their adjustment would require a lot of time, so we agreed students could practice the original German exercises.

This lesson took place on 16/12/2010 in the IT classroom. The students chose whether they wish to work in pair or alone. We recorded the class with the video recorder and the camera again. I noticed in the research diary (16/12/2010): “We can see the improvement in students’ handling learning, the exercises are motivating and students specially look forward to collecting points. They easily solve the worksheet. There were bugs in some Meier’s applets – warned the students how to avoid them. Most students completed the tasks in 45 minutes so we continued by the workbook.” After reviewing the video recording I wrote: “This time the teacher speaks more to the entire class, and therefore they call for the individual assistance less. The atmosphere is relaxing, students help each other a lot more.” (research diary, 18/12/2010) I carefully reviewed and analyzed the first 20 minutes of both recordings (until then the students have not finished yet) and separated two categories: teacher assists students (individually or in pair) and teacher talks to the class aloud. I counted how many times each category appeared and the duration.

⁵ <http://www.realmath.de/Neues/10zwo/trigo/sinuseinf.html>

⁶ <http://free-bj.t-com.hr/zbjelanovic/apleti/sinus.html>



On the first recording (25/11) the teacher assisted the students (referring to the quiet conversation between the teacher and one or two students) 28 times in the total duration of 12:42 minutes (63,5%), which means students had teacher's assistance an average 6,35% of the time, so the rest refers to individual activity. On the second recording (16/12) teacher talked to the entire class aloud 9 times in the total duration of 1:09 minutes and assisted individually 17 times in the total duration of 6:49 minutes (34,08%), which is an average of 3,41% per student (because of 10 pairs).

At the end I incorporated all the questions and dilemmas that we encountered during the process and made Likert scale which will serve as a critical reflection for the data analysis and the verification for the realization of our initial educational values.

	Statement	Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
1.	Individual computer-assisted learning enables easier processing			20%	65%	15%
2.	Individual computer-assisted learning is more interesting then teaching in our classroom			25%	45%	30%
3.	The computer allows me to easily come to conclusions on my own			10%	70%	20%
4.	The computer helps me visualizing mathematical concepts			15%	40%	45%
6.	The computer makes me think more than teaching in our classroom			10%	70%	20%
10.	Progress at own pace is the advantage of individual computer-assisted learning			25%	50%	25%
11.	The computer easily and automatically plots graphs so later I have problems with plotting graphs on a paper	5%	20%	40%	30%	5%
12.	The digital material available on the Web allows me to repeat and clarify at home what I did not understand well in the school			10%	30%	60%

15.	The feedback on the accuracy of our answers makes it easy to go through the content			10%	75%	15%
17.	The worksheet for writing answers with pencil helps me go through the content of the digital material better			15%	60%	25%

Table 3. Likert scale (taken from the questionnaire 3).

The interpretation of the action research

A starting point for this research was the theory of constructivism which emphasizes that knowledge is produced by the activity of student and the practice which shows it is not easily feasible in traditionally organized teaching. In practice, we noticed some students have serious problems in processing mathematical contents so we asked ourselves how to achieve the activity of all students in the class and how to encourage independence in learning. These were the initial values from which we started. As Kurnik (2008) proposes, we wanted to achieve that students come to their own knowledge using their own strengths so we opted for computer-assisted learning. However, we do not mean the using of computer in frontal teaching with LCD projector or smart board (Figure 1), we ask from everyone to work at his/her own computer (Figure 2). If we add Pólya's heuristic learning approach (Pólya, 2004), the individual discovery learning can be the one of solutions for the initial problem.

Considering we collected quite a lot of qualitative and quantitative data in the research, their synchronization will serve as triangulation process to avoid one-sided approach of qualitative data analysis so we could reach valuable conclusions.

On pictures 2, 3 and 4 it is evident that we have reached the **activity of all students** in the class. This is confirmed with the results of written exams that are much better than usual. In short test (Figure 3) one third of students were assessed with an excellent, while in the third written exam (Figure 4) 80% of students passed the exam. As advantages the students emphasize the progress at their own pace (75% of students, Table 3, statement 10) and the fact they can repeat the same procedure many times until they clarify the problem. 90% of students (Table 3, statement 6) believe the computer makes them think more than in traditionally organized teaching. We assume all of this affects higher level of students' activity in the class. Furthermore, we notice students visit Web sites with learning materials at home on their own initiative (questionnaire 2, 61% of students) and availability on the web enables them to clarify at home what they did not understand in school (90% of students, Table 3, statement 12).

The analysis of video recordings (Figure 5) indicates the **increase of independence**. We differentiate between the independence in learning process and independence in reaching conclusions. In the first cycle (Figure 1 and 2) we noted

students are more independent in reaching conclusions (28%) than in computer-assisted learning in general (6%). This can be explained by the fact that students had difficulties following the instructions at first so they called the teacher for assistance or consulted other students more often. After understanding the instructions they easily come to conclusion with the assistance of computers. Students regard individual coming to conclusions very stimulating (Table 2), and this is what we aspire.

Qualitative data (photos, videos, critical friends' statements, group interview) indicate the value of computer-assisted learning. 75% of students believe this way of learning is more interesting than traditional teaching (Table 3, statement 2). It enables easier processing of the material (80% of students, Table 3, statement 1) because the computer can generate many different examples in a short period and helps with the visualization of mathematical objects (85% of students, Table 3, statement 4). The computer provides instant feedback that helps students follow the digital content (90% of students, Table 3, statement 15), and exercises with collecting points are particularly motivating.

However, we observed the disadvantages of this way of learning. Students learn by using computers but they were assessed by using written exam (on a paper). We tried to alleviate this contradiction using the worksheet that students fulfilled during studying the contents of digital material. 85% of students (Table 3, statement 17) stated a worksheet helps them to follow the content easily. Even better solution would be to develop computer-based assessment. Furthermore, we noticed some students had difficulties with plotting graphs on the paper because the computer plots graph automatically (Table 3, statement 11, Kralj's comments). We spotted the problem of understanding the instructions, and it mostly occurred with those students who usually have difficulties with understanding the task. The solution to this problem can be found in heuristic learning approach and in the art of discovery (Pólya, 2004) so it should be dealt continuously regardless of the organization of teaching.

The changes I introduced in my teaching practice within the process of action research are the result of various factors: the readiness of both the students and me to act on the basis of initial values, continuous cooperation and appreciation of everyone's opinion, the responsibility of students for their own progress, the support and advice from critical friends and the self-critical consideration of their comments. Unlike the previous generations of students, these students approached computer-assisted learning more seriously which was greatly contributed by the process of action research and the reflection after each cycle in particular.

After analyzing the collected data critical friend Željko Kralj made the general conclusion: *"The computer-assisted learning transforms student from a passive observer to become an active participant, and therefore his/her motivation for learning increases."* However, this does not mean our action research is completed, now it opens new perspectives, we spotted problems and potential solutions and opportunities for improvement. The issues that we will deal with in the future are:

- the implementation of Pólya's heuristic learning in a dynamic learning environment,
- assessing the students using computers and interactive digital resources,
- developing new and improving existing digital materials (this was the second aim that was achieved in the framework of action research, but for the further implementation of this way of teaching it is necessary to provide quality digital learning materials).

Conclusion

Discovery learning mathematics with the assistance of computer and dynamic geometry software *GeoGebra* using the interactive digital learning materials that are methodically and didactically designed, ensures the activity of all students in the class, increases the motivation for learning mathematics, encourages individual coming to conclusions, as well as the cooperation between students. It is very important that student learns alone or in pair on the computer trying to discover new knowledge. This knowledge is going to stay longer in his/her memory and student is going to learn how to think, he/she is going to resolve a new problem much easier and master the art of discovery.

However, its feasibility in teaching practice depends on the school conditions. It has been shown that students are willing to cooperate on implementing changes in teaching, but if Math teacher is not able to use the IT classroom for his/her classes, he/she will not be able to implement the ideas described in the paper. However, I hope the reading of this article will encourage other teachers to try to introduce changes in their teaching practices in accordance to the school conditions they work in.

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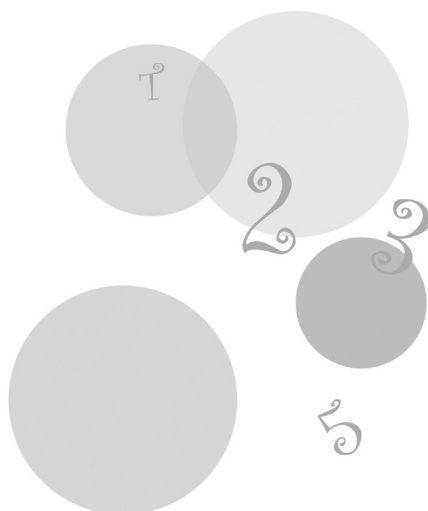
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5.

What difference can a math teacher make



In the fifth chapter the authors comment on the general issues of accepting mathematics as a vital scientific field by the broader community. Certain research suggests that a child's attitude towards mathematics largely depends on the attitude of their parents, but also on the attitude of the teacher in the first four years of primary education. Issues connected to mathematics school books are brought up as well, and the chapter offers comments on the teacher perspective in Croatia.

Low success in mathematics teaching: causes and consequences

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Abstract. The poor performance and low level of knowledge in mathematics is not an empty assumption, but the fact that is corroborated by the statements from the most reports taken from both the primary and secondary schools. Most of the surveyed students consider mathematics as a difficult, incomprehensible, uninteresting, and too demanding subject.

Undoubtedly the “culprits” for the poor success in the field of mathematics are in a great number at all levels, and making a classification hierarchy or a scale would be ungrateful and irresponsible. In fact, there is no specific and clearly defined position that the “culprit” could be identified; thus, as such should be eliminated or reduced to a minimum. The most experts can claim that the reasons for that are inadequate overbooked curricula, too many departments, poor professional qualifications of teachers, poorly equipped school, bad textbooks, outdated teaching methods and didactic forms of work, poor family conditions due to a low standard of living, and so on.

From the above, as well as other unmentioned reasons, it would be ungrateful to extract some specific factors, because a failure in mathematics is a function of all these factors.

Most of these elements are said to be caused by extracurricular activities, ignoring the fact that the main potential culprits are found in the school’s teaching methods; therefore, there are few answers to simple questions be found first: “Who?, How?, What?” a teacher teaches. There is no doubt that any good answer to these questions would clarify the concept of failure in mathematics.

Many previously published analysis indicate the above statement, based on a series of tests carried out, interviews conducted on pedagogical documentation, as so in our region as worldwide. We are also witnessing a disparity in grading system and in different methods that teachers use for the same subjects, not to mention for the different subjects. The fact is that elementary school graduates go to high school having high grades in mathematics, and high school graduates go to college with very high grades too.

During the studies in a high school, which is especially evident in the first semester, the results are much weaker than the results passed

from primary school. To the end of the year the grades improve slightly, but again with the notable difference between the closing of the eighth grade class in elementary school and the first grade of high school. The disparity in the mathematics' grades not only appears as a product of different criteria when assessing students, but it may influence other factors that we have intended, if not entirely then at least partially to reveal here. The research results should enable practical action in the field of mathematics with which would, at least the part, of the cause of the poor success in teaching mathematics be dismissed.

Keywords: school, mathematics, success, lessons, plans, methods.

Questions whether society development ratio is direct consequence of education, or whether education ration is contingent upon society development, are disputable and open for discussion. School, as an important factor in a system, directly or indirectly influences the societal existence, development and change in its broadest sense.

Global discontent, a result of limitations existing within any social systems, and among other factors, a consequence of unsatisfactory education process, has affected our region, and in a place where quantity surpasses quality – little is left for the future generations.

Educational institutions have become “large in number, low in quality” which again directly or indirectly influences the existing social relations. Repair or even destruction of a whole series of faulty goods is compensable, however, a generation of badly-educated students is non-compensable and extremely dangerous; a solution includes an extensive effort, time and financial support.

Teaching is continuous process happening at any time and any place in a number of forms, and communication and interaction are important ways of information transfer among students; they also include establishing connections, exchanging information, strengthening relations, and in a way grow intellectually and culturally.

Teaching as a continuous process is unsustainable without interactions between students, students and teacher, students and teaching material, and it can be concluded that such interactions must be cherished and improved. Namely, absence of interactions between the students, or students and teacher, can have negative effects during and after education period. Such interaction influences brain activity, self-esteem, self-confidence, and finding better answers and solutions. Rising the level of mathematics culture appears to be an inevitable phenomenon caused by development of mathematics which, on the other hand, develops in regard to the needs of society and other sciences. The chasm between the mathematic culture cherished through teaching process and applied mathematics, mathematics used for material and spiritual well-being, can be found in forms and methods that are dominant in the school teaching systems.

New teaching plans and programs do not threaten the exact of mathematics, but their redundancy and reduction of hours yield mathematics to be perceived as a difficult science discipline. Undoubtedly, mathematics has made a “deep” and “wide” breakthrough into exact sciences, and then into a number of social sciences,

however, it is not possible to place all the science in teaching plans and programs for children.

Language, symbols, operating systems, rigidity as well as other teaching methods are no longer recognizable exclusively in the work of exact sciences researchers. Functional and structural observations are also used in a field of social sciences (economists, sociologists, lawyers, psychologists, and others). The fact that development of a science is seen through the rate of mathematics applied, most vividly describes the importance of mathematics.

The massive approach to education is a trend that has happened as a direct consequence of a highly industrialized society that is constantly developing, with demands for a greater number of experts in all fields of social development. On the other hand, elements of a theory, problem aspects, disparaged knowledge gain, neglect of social aspects, point at – if not the root changes – then partial changes in a teaching process.

Education as the fundamental element of any social act cannot in any case be excluded from science and technology development, especially bearing in mind the general discontent with educational systems. In order to implement faster and a more economical way of education, mere knowledge of what to study is no longer sufficient, but how to study takes the lead. In 1613, Rathke suggested teaching skills to be implemented in a teaching process. Regardless of the number of offered solutions, a unique teaching model was not found, which suggests that there is not one, but that it is necessary to use various, intertwined methods, all to achieve better results.

When and how to use certain methods in a teaching process is primarily a matter of the material that students need to learn, and of a teacher that mirrors the grandeur of an excellent methodist, didactic, pedagogue, and teacher. On the other hand, the fact that life of future generations will probably be different from the life of present generation, represents a nightmare especially for pedagogues, since it is unknown what the future holds for it depends on many factors.

A teacher is an irreplaceable participant in a teaching process, but there is a question to what extent his presence is really necessary. Is there a real reason to prevent self-teaching and letting students build their own world? Why not replace a lecture-reception form of education with a simpler dialogue?

Year in, year out, same old problems occur, problems related to different levels of previously acquired knowledge that children have at enrolling a school and this is most noticeable when it comes to mathematics. Why? Earlier practice was not introduced with the concept of “the prior knowledge”, and the acknowledged fact at the time was that “the role of mathematics in lower grades of primary school, or during the education in primary school in general, was determined by its function”. Namely, studying mathematics, students are not only being prepared for a certain vocation, but they are being educated in general, and it can be said that the primary role of teaching mathematics is embedded in its educational component. Besides this, in lower grades, mathematics has a significant preparatory function. In other words, it prepares the students for higher levels of educational, including higher grades of a primary school and secondary school (Markovac, 1992:17).

It is not a novelty that preschool children can spontaneously think and count, and that cognitive ability of language acquisition, ability to calculate and write a number can also be congenital and universal. Mathematics and other related sciences give great possibilities when it comes to developing logical thinking. Mathematics demands cognitive activity based on an analysis, abstraction, generalization, and conclusion. "It is clear nowadays that we entered the 21st century studying, knowing that happy will be those who learn with ease, not afraid of the unknown and new, able to solve problems and accept the state of the art technology. It all point out that children from the early onset must be prepared for the learning, to make them like learning and by doing this preschool pedagogy gets its value and significance" (Suzic, 2006:10).

These two streams of pedagogical thought explain a lot, and even if they are close by, let us say "birth", it can be noticed that methodology of the 90ies did not emphasize the preschool math studying, while the 21st century expectations are at a much higher level. Prior preschool teaching was not officially prejudiced, but it existed as a formal, informal and technical. The following definition serves as the best example that preschool teaching existed even in the earlier stages of human evolution: "conscious, with purpose, and more or less organized individual and social development of a preschool child is a socially determined and individually marked system of activity, purpose and process influencing a complete development of a child from its birth until enrolling a school" (ibidum, 10).

Before discussing the importance of starting with math in early childhood education, it is necessary to explain certain principles of preschool education. "Starting from the most human desires, freedom and happiness as the highest of them all, principles of preschool education can be qualified as follows:

- principle of optimizing child potential developing,
- principle of play activities,
- principle of non-violent communication,
- principle of spontaneity and organization,
- principle of suitability," (Suzic, 2006:10).

These principles are completely clear and concrete, even though the questions related to the aspect of humanity and pedagogical justification for pushing the border of technical education are not answered in total. Namely, answers are greatly difficult and complex, though prior experiences related to preschool education in France clearly point at the prevalent understanding of a matter according to which the countries with education system that includes children enrolling school earlier than usual will be dominant, or to put it differently, superior to others scientifically and technically.

"If freedom and happiness are the supreme principles of preschool education and if an organized preschool system of education will give children more freedom and happiness, then using the knowledge of our civilization is justified"(Suzic, 2006:12). When it comes to math, the early start should be used in such a way to educate a child psychologically and intellectually, especially regarding development of logical thinking, attention, memorizing, noticing, concluding, realizing connections between elements, relations, etc. Through this comes to light the functional component seen in developing the psychological and intellectual abilities in

children, as well as the educational component that gets stronger with forming and perfecting the positive characteristics of student's personality.

Even if each principle, directly or indirectly, influences the basic understanding of mathematical thought, it is not necessary to go in depth of all included in preschool education through the principles, but more discussion will be about the cognitive process of developing the mathematical thinking.

Thinking, process covered with abstruseness and secrets, can be accepted as a consequence of instinct known as curiosity, an urge for learning about the world we lived in. A child quite early starts developing its mind, its logical thinking by identifying, comparing, differentiating, etc. Discovering the categorical relations (cause-effect, object-characteristic, part-whole, etc.) and knowing the purpose, a child's thinking starts to develop and advance (Slatina, 2005:179).

It is possible to roughly determine the levels and types of thinking regarding the age. For instance, there is a division of thinking in the broadest logical sense. A resulting type of thinking process is a consequence of questions "why? with what? how many?" however, the answers to the questions do not satisfy children's curiosity as they are not formed with a concrete importance. As an opposition to this, there is a definition of thinking in its narrowest sense – a consequence of unilateral ideas that are the result of adopted mentality. A third kind of thinking is the result of analytical and synthetic reasoning, and such reasoning is the consequence of existing problems, puzzling moments, obstacles, etc.

It is an indisputable fact that development of child's brain requires not only congenital, or genetic factors, but also external stimuli experienced through touch, speech, pictures and it can be concluded that surrounding shape the brain, or to put it differently, external stimuli in some way connect brain cells and neurons.

Researches bring new facts pointing at the prior mistakes made when it comes to children education. Gordon Dryden (2001) explains why babies need to watch sharp contrasts. "In the first months after the birth, a child's brain establishes its optic tract. A child's cortex has six cell layers that transfer different signals from retina through optic nerve, and to the brain. For instance, one of the layers transmits signal for horizontal lines, while the other for perpendicular. Other layers or columns deal with circles, quadrants and triangles. If the child's saw only horizontal lines, then it would hit objects while walking or crawling, because its optic tracts would not process perpendicular lines" (Dryden, 2001:242).

Ronald Koutlack (1996) claims that even in case that a person's brain is perfect, that person will not be able to see if the brain does not process visual stimuli before they reach two years, and will not learn to speak if does not hear the speech by the age ten, and hence the suggestion that babies should be exposed to strong black and white contrasts, and not to cover the walls with monotonous wallpapers (Koutlack, 1996).

According to prior researches, especially the one by Birkenbihle (1989), it is possible to differentiate two brains "older" and "younger." Biological program, a hardware, is placed in the old brain and its function is to control conclusion, pain and discomfort avoidance, motivation and lowers the urge for escaping, surviving

instinct, urge to satisfy vital needs, manage and control behavior, blocks the thinking, reality awareness, body movements control: it is a place where the feelings and memories are stored, a place of Freudian “id” and child’s “I” (Berne), our super-conscious, subconscious, and probably much more yet undiscovered.

On the other hand, there is a “new” brain which is an integral part of both hemispheres, and represents software, with roles and tasks related to connecting, analyzing, concluding, think about the self, self-exploring, satisfaction, intelligence and creativity (Brajša, 1995:53).

According to Piaget (1970), development of thinking can be roughly divided into four phases, in such a way that every new phase is closely connected to the previous. Based on the experiments, Piaget makes a scale based on age and lists the first period, or sensor-motoring phase, lasting from the birth up to eighteen months. The basic characteristic of the first period is that a baby understands a connection between a feeling and an action, that an object existence is proved in seeing it, while in the mature stadium, a baby becomes more aware of objects without seeing it, and it also becomes aware of reversibility.

The second period (between 18 months and the seventh year of life), Piaget calls per-operating period that he divides into two sub-periods, 2a and 2b. Both have their characteristics regarding the child’s behavior and the world seen through their eyes, especially in 2a, when the problems are solved intuitively, while the logic takes place in the phase 2b, a period of collision between a child’s perception and logic.

The third period (between the seventh and twelfth year of life), so called concrete-operating period, is the period when logic takes the lead, even though the period is marked by close perception and earlier experience. A child understands a term commutativity, summing and reversibility, enabled to conclude that, for example, $7 - 3 = 4$, because $3 + 4 = 7$, even though it does not highlights the opposition between the given operations. It is interesting to mention that a child in this period understands a term transitivity given in forms such as $A = B, B = C \implies A = C$, or $A > B, B > C \implies A > C$, but when it comes to tasks such as “Emir is higher than Goran, Goran is higher than Zlatan: Who is the highest?”, children are not prepared to give to correct answer. This inability Piaget sees in the fact that logic takes more significant role in thinking, but also that the logic is still largely based on finding a solution based on perception.

The fourth period is marked by abilities to prove abstract stands by deduction, logic, and so forth. It is thought-provoking that children think only hypothetically, without understanding a possibility of the practical realization of a problem, for example: if 15 eggs cost 2 KM, how many eggs can you buy if you have 1 KM? Children usually say 7,5 eggs, and children do not interest if such an answer is even probable.

The questions of relevance can be shown in the following example:

What is more important for children, realizing that $7 \cdot 2$ equals $2 \cdot 7$ or memorizing that $2 \cdot 7 = 14$, or in fewer words, is it calculating more important than realizing? And here lies the question, here among supposedly irrelevant things.

“When studying the calculus, understanding is required, when studying understanding, calculating is required” (Liebeck, 1984:7).

Without taking into account demands of galloping, and ruthless phenomenon called globalization, it should not be forgotten that every child should be given an opportunity to realize the mathematical problem visually (lego, sticks, tangram, abacus, etc.) and based on the visual be able to sketch the required form (quadrants, triangles, circles, etc.) and join the sketches to symbols, or numbers. Jean Piaget, describing the Aebly's work within the spheres of operating methods, emphasizes the conditions of education, which can give accelerating or decelerating effects, and it is recommended that symbols are used in order to show the significance of the concrete display (Aebly, 1982, chf. Piaget, 1983).

There is a fact that “children are born smart” and a large number of children even before starting with school are familiar with the elementary math operations such as summing, multiplying, dividing, and all please the parents who are greatly responsible for their children's physical and intellectual development. A large number of high school students, due to the abstract notion of the concept, understand the operation $2a + 2b$ as $4ab$.

Galperin, Lompser, along with Bruner continued with the research based on Piaget theses that all cognitive activities are based on a transfer and copy of realistic activities of the real objects, and came to a conclusion that mental activities take place on several related levels.

1. The level of practical and objective activities that enable the cognition of an object.
2. The level of direct insight. It is highly important to emphasize the fact that spotting an object is not sufficient solely, but also matching the object with the related characteristics.
3. The level of indirect insight. Activating prior memorized and comparative approach to an object happens at this level.
4. The level of lingual-conceptual approach, where mental activity happens primarily in terms of notions, or concepts; in other words, within abstract, indirect structures.

The fact is that students can easily identify the number of parallelepiped in terms of abstract thinking, rather than realizing things in terms of doing. To put it differently, abstract-concept processing is much more prominent if compared to visual-object processing. Developing the theory about the children's thinking and operating processes, Piaget emphasizes the influence of Lompser, and lists four cognitive levels that, on the one hand, represent the genetic order -that the higher level stems from its “predecessor”. On the other hand, lower levels never cease to exist, and new planes of reasoning influence the older ones changing their characteristics. During development, relations between the planes change constantly (Lompser, 1972, chf. Piaget, 1983).

Prior researches, as well as practices, have shown that combined operations in mathematics give better results when solving a task if compared to solving it using only one operation. For example, when it comes to multiplying, it is recommended

to use summing and subtraction, as shown in the following example:

$$6 \cdot 9 = 9 \cdot 6 = 9 \cdot 5 + 9 \cdot 1 = 10 \cdot 6 - 1 \cdot 6$$

Tasks presented in this or similar way, educe the sense of flexibility, combining, freedom, resulting in easier and simpler learning the multiplying table, since the students are offered a number of ways how to achieve it.

On the other hand, the receptive method of learning has proved to be extremely efficient, and it allows students to search and find the best ways to reaching their goal. It is important to emphasize that learning new math theories, skills, operations, new terms and their adequate application in problem solving, must be based on prior knowledge. Namely, math skills cumulativity and its functionality is demonstrated at this, and similar situations.

It is important to say that rarely only one learning method is used; variety of forms are intertwined and complete each other, and in math the following forms are met:

1. Associative learning (learning based on signals and stimuli that transform into automatic knowledge, such as multiplying table, fractions, equations, etc.).
2. Discriminatory learning (comes to tending the ability to differentiate between the notions such as quadrants, etc).
3. Math term learning (certain terms must be understood axiomatically, and then move to the more complex terms).
4. Learning the math rules (defining the problems using theories and theorems).
5. learning the heuristic rules (learning with understanding, making distinction between the receptive and cognitive, realize the given data, and what is required; what is the best way to solve a problem?).
6. Math problem solving (learning through solving math relations, solving typical, and constructive math problems).
7. Insight learning (Bandura, 1977:31).

There are many dilemmas when it comes to alternative beginning of planned and cognitive children's learning. Dryden Gordon analyzed the problem as well, describing the work of Harvard professor Burton L. White, and claimed that, in the history of Western education, there was not a society that recognized the importance of early age education. Also, there were no preparations or help offered to families as a goal to start early development of children. Professor White also pointed at the fact that the most important period is the one between the moment of a child start walking and the second year, and emphasized the importance of early age education as fundamental and crucial for development of speech, curiosity, intelligence and sociability (White, 1991; chf. Dryden, 2001:253).

Optimizing the child's potentials development includes creating conditions and organizing activities that will help a child in developing its capabilities, competences, and personal characteristics needed for free and happy life in social and natural surroundings, a place where a child will spend its life. It is difficult, and

almost impossible to predict the conditions and life space of a person, since it is necessary, from the early childhood, to prepare a person for education and developing the competences, for self-training. It is certain that the 21st century civilization will be marked by educated society. Constant development of competences and lifelong education become the condition to freedom of a modern man.

Results of these researches, and these thoughts, would be inhuman to complete without asking a questions asked numerous times: does the early childhood takes the childhood form children? The answer could be the following statement: As long as a child has a blast while learning, there is no danger for its mental health, and if the child cannot find learning fun, teaching should be stopped.

Low success and the low level of math knowledge is not a presupposition, but a fact supported with reports from primary and secondary schools. Most of the answers students gave in a survey picture mathematics as difficult, boring, and too demanding.

There are a lot of “culprits” accused of low success in fields of mathematics, but qualifying or making a hierarchical ladder would be ungrateful and irresponsible. Namely, there is no specific and well-defined stand to identify the “culprit” and as such eliminate or reduce the number of them as much as possible. Something that really stands out from the reports are much mentioned inadequate and overfilled teaching plan and program, too many students in classrooms, unskilled teachers, bad textbooks, old methodical – didactic forms that are no longer used, bad family conditions caused by low life standards, etc.

Low success in the mathematics teaching is the result of many factors, and it would be ungrateful to name only few.

Possible “culprits” should be found in schools, classes, ways the teaching is conducted, and it is pure necessity to find the answers to questions such as “who? how? and what?” is taught. It is out of doubt that one decent answer would at least partially resolved the cause of low success in math teaching.

Many prior analyses, based on tests, surveys, pedagogical documents, in this region and in the world, favor the earlier statement, as well. Also, we witness discrepancy in grading the same subject by different teachers. The fact is that in a large number of students after completion their primary school, enroll secondary school with the highest grades, and similarly students enroll a faculty with the highest grades after completing their secondary school.

During the high school education, evident in end-term classification, the results are much lower than those in primary school, with the slight rise in success by the end of the year. However, the results are still lower than those from primary school with improvement seen at the end of the year, but there is clear difference in the final grade in the eighth grade and the first year of secondary schools. Discrepancy in math class grading is influenced by a number of criteria, and the factors are only partially revealed. In order to solve this problem, interactive teaching method offers something that might help up to a certain degree, and Ante Vuksanovic claims the following: “A solution to the problem appears as intellectual and practical form, and through practice students contemplate intensely, research, they act, find solutions, and study” (Vuksanovic, 1977:311).

Low success in math teaching is only partially mentioned, and it is expected to find the right solution through identifying, preventing and removing the causes. Scientific research should offer some indicators that will help finding ideas for adequate didactic and methodical actions to apply in math teaching. The results should make practical teaching possible, and that would to an extent remove the cause of the low success. It is completely unjustified to perceive interactive teaching as the solution to all the problems found in education process, and math teaching as well. However, interactive teaching if applied, and combined to other teaching methods should definitely improve teaching on many levels.

Failure in math classes, as well as failures in other spheres of life, have their own characteristics that are easily recognized. Students' passive behavior, no matter what method and material used, vividly describes the causes of students' failure in math and other subjects. Of course, everything mentioned should not be ascribed solely to math teaching, for many factors influence students' psyche, manifested through ignoring studying in general, and given the fact that such behavior is not so common, it should not be taken into count. There are two types of characteristics – qualitative and quantitative. Regardless of the type, they are easily recognized especially through direct teaching. The fact that quantitative properties are marked by cummulative can be seen as negative, in other words, gradual increase of ignorance in the area of mathematics, is surely a direct consequence of nature of mathematics as science.

Namely, there are many factors that make math so difficult – terminology, measures, parameters, operations, symbols that are connected and compatible; all result that every new math discipline is conditioned through prior acquisition of definitions, theorems, theories, operations and facts related to other sciences. In other words, mathematics makes a whole where every part matters, and there lies its gravity.

On the other hand, the problem with studying math is that the symptoms do not appear suddenly, but their existence is largely hidden and is in a function of time as the parameter. Rationalization and other subconscious mechanisms show their influence, which altogether result in low success of students in math.

According to the Polish pedagogue Konopnicki, there are four stages in teaching process and the failure birth.

The first stage is characterized by subtle gaps in students' knowledge, they are caused by the reactions of students manifested through defensive attitude toward school and classes. As the gaps grow, which is inevitable, the loss of motivations appear, and consequently – the loss of will to study.

The second stage is manifested in the inability to be fully focused on teaching process, and increased problems non-monitored work. Regardless, the magnitude of symptoms, teachers, or even the parents, do not pay much attention.

The third stage is characterized by students' indolent approach to teaching and school; students neglect their obligations resulting in low grades, which is the very first indicator of a serious problem. Increased number of low grades results in student's passive approach to the process of teaching. A student has difficulties

coping with a number of problems, and without help from the outside world, one does not get back to the normal state.

The feelings of helplessness and rejection are manifested in the fourth stage, and is compensated through other forms uncommon for the teaching process. In regard to the number of low grades, a student is being suggested to attend the same course again, but that is not the only option, if the students decide to study with teacher's help, or instructive teaching (Knopnicki, 1969:452).

Negativities are difficult to prevent, but anything should be done to decrease their number. Teachers, pedagogical and social services, and parents can participate in preventing the problems. Even if it is impossible to prevent them completely, there is still something to be done – a constant monitoring over student's progress in teaching process. Namely, noticing the problem at time, could help choosing the right steps for preventing failure.

In short, failure in math teaching is manifested through:

- unsatisfactory prior knowledge of basic terms, rules, and mathematical laws,
- insufficiency knowledge of the math language and its application,
- insufficiency of math operations automation,
- bad or non-existent ability to memorize math formulas.

Interactive teaching makes it possible that each student is active, resulting in changing views and ideas, and ambiguities. That kind of approach to teaching influences the abilities and techniques of listening and demonstration of ideas, which all help setting the ground rules in math. Teaching is conducted in such a way that a student is subject and not passive observer. Memorizing happens spontaneously and creative thinking is in a way emphasized. Co-operation takes place, which is an excellent precondition for eliminating factors that cause unsatisfactory success in math teaching.

Mathematics as the key subject of this project contributes the implementation of a number of pedagogical effects, and all together form the personality prepared for the 21st century life, of which Suzic says: "If the first generations of primary school are being taught interactive methods, pair work, control their emotions, preset their accomplishments, and to find information as quick as possible, and, of course, to use them. Above all, those will be open-minded people, ready to cooperate, and support others. That is the way to accomplish peace that is the ultimate wish of all people in this region" (Suzic, 2001:87).

The cause of low success in math teaching

The exact or abstract make the mathematics so difficult to comprehend. The very first encounter with something such as the plate, round, circle, cylinder, can seem abstract to children, while the formula $E = mc^2$ causes the same effect with the grown people. The number two can also be but an abstraction, until adding two elements to mark the abstract notion: two students, two coins, two books, etc. All mathematical operations, including the elementary ones – summing or subtracting,

are more abstract if compared to the notion of the number. Difficulty of correspondences between concrete and numeric marks, and as the ultimate goal of the first phase of abstracting, is unavoidable phase in understanding mathematics, and it can be accepted as one of the first causes of low success in math teaching.

If asked “how to teach” all schools would answer the same – that it is conducted according to principles of logic, psychology, and philosophy. It is the truth that small number of schools support creative thinking, that is to say, the ability to open the mind (free from other ideas) and search the new ones, to look at things in a different manners and directions. The human mind is capable of conceptualizing, analyzing, speculating; it is able to find numerous solutions including those that are critical, silly, convergent, divergent, bizarre, reflective, visual, symbolic, numerical, metaphorical, mythical, poetic, verbal, non-verbal, practical, ambiguous, constructive, realistic, surreal, focused, creative, concrete, objective, subjective.

A solution to any problem, the brain finds in empirical data search – in order to find the exact or similar situation solved before. Inadvertent limitation of brain activity is conditioned with prejudices, taboos, which is closely related to national, religious, psychological, or other feelings. But, let us ask: Are the one who others expected to be? The parents, teachers and the milieu transmit their expectations through the words, views, mood and body language in such a way that those expectations become student’s expectations, as well.

The schools of all levels already did what they wanted, or are still doing, so it can be said that we are preordained, or “sentenced” to direct answers, left without an opportunity to find other ways or solutions. There is no doubt that every person claim that their school, secondary or higher, is the best, and if not that, then at least one of the best. This kind of opinion should be accepted as normal, or usual, because seldom a person has the clear picture and cannot be objective, just as there is not one teaching, religious or health system that is perfect for everyone. Fear and subjectivity lies in all of us, including the students. In order to overcome such emotions, it is of utmost importance to include more fun in teaching. Fun and entertainment make great working atmosphere, which is a condition for creative that our schools lack; this should also be applied when it comes to math teaching, since many claim that it is filled with abstractions, which makes it difficult to comprehend, and as such math is repulsive to the majority of students.

Language, as a means of communication, also, in essence, is an abstraction, but unlike mathematics, abstraction in mathematics is not necessary when it comes to explaining certain terms, and as such, does not require hierarchy of concepts. Abstraction is dominant in math and probated with prior knowledge about abstractions, and all in order to solve more complex operations. Teacher’s competence to make the connection between the real world and the hierarchy of abstract concepts could make learning math easier.

It is an indisputable fact that students come to primary school with a high dose of interest for numbers and math. As the years pass, that spontaneity and satisfaction decreases, and they leave their schools with great disgust toward math and notion that mathematics is extremely boring and difficult to comprehend. All subjects, math included, are usually perceived as highly complicated activities, that

exist for no obvious reason, and students fail to see them as a means of understanding the world they live in. Consequences are serious, for they point out how their lives are limited, and how small the number of national talents really is, and it should not really be. According to the SCANS (The Secretary's Commission on Achieving Necessary Skills) report, titled "What Work Requires of Schools", that more than fifty per cent of the American youth leaves the school without knowledge required for finding and keeping good working position. "The youth will pay the price. They are facing the sad perspective of a dead-end work, which will be interrupted only in periods of unemployment" (Dryden, 2001:271). According to estimations, less than half of the adults acquired the minimal literacy that is required. The number of those successful in math is even smaller, and schools nowadays only indirectly deal with listening and speaking. The working class is not well-educated, lacks practice, and is not fully qualified (ibid.)

When it comes to the math teaching in this region, the situation is not better – actually it is even worse. It would be great if the guilty party should be found in schools, regardless of the social structures that spawn the exact schools, and itself preordained to create failures. The worst thing would be to expect new techniques to right all the wrong, and prove successful in the society where the majority of its members is doomed to fail. Schools today are what they are, not able to change, and ironically enough, they represent the essential hope that things will change for better.

It is evident that continuing the education, there is increase of low success rate, especially in math. Namely, a greater part of mathematics in the first year of secondary school, is basically the same subject matter taught in the primary schools, but with more direct approach to task analysis, and its theoretical background. For example, in case of primary school math, here comes to light the fact that good and quality accomplishments are the result of quality and complete prior knowledge.

Mathematics by nature falls into a group of sciences that requires great intellectual effort, and is recognized as a difficult teaching discipline. Saying that children know a lot, but understand very little, support the theses that difficulty lies in unsatisfactory intellectual development of the students which is, of course, incorrect. Implementation and realization of the math teaching material through the methods, can be also said to represent one of the crucial elements that contributes the difficulty of math.

Social environment, surroundings, the place where a student lives, life style – all are listed as possible factors that contribute to the low success rate in math teaching. As one of the main reasons to low math knowledge is the number of hours prescribed in the teaching plan, and the number is small if compared to the European standards, as well as the number of students in classes, which affects the quality of teaching. Now, there is a question where to find the major culprit. The most obvious solution to the problem would be to "accuse" the outside factor, but all prior researches point out the fact that the reason is complex and found within the schools.

Failure or success in math teaching is closely related to the student's maturity, organization and realization of teaching process, math teaching methodology, student's environment, vocational and pedagogical capabilities of a teacher, student's

talent, and other factors (Frommberger, 1955:168). But what should be placed on the first place of the hierarchical ladder of factors, or causes to low success rate in math teaching? Every listed factor has its effect. There is a way to prevent the failure in math teaching, and certain steps should be made: math teaching process must be adjusted to the students' need, students must be motivated, their energy must be developed, they must be taught how to study, to develop the ability to understand abstract concepts.

A number of high school teachers demonstrate their and supremacy over students, claiming that education is not obligatory which creates in students repulsion, fear, frustration and other negative effects. Mathematics teaching methodology, like it or not, clearly show our automatism. Being aware or unaware of the problem, in order to solve it the first step must be made, then the second, and so on. This road to success must be closely connected with the cognitive structure of the child's central nervous system.

Implementing the interactive teaching as a novelty and a new model of education in schools should, without a doubt, yield positive results. "Innovation school is perceived as a continual process, that will be constantly improved. It does not take only the best, that with high quality, rational, pedagogical, but the innovation school creates new, improved and all in aims and contents of the teaching process; in school development, methods, media, technical and technological solutions; in more sensible and efficient forms of teaching, in methods and actions" (Miljević, 2003:99).

It can be concluded that from the very moment when teachers choose the significant, adequate, and in a way, exciting topics, in math and other scientific discipline; when they start to favor group work in all forms, support the competitiveness in students, focus on the research based work. The focus should not be set on mere memorizing the rules and formulas, and the moment they realize that the students know what is actually being expected from them, the larger number of students will begin to study.

Mathematics with its language, operations, symbols, theorems, proofs, logical operations, and with everything needed for successful functioning demands continual improving. "Piling up" of the studying material cause panic in students because they are convinced that they cannot catch up, which all result in students' passive attitude toward math and repulsion toward teaching.

Passivity is not only manifested in math, but also in other school subjects, which affects the student's mindset. Decrease in math learning directly causes a decrease in mental abilities, not to mention other consequences.

Due to unsatisfactory activity of teachers, pedagogues, teachers, parents, there is the syndrome of low self-esteem found in students, doubt in their abilities and themselves.

The first phase of indifference in regard to the math teaching, as well as other subjects, is not necessarily the result of piled up problems in conducting the teaching, but can be the consequence of other factors, that can be placed in the following categories:

- pedagogical,
- psychological,
- social,
- economic.

As previously mentioned, one of the most common reasons for such behavior toward math teaching process is piling up the teaching material, quantitatively and qualitatively. Quantitativeness is recognized in the absence of developed working habits, sense of obligation, ignorance when it come to the basic mathematical facts. Quantitativeness manifests through the lack of logical reasoning, reversibility, etc.

The lack of quality and quantity in math teaching process, results in passive attitude toward math, inability to comprehend the material, and all leads to math losing its prime function in the educational process.

Students very often close themselves in their own worlds, and try not to attract the attentions, and they are often exposed to ridicule due their lack of knowledge. With time, they get less and less attention resulting in student's alienation and, apathy, and the loss of confidence and ambitions.

Social consequences are obvious and manifested through asocial behavior, easily recognized in arbitrariness, absence, disrespect toward teacher, colleagues, and aggressive behavior.

When it comes to economic consequences, one of them is retaking the class. Undoubtedly, mathematics appears to be one of those subjects that takes the lead when it comes to the reason why the students must repeat a year at school. The conclusion would be that the higher success rate in math teaching would improve the financial status.

Eliminating the failure in math teaching

The first step toward the eliminating or lowering the rate of failure at teaching, as well as in developing the interest and will in students, is ascribed to teacher. A teacher's task is to be fully committed and persistent. The primary activity would be identification the failure causes. They can be recognized through:

- partial or complete lack of mathematical knowledge (concepts and rules),
- weak or non-existent knowledge of mathematical language,
- weak or non-existent knowledge of math operations,
- passive attitude,
- weak or non-existent working habits,
- lack of sense of responsibility.

In essence, all those reasons could be sum up or ascribe to the lack of intellectual, aesthetic, and physical education. Student can demonstrate only one of the reasons, but they are usually expressed in "clusters". Some of the reasons are hidden, and some of the become more prominent with the growth of negative

effects. Identifying the causes can help finding the adequate measures for their prevention and elimination.

Elimination of causes and the full engagement of students into teaching process in a number of ways:

- remedial teaching,
- additional teaching,
- repeating a year at school.

Remedial teaching is similar to the regular teaching, except the method and number of students that participate in it. It is organized during the school year, when it is necessary, and is focused on students with difficulties in understanding the teaching material. Remedial teaching aim at the filling in the gaps occurred within the regular teaching. Experiences have shown that this type of teaching is extremely productive and students advance. The effect of the remedial teaching would be even greater if the greater number of teachers are trained, because prior practice has show that the work within remedial teaching does not differ from the regular teaching, even though such type of teaching should be adjusted to every student, and that is exactly what makes it so specific.

Additional teaching is organized upon finishing the school year, and the main goal is to help students acquire knowledge with additional hours in class. Basically, additional teaching is conducted by the same teachers, because they are familiar with the problems a student had during the regular teaching. However, the results are not so impressive, and the reasons are numerous, primarily psycho-physical in nature. Namely, additional teaching demands extraordinary engagement both teachers and students', manifested through demand to acquire a great deal of knowledge in a short period of time. I order to achieve this, modern teaching methods must be applied, as well as qualitative didactic – methodical teaching forming.

Repeating a year at school is the last thing to be done, but suggested when all other measures yield negative results. These final measures are applied in those cases when going to the same grade, student gets the opportunity to acquire mathematical knowledge necessary for the future advancement. Suggesting the repeating a year in school, no matter how unpopular, proved effective for generations of students. On the other hand, repeating a year has many positive effects, especially those regarding students' habits and skills. "Theoretical base of repeating a year, as a measure, way and method for eliminating the failure in math teaching, in primary and secondary school, is a fact that students must have some kind of knowledge in the area of mathematics to be able to become an active participant in the math teaching process in the next year at school they attending" (Markovac, 1978:98).

Repeating a year at schools is suggested only as a pedagogical measure for those students that fall behind in regard to their psychophysical development, illness, or other reasons for class-long absence, and all that result in inability to participate in math teaching process.

Prior researches, based on the students that repeated a year at school, have shown that repeating a year at school does not yield expected results, students fail to fill out the gaps in math skills, and do not increase their intellect significantly.

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Teachers' attitude toward mathematics

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Abstract. The paper points out the implications of teachers' attitude toward mathematics on the quality of their teaching and pupils' results. We wanted to investigate what attitudes towards mathematics have primary school teachers to make the results might try to discover some of the causes are often unsatisfactory results of students in mathematics. The attitudes of teachers to mathematics, we examined the anonymous survey on a sample of four hundred teachers from various parts of the Croatian and Bosnia and Herzegovina. The results showed that despite the positive attitudes of the teachers explicitly declare, in their work they have many prejudices, as to mathematics, and toward pupils. Raising awareness of this problem can eventually affect their changing, and thus indirectly raise the quality of mathematical education in the classroom period.

Keywords: teacher, math teaching

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Teacher and textbook in geometry education

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Abstract. This work deals with the role of teachers, curricula and textbooks in geometry education. The mathematics education is the meeting point of two different “types of mathematics” – mathematics as a scientific discipline, and the everyday mathematics that is familiar to pupils. Therefore the role of a teacher as the expert and mediator between these two cultures is particularly important. The subject of geometry is especially interesting for such analysis, because the geometry tasks include practical as well as abstract ideas.

It is thus necessary to pose a question how the abstract contents are connected with practical problems in geometry classes. The analysis of mathematics textbooks can partly answer this question. This article presents the research results of the geometrical contents in mathematics textbooks in grades 6 to 8 of the compulsory education in Croatia.

The results show the domination of tasks with operational requirements, and the lack of tasks with argumentation and reflection requirements. The problem of connecting everyday mathematics with school mathematics is emphasized by the lack of tasks with realistic and authentic context. Therefore the article discusses the role of textbook, teacher and curriculum in the intercultural perspective of geometry education.

Keywords: Analysis, geometry education, teacher, textbook, intercultural perspective

Introduction

Many research results showed that the textbook has a central role in mathematics education (Pepin and Haggarty, 2001). The textbooks are used in Croatian mathematics education to a huge extent as well (Glasnović Gracin and Domović, 2009). Mathematics teachers surveyed in this research stated that they mostly use textbooks for instruction preparations, while their students mostly use textbooks for exercising and for homework. A teacher has the role of a mediator between the textbook and students. These results refer to mathematics education in higher

compulsory education in general, so we can apply it to the geometry education as well.

Teaching geometry has always been a part of mathematics education. The oldest known mathematical papyruses contain geometrical problems which arose from everyday needs. Actually, the name “geometry” reveals the ancient need for ground measuring, i.e. appliance of mathematics in everyday life. On the other hand, mathematics in Ancient Greece established another insight to geometry – abstract, deductive and philosophical approach to the geometry, which was far from just appliance of everyday situations (for example, The Euclid’s Elements). The Euclid’s Elements were the main source for teaching geometry in Europe in medieval period, but also later in some European schools (Dadić, 1982; Jembrih, 2008).

Therefore one can notice two-sided approach to geometry in the education – geometry that arises from everyday needs, and geometry as an abstract discipline. These both aspects should be present in geometry education. Furthermore, they should be well connected to each other. Mathematics teacher and textbook play an important role in the process of connecting these two aspects of geometry education. The idea of connecting these “two geometries” refers to a concept of intercultural perspective of mathematics education (Prediger, 2004).

Intercultural aspect of mathematics education

The intercultural aspect of teaching mathematics refers to the relationship between mathematics as a scientific discipline, about mathematics education and about mathematics in everyday culture. Susanne Prediger in her book (2004) finds mathematics education as an intercultural process between everyday culture and mathematics as a part of scientific culture. Connecting these two aspects of mathematics should be one of the aims of mathematics education. It can be achieved by appliance of mathematics in everyday problems. Heymann’s concept of “Mathematik als Inventar der Lebenswelt” (Heymann, 1996) matches this idea. Also, between everyday mathematics and mathematics as a formal discipline there is another separate „culture“: the culture of mathematics education which has a long tradition and its specific characteristics, language, phrases etc.

Applying mathematics in everyday life is one of the aims of all school curricula. These aims are mentioned in Croatian curricula as well (MZOS, 2006; 2010).

But since mathematics culture does not have its specific geographic place, the role of teacher and textbook is very important in connecting everyday mathematical knowledge that a student brings from home, and the knowledge that is to be learned at school. This concept specifically reflects the geometry education. Teacher and textbook strongly affect the “contact success” of everyday and mathematical culture. The following two chapters include results and discussion about the role of textbook and teacher in geometry education.

Textbook in geometry education

Croatian Educational Plan and Program for Mathematics (MZOS, 2006, p. 238; MZOS, 2010) points out the importance of conceptualization of space relationships and objects in mathematics education, the importance of applying mathematics in everyday life, as well as using modern technologies in mathematics education. Therefore it was interesting to research the mathematics textbooks requirements in Croatia.¹

The analysis encompasses all examples and exercises from the three most used textbooks in the 6th grade (school year 2005/06), in the 7th grade (school year 2006/07) and the 8th grade (school year 2007/08). These textbooks were used by more than 80% of pupil population in every mentioned generation². Every example and exercise was analyzed according to various parameters. The main aim was to find out which competencies are required from Croatian pupils within each particular textbook item: Are the presentation, computation, interpretation or argumentation skills required? Also, the research considered the complexity requirements: Does a particular item require reproduction, connection or reflection knowledge? The reflection competency refers to deliberating mathematical relationships that are not directly visible from the given problem, deliberating the mathematical models, interpretations, argumentations and reasoning within the given situation. The base for this instrument is taken from the Austrian educational standards for mathematics (IDM, 2007). Besides that, the analysis encompassed the question type (open-constructed or close-constructed question). The intercultural dimension was researched through the context requirements: is the context within the particular task intra-mathematical, realistic or authentic. The realistic context within this study is defined as a situation that imitates the real life situation. Authentic situation does not imitate, it is actually taken from the authentic data. Example for the realistic situation: "Ivan has a computer monitor with side lengths 33 cm and 24 cm. What is the area of his screen?" Example for the authentic situation: "Measure the side lengths of your computer monitor or TV. Then find its screen area."

The geometry education from the 6th to 8th grade in Croatia encompasses the following topics: triangle, quadrilateral, polygon, circle, congruency and similarity, isometric mappings, Pythagorean Theorem, space and geometric solids (MZOS, 2006). This paper presents the results for some of these topics.

The analysis results of the topic Triangle in the 6th grade show the significant difference in the offered items amount in the researched textbooks (Table 1). The number shown in the brackets of the first table row refers to the total amount of tasks within the particular topic. The percents refer to the portion of a particular activity in the total. The first four table rows contain presenting, computation, interpretation and argumentation activities. Since one textbook problem could require different activities, it is to mention that the sum of the activity portions does not

¹ The analysis is conducted within the author's PhD research at the Institut für Didaktik der Mathematik, Aplen-Adria Universität, Klagnefurt.

² In this paper the textbooks were given codes A, B and C. Readers interested in original are invited to contact the author.

necessarily give 100%. The analysis results show that the activities of computation and operation dominate in the Triangle topic (Table 1). Calculations refer to finding triangle circumference and area, while operating mainly refers to executing the construction steps. The argumentation, reflection and open answer requirements are not required at all (or are barely required) in the analyzed textbooks within the triangle topics.

Triangle	A6 (568)	B6 (677)	C6 (837)
Presentation, modeling activities	38%	30%	46%
Calculation, operation activities	59%	68%	58%
Interpretation activities	29%	41%	37%
Argumentation activities	0%	3%	5%
Direct rule application, reproduction knowledge	49%	50%	48%
Connection knowledge	50%	49%	52%
Reflection	1%	1%	0%
Closed answers required	98%	97%	96%
Intra-mathematical context	98%	97%	99%
Realistic context	2%	3%	1%
Authentic context	0%	0%	0%

Table 1. Requirements of the Triangle topic (6th grade).

The results also indicate the domination of intra-mathematical items, i.e. the domination of symbolic items that are not put to a context. Similar results are obtained for the topics Quadrilaterals and Polygons. The lack of reflection and argumentation requirements can be also noticed in the topic Circle (Table 2), as well as the domination of closed constructed answers with intra-mathematical context. Computation and operation skills are emphasized comparing them to presenting (construction) and interpretation skills. This result can be connected to the requirements of topics Circle Circumference and Area where dominate computation tasks. Direct applying formulas can be also connected to the domination of reproduction requirements, especially in the textbooks A7 and C7 (Table 2).

Circle	A7 (753)	B7 (306)	C7 (381)
Presentation, modeling activities	25%	27%	26%
Calculation, operation activities	62%	71%	59%
Interpretation activities	22%	10%	28%
Argumentation activities	0%	5%	3%
Direct rule application, reproduction knowledge	72%	45%	58%
Connection knowledge	28%	55%	40%
Reflection	0%	0%	2%
Closed answers required	100%	94%	93%
Intra-mathematical context	96%	92%	79%
Realistic context	3%	6%	17%
Authentic context	1%	2%	4%

Table 2. Requirements of the Circle topic (7th grade).

The content related to Pythagorean Theorem is historically related to needs of everyday mathematics. On the other hand, it also successfully connects everyday mathematics with formal side of mathematics (through the logical steps of the proof of Pythagorean Theorem, or through the inverse of the Pythagorean Theorem etc). The Table 3 presents textbook requirements in the Pythagorean Theorem topics. All three analyzed textbooks emphasize the domination of computation requirements. Presenting and interpretation are required to a small extent, while argumentation, reflection and open answer types are not required at all. On the other hand, all analyzed textbooks emphasize tasks with connection knowledge requirements. It is interesting to notice that the emphasis is not put on the context tasks, in spite of the potentials of using Pythagorean Theorem in the everyday situations.

Pythagorean theorem	A8 (797)	B8 (474)	C8 (723)
Presentation, modeling activities	8%	8%	7%
Calculation, operation activities	92%	89%	95%
Interpretation activities	13%	13%	13%
Argumentation activities	0%	4%	2%
Direct rule application, reproduction knowledge	41%	45%	37%
Connection knowledge	59%	55%	63%
Reflection	0%	0%	0%
Closed answers required	100%	96%	98%
Intra-mathematical context	91%	89%	98%
Realistic context	9%	8%	2%
Authentic context	0%	3%	0%

Table 3. Requirements of the Pythagorean Theorem topic (8th grade).

Stereometry education has always been part of geometry education. In the beginning of the first grade Croatian pupils learn to recognize basic geometric solids, in the 3rd grade they learn measuring volumes, and in the fourth grade the basic elements of cube and cuboid (MZOS, 2006). After that the space geometry comes not before than the end of the 8th grade (with topics: relationships in the space, geometric solids, formulas for volumes and surface areas).

The Table 4 presents the textbook requirements from the topic Geometric solids in the 8th grade. The results show the domination of computation requirements and closed-answer types with intra-mathematical context. About one third of each textbook items required interpretation activities as well. Interpretation here mainly refers to sketch interpretations in order to solve the problem (for example, searching the relevant right-angled triangle in the pyramid figure). Therefore the geometric solids items put emphasis on the connections knowledge.

The textbook problems with volume or surface area requirements are pretty suitable for composing problems from everyday life (realistic or authentic). But however, the results show that such tasks are mainly intra-mathematical (Table 4).

Geometric solids	A8 (1048)	B8 (913)	C8 (726)
Presentation, modeling activities	4%	4%	3%
Calculation, operation activities	86%	84%	92%
Interpretation activities	32%	30%	33%
Argumentation activities	0%	0%	1%
Direct rule application, reproduction knowledge	59%	59%	34%
Connection knowledge	41%	39%	65%
Reflection	0%	2%	1%
Closed answers required	100%	96%	97%
Intra-mathematical context	89%	75%	94%
Realistic context	8%	19%	5%
Authentic context	3%	6%	1%

Table 4. Requirements of the Geometric solids topic (8th grade).

The curriculum and textbook analysis shows that the geometric solids are represented in the first grade on the recognizing level, and after that in the 8th grade with the strong emphasis on computations. Such an approach could influence the spatial ability competencies at the Croatian pupils. For example, Čižmešija and Milin Šipuš (2009) present the problem with spatial ability in the population of students of mathematics teacher education programmes.

The obtained results of textbook analysis suggest that in the geometry education dominate computation in the pure mathematical surrounding without context. Also, the results show the lack of geometric problems with argumentation requirements. Many existing tasks could be extended by additional open-answer task such as “Explain it” etc. It is important to mention that the reflection competencies, open answer types or argumentation requirements are not particularly prescribed in the current Educational plan and program, which emphasizes the computation and operation requirements (MZOS, 2006). The analysis shows that the textbook requirements match these curricular requirements.

On the other hand, the Educational plan emphasizes the application of geometrical contents in everyday life, but the analysis shows that textbooks mainly offer intra-mathematical tasks with no context. At this point we notice the discrepancy between the curricular and the textbook requirements.

Teacher in geometry education

The traditional approach given in the Educational plan and in textbooks affects the teachers’ practice, attitudes and beliefs. In the research of Glasnović Gracin and Domović (2009) the surveyed mathematics teachers declared that textbook contents directly influence their instruction in the classroom. The research of Baranović and Štibić (2009) showed that surveyed mathematics teachers in Croatia “*put the biggest emphasis on solving mathematical tasks and the development of logical thinking, while the application of mathematics in everyday life and critical*

thinking about mathematical concepts and procedures they consider less important" (ibid, p. 96). This finding matches the deficit of intercultural and reflective requirements in mathematics textbooks.

We can conclude that the teachers' instructions depend on curricular and textbook requirements, but it is always to remember that teachers' personal beliefs and attitudes are very important for instruction as well (Leder, 2002). Firstly it is important to consider which beliefs mathematics teachers have towards mathematics, because the teachers' opinion towards mathematics reflects to their instruction in mathematics education (Cross, 2009). If a teacher considers mathematics as mainly a set of procedures, computations and operations, his/her instruction in the classroom will follow these beliefs. It is also well known that these attitudes and beliefs are very strong and change slowly. The mathematics education reforms will not be effective unless we try to influence changing teachers' beliefs. It is a long process with the beginning already in the university programs for future teachers.

The concrete ideas for teachers' contribution to geometry education in the intercultural and reflective manner are: implementing more authentic problems and projects, as well as encouraging explanations and argumentations about geometrical concepts. Also, the potentials of computers, especially dynamic geometry software could be used for discovery learning and for the insight and understanding the geometric ideas (Schneider, 2002). Besides that, a teacher who knows well his/her class can contribute to interculturality by using different students' potentials as well as encouraging ideas instead of routines.

Conclusion

The insight into the Educational plan and program (2006) shows the domination of requirements for computation and operation within the geometrical topics. These requirements match the requirements from the analyzed mathematics textbooks in the 6th, 7th and the 8th grade. These results also correspond with the survey results on traditional character of mathematical instruction in Croatia (Baranović and Štribrić, 2009). On the other hand, the Educational plan and program clearly emphasizes appliance of mathematics in everyday life. However, the textbook analysis shows the significant lack of context items. Therefore one can conclude that there is a discrepancy between curricular and textbook requirements in the intercultural domain.

The obtained results suggest stronger connecting of everyday situations with abstract geometrical contents, encouraging argumentations and explaining, reflective knowledge, and understanding geometrical concepts. The new curricular document (MZOS, 2010) brings significant steps forward concerning these suggestions. Besides the curricular level, the intercultural dimension should be encouraged at other levels as well, for example within university programs for future mathematics teachers, in instruction, and in the field of textbook development.

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Mathematics as a factor in the development of a well-educated individual

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Abstract. The focus of this research is the influence of mathematics on the development of a well-educated and desirable individual. A research including the way students regard these characteristics after completing general education has been conducted. Upon inspection of all the aspects of math teaching and their effect on the formation of desirable personality characteristics, the results have shown that math teaching is in fact a perfect way to influence the development of a well-educated personality. As is the case with the changes, there are no recipes here, all that can be relied upon are the good experiences of the teachers who are the ones responsible for making math teaching and influential part of formation of a young desirable personality for the 21st century. At the end of this paper there are some recommendations to help teachers make mathematics teaching more interesting and efficient in the mission of developing the characteristics of desirable young individual.

Keywords: a well-educated young individual, math teaching methodology, a desirable mathematics teacher.

Introduction

During great changes in curricula, not only in Bosnia and Herzegovina, but also in the world, when new standards are being set and decades old programs being altered, it requires a great deal of thinking about the way math teaching should be conducted in our time. Research and studies all over the world in the area of math teaching methodology brought to light important conclusions regarding this matter. These conclusion have the effect on curricula in the countries where these experts work. In this regard, the members of the European Union came to a mutual image about what kind of person they perceive, educated and desirable for the 21st century. It has been concluded that the desirable qualities or outcomes after general education are as follows. A well-educated young individual:

- an independent student and decision maker,
- has good report with his peers and grown-ups,
- is literate and successful in communication,
- easily adjust to new situations,
- is an adventurer and willing to engage to new experiences,
- co-operates as a part of the team,
- has a sense of responsibility and disciplines,
- has moral and spiritual sensibility,
- is able to function of a team,
- is prepared for challenges which society offers,
- is tolerant and overcomes stereotypes,
- has a sense of prosperity and is able to lead a secure and fulfilling life,
- is a student who takes initiative, who is also able to claim control and responsibility regarding the aspects of his/her own studying process.

These demands, or better yet, desires, are inspired by the changes in society we live in, in which the education system must be changed as well – and math teaching is no exception. Mathematics has long been considered as the queen of sciences. However, that does not mean that is not subject to changes which permeate concepts, as well as approaches, especially those regarding methodology. For a long time *“math teaching methodology in different countries and math circles had not been accepted as a scientific discipline, there were no investments in researches in this area, not in many places were there postgraduate scientific studies in math teaching methodology which would give birth to experts and guarantee the appropriate teaching quantity and quality plans throughout the entire education vertical structure”* (Glasanović Gracin, 2010). This is where things must be changes because the modern trends point out that math teaching should be described as student-oriented, meaning the dominant part of the teacher should be in the background, and the student’s activity in mathematics class should be increased through different types of work (pair work, group work, work on projects, studying through research, etc.). By applying this, a teaches is no longer in the position of the main conductor of knowledge, but becomes a coordinator, and organizer of the teaching process, or, in the words of Delors (1998) he/she *“leads the process, an does not form it”* (Delors, 1998:162). It must be understood that the educational services are the most subtle and most important in the world of services. It is necessary to rid oneself of stereotypes about the relationship between teacher and student. Kotler (1991) claims that *“we [teachers] do not offer satisfaction to the buyer [student] by serving them, rather they make us satisfied by giving us the opportunity to serve them”* (Kotler, 1991:23). Those who work in teaching profession should be clear about their mission which has not changed in centuries to bring up, to educate, and to strengthen the students – the future carriers of social developments.

This research has given a review regarding above mentioned demands which embellish a well-educated individual as well as the role and ability of math for the achievement of those. Likewise, there is an over-view of research among first year students of the University of Tuzla. Their level of satisfaction regarding the achievement of afore mentioned outcomes upon completion their high school education. Based on personal knowledge and experiences of probationers, as well as

contemplations of different theoreticians, it was possible to extract certain recommendations guidelines for teachers, which include the improvement of the role of the math teaching methodology in the formation of a desirable individual for the 21st century.

Paradigm and methodology of researches

The aim of research in this paper is to identify the present state of efficiency of general education in connection to reaching general, desirable, educational goals, as are defined in the countries of the EU, and create a connection between them and the application of different methods and forms of work in mathematics teaching.

The proposition of this research in this paper is: establishing the students' level of satisfaction with their general knowledge from the aspect of acquiring the characteristics of a well-educated individual, and to comprehend, based on this research, and theoretical contemplations of different authors, the possibilities of math teaching in reaching those objectives – creating desirable characteristics of a well-educated individual.

According to this, a following **hypothesis** is set: “Upon completion of general education, good results are achieved, to a satisfactory level, in forming the competences of desirable personalities.

Math teaching should bring a great contribution to this procedure. As follows, based on theoretical and practical experiences of researchers and students, recommendations will be provided for a more efficient role of math teaching in the formation of competences of a well-educated personality in high school education.

It was necessary to determine “**student test group**” which would guarantee, based on their points of view, the existence of the very same points of view and opinion in all students, after they have graduated from high school, “as a basic group or population” (Muzic, 1979:534).

The test group of participants in this research was selected purposefully (purposeful sample) since only first year students of in schools for teachers, 292 of them, were taken into consideration because they have a more sensitive relation toward teaching and its objectives. To sum up, such test group was selected because it is expected to “learn the most from them”, that is, gain the most useful information (Merriam, 1988:61).

In this research, a **survey** was conducted, something that Muzic (1999:82) equalizes with “*the process of simultaneous written data acquisition from a greater number of people (test subjects) about something they know, feel or think*”. Considering the fact that it is not functional and very expensive to acquire answers from the entire basic sample group (all freshman students) it was decide to conduct a survey in order to reach the answers to posed questions from the selected sample, as a subgroup of the basic sample, “*in order to determine a particular distribution of behavior, thought, and principles in the basic sample*” (Ristić, Z. 2006:350).

The survey was conducted using survey questioners of twelve questions with the Likert scale and one open question, and or better yet, possibility for students to

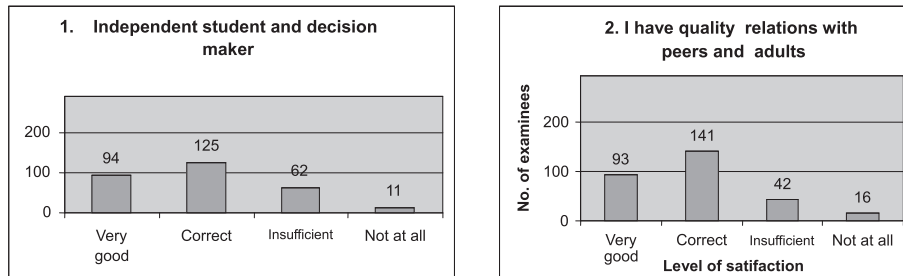
say whatever they want regarding the subject in question. So, the questioner covers eleven “wishes” from the list of demands needed for “a well-educated person”.

The limitations in the research exist in connection to generalization. The test group in the conducted research was chosen spontaneously. As such, it cannot guarantee the possibility of generalization (including wider population), that is, the parameters of Bosnia and Herzegovina. However, that was not even the intention of the researches, since the Bosnia and Herzegovina parameters are treated in variety of intensities, in the implementation of educational reforms, from the segment of curricula, up to securing quality and professional and vocational development and strengthening of teachers as well as the management in schools.

The results of the research

Data acquisition was performed through questioners. The results for every question from the survey are represented through graphs, where Microsoft Windows Interface was applied. Based on this and the answers to the open twelfth question, as well as through experiences of different authors, the results yielded suggestions to improve the role of mathematics teaching in creation of personality competencies, a personality which is, according to the opinion of experts in the EU countries, “a well-educated personality”.

Mathematics and the independent student- decision maker



Graphs 1. Answers to questions one and two.

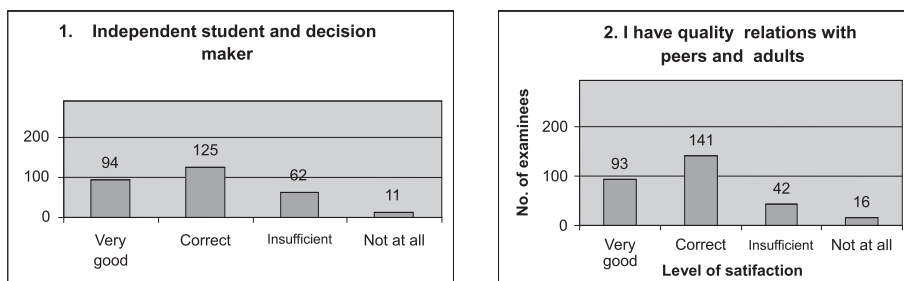
The research has shown that first year college students, not so long ago high school students, 75% of them believe that they have achieved independence and capability, or great capability to make a decision on their own, after the completion of high school. Twenty-five per cent of them believe that they have not acquired that competence sufficiently, or almost not at all. They think that “*nevertheless, even after finishing high school certain decision cannot be made by oneself, advise form elders is needed*” (from the questioner) because “*after finishing high school nobody is that much capable of starting living on their own, or taking care and responsibilities for themselves. Assistance of parents is inevitable in all segments, even though we are much more reliable and take life more seriously*” (from the questioner).

Solving problems from different areas of math is filled with opportunities for individual work and decision-making. Traditional math teaching mostly relied on

the teacher, as a lecturer, and students who individually solve tasks afterwards. In other words, the student have the passive role while acquiring new knowledge. Strengthening students in individual work and research, problem solving, choosing studying methods and presentation of accomplishments, leads to the creation of feeling of security and ability of individual, independent, and argument presentations in the student, which is in the end needed for making real life viable decisions, which would be accepted by all those concerned.

Mathematics and peer relations, readiness for team work

Traditional math teaching was mostly based on the introduction and the main part, where the teacher dominates, with co-operation from the same students all the time, who fill the empty space, and the conclusion of the class, which consisted of short revision of subject matter and giving homework, sometimes pointing out materials. To put it simply, group work was conducted only in the occasions of practice teaching classes or visits by the principle or councilors from pedagogical institutions. Certain studies have shown that it is much more efficient to learn peer-peer learning, through group work. Of course, apart from individual and group work, as well as pair work, the frontal approach has its place in teaching, depending on the subject matter and special aims of teaching units. However, if individual development has an effect on the development of independence in students, as well as, sensibility regarding right and rational, but also routine decision making (Agić et al. 2006) then group work is “responsible” for the development of sense of team work, but not just any kind, this is efficient team work, and for successful communication and good report with peers as well as adults. Group work is especially important, for subject matter revision (homogenous groups), but also for teaching classes (heterogeneous groups). As it true that every specialist doctor is a doctor, and that not every doctor is a specialist, such is the case for groups and teams. Every team is a group, but every group is not a team (Belbin, 1996). Employing group work frequently, with a clear objective, skillful enticement by the teacher, will surely leads to development the above mentioned competences, very much useful for living. Applying group work is not efficient if the goal is not transformation into team work, which according to Belbin (1996) is characterized by a small number of group members, joint approach to the task, complementary knowledges and skills, commitment to a mutual purpose, concrete work aims, joint responsibility, enthusiasm, joint creed and identity, joint experiences and mutual



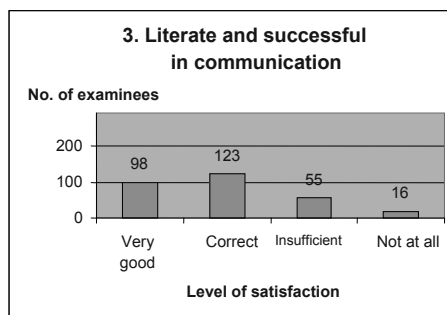
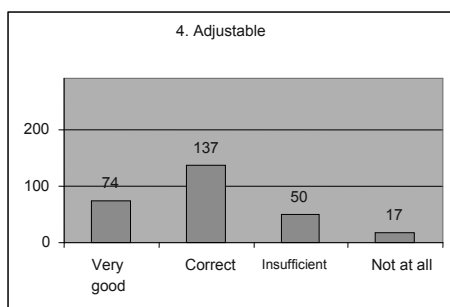
Graphs 2. Answers to questions six and five.

amicability of the members. English scientist **Belbin** (1996) proved that **teams comprised of the best experts in a certain area are far less successful than those consisting of various members**. This is something that should be taken into account, while forming groups – students should never be divided into groups using the principle “two desks=one group”, but using the criteria that a group comprises different types of students from the aspect of their concept of studying and the levels of their abilities.

The research has shown the high level of agreement between examinees with the claim that “primary and secondary education help students to **form good relations with peers and adults**,” even eighty per cent of them highly and very highly agree with this claim. However, every fifth student does not believe so. It was possible to notice an attitude in the questioner: “*When it comes to me, I can say with great certainty that, upon completing high school I became an independent decision maker and I have developed my relations with peers, as well as people of all ages, and I have manage to function*” (from the questioner). The information that over seventy-nine per cent of examinees are able to function as an efficient team member can be deceptive, because the question remains whether students know the theory about the teams and team work at all, since that is not included in the curriculum of the formal education. That piece of information indicates a need to include at least in the curricula teaching colleges sum content regarding education management, more precisely, HUMAN RESOURCE MANAGEMENT, where team work is included – it is very much necessary to pay much more attention to group and team work. Of course, support should be provided for teachers when it comes to significance and application of teams, in order for them to be able to motivate the very same among their students during the realization of usual curriculum. Extra-curricular activities and work of different work groups, definitely offer more opportunities for team work organization. It should be kept in mind that work with teams begins with the lowest levels of education.

Mathematics, literacy and successful communication

Together with successful communication and making good, rational decisions, literacy is an important companion, which can be very well affirmed in mathematics. We should part ways with the stereotypical ways of thinking and old definitions of mathematical literacy which stems from syntax and phonetics as opposed to



Graphs 3. Answers to questions three and four.

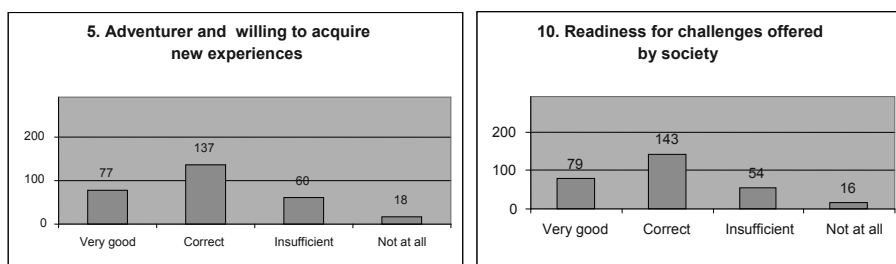
mathematical rhetoric and way of thinking, argumentative opinion expressing, discussion about various possibilities – without and it that is the way we do it, even in, lower grades of primary schools, discussing solutions of equations and non-equations, but discussing through the system “what would happen if. . .” etc. The research has shown that students are greatly satisfied with acquired literacy in secondary school. Over 75% believe that they are satisfied or very satisfied with that acquired skill. Nevertheless, the fact that the other 25% is not sufficiently (19%) or not at all (6%) satisfied with acquired literacy upon completing secondary school, cannot be ignored. Communication and literacy are very tightly connected.

One of the examinees says: *“I am successful in communication, but I don’t think I’m literate enough”* (from the questioners). When it comes to mathematical literacy, it is quiet clear that surpasses the meaning of the linguistic literacy, and reaches deeper into understanding problems, which are the basis for a more effective contemplation, bringing conclusions, and finally decision- making. OECD describes mathematical literacy as *“the ability of an individual to recognize and understand the role which mathematics hold in the world, to make well-founded decisions and to apply mathematics in ways which correspond to the needs of life of that individual, as constructive, intrigued and thinking citizen”* (Brash Roth et al., 2008:124). An interesting information was acquired during research and is related to adjustability. A great majority of students believe that, upon completing high school education, they acquired in a great or good enough measure, the skill of adjustability. Even 25% believe that they are able to adjust very well, while almost 47% of students are satisfied with this acquired skill. The question of the role of math in that process remains. It is certainly not negligible. Again we return to the role of a teacher who diligently applies the principles of suitability, where he or she knows how to adjust not only to the levels of development of students, but also to their momentary psychological state and specific demands.

Mathematics and readiness for adventure and exploring the new experiences

Math teaching is the perfect tool to develop the sense for adventure, and wish to explore things in a child. In this case, a suitable method is learning by revealing, which refers to the possibility for “students to individually, through experimenting, reach new knowledge, ideas and solutions to problems” (Glasanović, 2010). it should be clear that during realization of classes by heuristic methods, scientificity and classical directionality toward definitions and evidence is neglected, but that does not mean that these classes are not systematically organized and useful. On the contrary, the teacher should organize such a class well, while being familiar with the appropriate methods and abilities – their own, as well as those of students (or example, while using computers).

All things considered, the research method and its appropriate application may invoke a sense of traveling into the unknown, via system “what if. . .” in students. It represents a condition for creating pre-requisitions in a young human being for adventure, i.e., dealing with the unexpected and unknown. The research has shown that students are, after finishing school, in a great measure ready to try new experiences (over 73% of them) and new challenges (76%) which can be seen in graph 4. *“I am satisfied with my accomplishments so far and I would gladly like to gain more knowledge, and face many challenges,”* stated one of the examinees, from the questioner.



Graphs 4. Answers to question five and ten.

Of course, the role of the teacher crucial here as well. He/she should support and encourage students in their creativity, but also control those actions which may have unfavorable results, because of which students should not experience possible problems due to their attitudes with which the teacher sometimes does not agree. New things cannot be tested if the students are required to use only well-oiled forms and clichés. On the contrary, we must provide support for them in expressing all their attitudes related to any sort of problem, and teach them to analyze all the possibilities and reach favorable results through the method of elimination. There will be no creativity and adventure seeking in students, if we set boundaries and punish incorrectly stated attitudes. This way we will have only yay-sayers and head-nodders who will be unable to contribute to themselves and community.

Teachers believe that for successful work with students the following characteristics are necessary:

- they should be sufficiently knowledgeable about the subject they teach,
- they are able to connect mathematics to other subjects,
- they like children,
- they are familiar with developmental problems in children,
- they have more knowledge than average people, they are versatile and witty,
- they have a sense of humor,
- they have strong will power,
- they develop positive interpersonal relationships, they are subtle and kind,
- they trust their students,
- they appreciate unusual ideas,
- they allow and encourage self-initiative.

From these characteristics stems the conclusion that teachers greatly appreciate those qualities in their colleagues which are compatible with the demands given at the beginning of this paper. Mostly appreciated by the teachers are the last several characteristics, which initiate direction toward development of positive interpersonal relations, responsibility, creativity and independence in students.

The very same research also refers to desirable characteristics of teachers from the aspect of the student:

- sense of humor,
- steadiness and sedateness,

- seriousness, kindness and amiability,
- civilized behavior, and openness and adaptability,
- watchfulness, compassion and readiness to forgive,
- affability, modesty and even temper,
- perseverance, hard work and modesty,
- making presentations interesting and humorous,
- not interrupting students when they respond to questions, **not reading from books during lecturing,**
- grading justly, frequently tests student's knowledge,
- is mild in grading, gives the grade publicly and allows students to concentrate.

It is obvious that the students greatly appreciate qualities which promote openness and steadiness, and kindness, which greatly inspires development of responsibility and respecting people in general. It is interesting that students find these qualities more important in teachers than their directionality toward subject matter continuous student testing. Taking into consideration all the desirable characteristics of teachers of both angles, it can be concluded that possessing and application of most of them guarantees development and acquisition of desirable qualities of the students, young men. A functional connection between points of these two groups of characteristics could be constructed. However, it would demand a lot more time and space. It can be easily concluded that this discussion surpasses the boundaries of this paper but that just appears to be so. Alongside with the researched subject matter, those who are being taught and those who teach or coordinate the process of teaching are crucial. In any case, in qualities from the previous subtitle are tightly connected with the characteristics and manner of work of teachers, as well as, their style of guiding students in the teaching process. It cannot be expected that students should be tolerant if they do not feel tolerance from their teacher and so forth. Also, it cannot be said that students do not learn anything, when it contradicts the very definition of the student, which has the verb study as its root. Teachers are there to find the new strategies of learning, which will successfully produce greater mobility in students.

Conclusions and recommendations

Taking into account this research, it can be concluded that the results have shown that students are satisfied or very satisfied with acquisition of almost all traits of a well-educated person. This satisfaction is highly or very highly expressed within the range between 70 – 86% in all questions asked. Therefore, it is possible to establish that the set hypothesis is confirmed, that is to say, that “after completion general education, in a satisfactory measure, good results are achieved in the formation of competences of a desirable personality.”

Some would be satisfied with this, however, answers to the open question, including free comments by students, point to the caution. A question could be posed whether some other researching method, such as interview or open question survey, would yield the same results. That, of course, may remain as a doubt and a

subject of additional research. Nevertheless, the question of making mathematics more useful in the process of reaching above set demands is much more important.

Having that in mind, recommendations are provided, which came into being as the fruit of examining the work of various theoreticians and practitioners, which can therefore be useful to the teachers while aiming at a more efficient achievement of above mentioned requirements.

Recommendations

It can be recommended that a math teacher does not have to be an engrained scientist and that in math teaching requires scientific methods to be placed up front. So, we should first employ heuristic thinking, presupposition and ideas in order to build and prepare the grounds for generalization or even for deducing tangible evidence. Merely pointing to expressing claims and proving them, without prior routing it in the culture of the student, has no justification if we aim at making them self-thinking and useful members of the community. In math teaching, students can first examine the new subject matter from all aspects, express all their attitudes related to the subject matter, and then, while analyzing all those aspects can they generalize and gradually transcend to “more strict” mathematical level. This manner of work provides the students with the possibility to work in their own pace and also yields more respect for differences between the students and their points of view.

Teaching is a living process which can not be completely planned and channeled. Each mathematical problem is unique and has its beginning and end. During the process of solving a problem one stumbles upon something of a revelation and creation. This is why it is necessary for the teacher to truly develop curiosity in their students, willingness for individual mental activity and to point out the ways to new revelations, in order for students to realize that “every road leads to Rome”. A creative math teacher in creative classes, has good chances to develop creative characteristics in their students, especially the sense of efficient dealing with unknown things and challenges.

Only cowards work the general case. Real teachers deal with examples (Bruckler, 2006).

It is widely known that students mostly look up to their teachers, from being influenced in their further professional direction, or by mimicking they acquire some of their characteristics. So, for students, mathematics as a science, at least in a great measure, is not a motive for the studying. The same author (Bruckler, 2006) claims that to pronounce theorem and then go forward is logical, but that is not teaching because it does not contribute to understanding. Incorrectness is not allowed, but simplifications are. That is to say, there is a greater interest in applying inductive teaching (from simple problems and examples towards generalization), then in deductive approach (stating claims, formulas, and then applying them) because the inductive approach puts the student and his or her activity and creativity up front, which contributes more to the acquisition of desirable qualities and skills of students. Basically, it is important to gain the students’ interest in mathematics.

Teachers are “the greatest culprits” for students’ interest or lack of it in mathematics. They are the ones who encourage students to practice math, with their

behaviour they can transfer to students **responsibility, relation toward work, respect, listening to others, adjustability, trying new things, tolerance**, and other very important traits which are crucial for imprinting the development of human personality.

This is why a teacher should be knowledgeable of various methods in order to create interest in students to adopt (not learn by heart) all the prescribed subject matter and in doing so acquire an appreciate level of mathematics knowledge and skills. To put it short, a desirable teacher should, in that case, be:

- a successful motivator,
- somebody whom students like,
- somebody who is trusted and not feared,
- a person whose intercession for the students' well-being is noticed,
- somebody who does not threaten students with the negative grades,
- a teacher who does not consider written exams as the only criteria for grading their knowledge,
- somebody who transcends upbringing values to students,
- a teacher who regularly follows the new trends in science and pedagogical practice,
- a person able to prepare "grounds" for formation of new concepts and analyze them with the students, instead of dealing strictly with definitions and theorems.

If a teacher constantly employs, same teaching strategies and the student is constantly unsuccessful, which one of them, in fact, is a slow learner?

Eric Jensen

Apart from all this, textbooks often place limitations for the teacher while he or she is applying creative approaches, creativity of a math teacher is often at a loss because of excessive reliance on the way how mathematical content is analyzed in textbooks (Kurnik, 2008). This is why it is necessary to work on studying strategies together with students who know how to use other resources. In short, it is necessary to forget the stereotypes about traditional teaching and performing an analysis class: introduction, revision of the necessary analyzed subject matter, the main part including analysis of unknown content, and the final part involving the affirmation of analyzed subject matter and distributing homework. Instead, teachers are advised to:

- open problem situation which will subtly include the necessary revision of subject matter for the new unit,
- demonstrate anecdotes, illustrations and examples,
- put forward motivational tasks,
- demonstrate the problem with an obvious example,
- provide an interesting information from history.

A teacher should remember their own time as a student. They should treat other people as they would like to be treated, or more precisely to treat children the way they would like others to treat their children – students.

In addition above mentioned recommendations, the following can also be useful:

- teaching as such should be applied in dosages (the student is not “a storage unit” to be filled with knowledge),
- various teaching methods should be used in a well thought out way of behaviour, action with the goal to make studying easier and to improve the outcome of studying,
- more attention should be paid to the obvious and applicability in teaching,
- it should be known that a teaching class is a creative stage of a teaching process,
- every student should be an active participant in a class,
- it is very efficient to study in cooperation in **groups or pair** and to use projects to turn groups into **teams**, while nurturing acceptable peer to peer correspondence; to state **opinion** using the power of arguments, instead of arguments of authority and power,
- successful can be described as “organized chaos”,
- students show greater interest in studying when new technologies are applied, which helps to achieve elegance in solving tasks, where students are instructed to notice it and use it,
- students should be encouraged by praise and treat them amicably,
- we should not allow students to “mire” with negative grade, but allow them to “improve” their knowledge part by part.

As stated before **team work** is very significant for successful work and good communication. It is clear is that every group is not a team, especially a successful one. Successful teams are characterized by:

- clear and transparently defined general and specific aims,
- a successful leader,
- individual and joint responsibility,
- respecting differences,
- open communication,
- efficient decision making,
- mutual trust,
- constructive conflict solving (Miljkovic and Rijevec, 2005),

from which it is transparent that a properly employed team work in math teaching can help students develop very desirable characteristics mentioned at the beginning of this paper.

Many teachers claim that their students do not take interest in anything, that they are restless, etc. That may be only partly correct. The word “student” stems the verb “study”. We should rather question our curriculum or our teaching methods. In such cases, the teachers are recommended to employ efficient teaching methods and to conduct themselves professionally. Well-planned teaching activities, which will encourage and sustain students attention, the teacher’s interest, almost elation (“relishing”) about the work they are doing, as well as teacher’s sensibility regarding mutual respect and understanding will most certainly yield particular results in

the process of mobilizing students during teaching, because the teacher who loves what he teaches transfers enthusiasm on students (Kovač, 2010).

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Mathematics education in Croatian elementary schools – the teachers' perspective

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Keywords: opinions of teachers, elementary schools, mathematics teaching, mathematics competence, students' activities, teaching materials and tools

The presentation is based on the findings of empirical research which examined Croatian teachers' perceptions of mathematics education. The research, which was carried out in 2010, included a representative sample of 625 math teachers (Grades 5–8) from elementary schools throughout Croatia. The administered questionnaire consisted of closed questions set mainly on 5-grade Likert scales and a few open questions.

The findings indicate that the traditional approach to math teaching prevails in most of its dimensions such as teachers' understanding of the concept of mathematical competence, teaching methods, teaching materials and tools as well as teachers' mathematics-related beliefs. For instance, when defining the mathematical competence of students, the teachers attribute less attention to students' learning outcomes which are characteristic of the contemporary concept of mathematical competence (PISA) in comparison to the traditional one: as most important aspects of mathematical competence, only 13% – 18% of teachers ranked students' reasoning about mathematical concepts and processes, critical thinking and exchange of ideas and results, whereas 32% – 35% of teachers emphasized hard and thorough work of students during math lessons, students' confidence in solving standard mathematical tasks and mastering the math teaching program.

The prevalence of traditional characteristics in math teaching is also evident from the findings on the teaching methods as well as teaching materials and tools used. According to the teachers' estimates, during mathematics lessons students most often resolve mathematical tasks: practice mathematical tasks similar to those at the exams ($M = 4.21$, $SD = .73$), resolve mathematical tasks using the procedures demonstrated by teachers ($M = 4.19$, $SD = .55$), resolve standard mathematical tasks ($M = 4.08$, $SD = .67$). In contrast to this, students most

rarely do autonomous research and experiments on mathematical concepts and laws/regularities ($M = 2.43$, $SD = .75$), explain their own ideas and conclusions in written form ($M = 2.46$, $SD = .81$) and carry out their own projects ($M = 2.73$, $SD = .81$). Equally, teachers most often use traditional teaching materials and tools (textbooks, handbooks) and most rarely ICT.

It is indicative that older teachers and teachers who graduated from teacher colleges tend to use contemporary teaching methods and teaching materials more often than younger teachers who hold a university degree (especially those who are at the age of 29 and younger). An exception to this is the use of ICT which is used significantly more often by younger teachers. These differences among teachers could be explained by a difference in length of teaching experience and different teacher education tracks: whereas older teachers have longer teaching experience and their teacher education gave them broader teaching knowledge and skills since they attended specialized teacher colleges, the younger teachers are less experienced and their teacher education took place at institutions (faculties) where the focus was subject mastery.

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We would like to thank our distinguished mathematicians (poster)

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Abstract. The previous years Croatia has lost a number of distinguished mathematicians who have contributed greatly to teaching mathematics. These contributions can be roughly divided into two crucially different, yet inseparable wholes.

Two leading figures in mathematics – prof. dr. Svetozar Kurepa (1929. – 2010.) and prof. dr. Krešimir Horvatić (1930. – 2008.) have in this area (Osijek) initiated, organized and coordinated the implementation and realization of supplementary studies in teaching mathematics during the 1970's and 1980's at the Department of Mathematics of the Faculty of Science in Zagreb. The supplementary studies gave the opportunity for a hundred participants to receive the diploma for teaching mathematics, which resulted in a far-reaching incentive for economic and scientific growth in Slavonia, Baranja and partially, Bosnia and Herzegovina.

The second group of mathematicians will be mostly remembered for the brilliant books they wrote, their journals and magazines, and activities done in order to popularize mathematics, as well as their scientific contribution to teaching mathematics. Those are the aforementioned professor Boris Pavković (1931. – 2006.) and academician Stanko Bilinski (1909. – 1998.), as well as Dominik Palman (1924. – 2006.), Vladimir Devide (1925. – 2010.) and Zdravko Kurnik (1937. – 2010.), all professors at the Department of Mathematics of the Faculty of Science in Zagreb.

The Organizing Committee would like to thank our distinguished mathematicians.

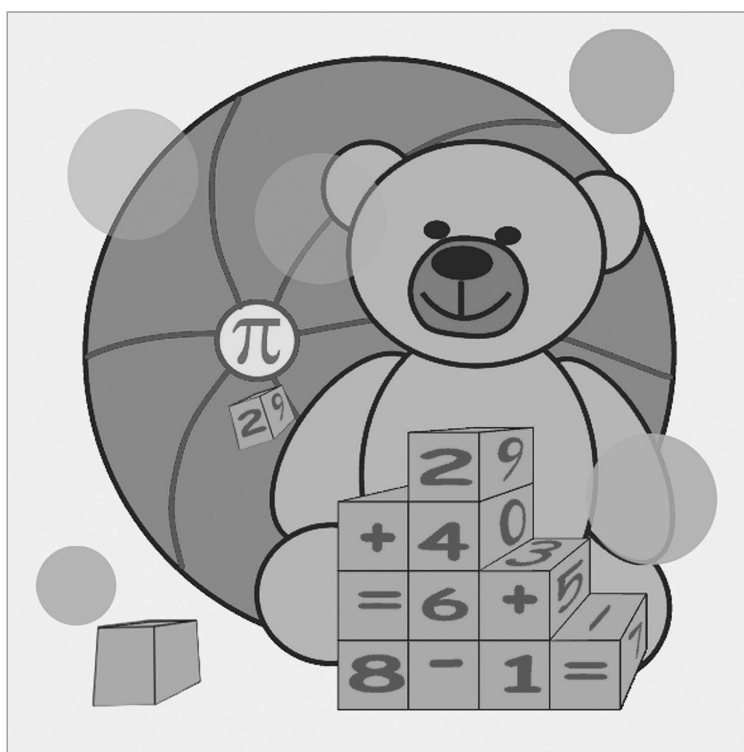
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Treći međunarodni znanstveni skup
MATEMATIKA I DIJETE
(Ishodi učenja)

monografija



Predgovor

U monografiji *Učitelj matematike* rabit ćemo termin *učitelj* za osobu koja poučava matematiku, a iz konteksta svakoga članka razumjet će se govori li se o učitelju u predškolskoj ustanovi, školi ili o sveučilišnom nastavniku.

Autori se u svojim radovima kritički osvrću na studijske programe prema kojima se obrazuju učitelji matematike. Analiziraju se rezultati istraživanja o razinama ishoda učenja na nastavničkim fakultetima s aspekta studentskih postignuća te predlažu rješenja i odgovori na aktualna pitanja iz nastavne prakse.

Matematika je jedno od devet akademskih područja za koja su u prvoj fazi projekta *Usklađivanje obrazovnih struktura u Europi*¹ dogovoreni zajednički okviri (stupnjevi/ciklusi, ishodi učenja, opće i specifične kompetencije, metode učenja i poučavanja, primjena ECTS sustava). U Hrvatskoj na razini vlade još uvijek nije postignut konsenzus u vezi s Nacionalnim okvirnim kurikulumom za područje matematike, iako je struka definirala ciljeve (engl. objectives), ishode učenja (engl. learning outcomes) i kompetencije (engl. competences) učenika/studenata na kraju svakoga obrazovnoga razdoblja. Stoga nailazimo na nedorečenosti kada je u pitanju matematička izobrazba učitelja, organizacija nastave matematike kroz pojedine obrazovne cikluse te učiteljeva cjeloživotna izobrazba i znanstveno napredovanje.

U prvom poglavlju monografije pod naslovom *Novi pristupi u izobrazbi učitelja matematike*, autori u svojim istraživanjima odgovaraju na pitanje zašto je učitelj matematike deficitarno zanimanje. Nadalje, autori u svojim člancima analiziraju zastupljenost i položaj matematike u programima studija kojima se osposobljavaju odgojitelji i učitelji matematike za djecu predškolske dobi, te osnovnu i srednje škole. Ukazuje se na neke nedostatke i propuste u programima učiteljskih studija te daju prijedlozi za iznovljavanjem istih. Predlažu se sadržaji i originalni postupci kojima bi se, prema mišljenju autora, mogla postići kvalitetnija izobrazba učitelja.

U drugom poglavlju autori izlažu rezultate svojih istraživanja provedenih među studentima učiteljskih odnosno nastavničkih studija. Istraživanjima su se ispitala razine studentskih postignuća u nekim, za struku, važnim vještinama i znanjima. Osobito važnima držimo rezultate ispitivanja koja su provedena s ciljem boljeg upoznavanja studentskih strategija rješavanja problema te njihovih sposobnosti prostornoga mišljenja.

U trećem poglavlju monografije autori svojim člancima nude odgovore na pitanje *Kako prema poželjnim ishodima učenja matematike*. Neki od autora u svojim promišljanjima stavljaju težište na interaktivni konstruktivistički pristup učenju i

¹ Tuning educational structures in Europe. www.tuning.unideusto.org/tuningeu, prva faza od 2001. – 2004.

poučavanju te progovaraju o važnom pitanju popularizacije matematike. Drugi su usmjereni informacijskim tehnologijama te istražuju kakav je odnos studenata učiteljskih studija prema korištenju ICT u nastavi, promišljaju o problemima, kriterijima i metodama odabira on-line materijala za nastavu matematike. Suvremeni uvjeti učenja traže osposobljenog učitelja i učenika/studenta za učenje na daljinu i primjenu inteligentnih sustava te se predlaže u iznovljenim programima učiteljskih studija pronaći prostor za postizanjem tih kompetencija.

U četvrtom poglavlju nazvanom *Učitelj i učenici u nastavnoj praksi*, autori u svojim prilogima progovaraju o rezultatima istraživanja i/ili dobrim primjerima strategija poučavanja matematike koji su rezultirali poželjnim ishodima učenja djece i učenika. Autori izvješćuju o tome da je među roditeljima, učiteljima i u društvu općenito prepoznata važnost ranoga matematičkoga obrazovanja. Također je sve više roditelja djece s teškoćama zainteresirano za uključivanjem njihove djece u redovitu nastavu na što nas obvezuje nacionalna strategija razvoja Republike Hrvatske². Stoga u predstojećem iznovljavanju programa treba planirati razvoj kurikula kojim bi se studenti učiteljskih studija osposobljavali za prepoznavanje, izobrazbu i podršku učenika s teškoćama.

U petom poglavlju monografije pod naslovom *Što može promijeniti učitelj matematike* autori u svojim člancima daju pregled uzroka slabog uspjeha u nastavi matematike, sagledavaju uzroke i posljedice takvoga stanja. Komentiraju neka opća pitanja o kojima ovisi prihvatanje matematike kao neophodnog znanstvenoga područja od strane šire društvene zajednice. Istraživanja pokazuju da odnos djeteta prema matematici značajno ovisi o odnosu njegovih roditelja i njegova učitelja prema tom predmetu. Analiziraju se pitanja vezana za udžbenike te komentira nastavnička perspektiva u Hrvatskoj.

Iz članaka zaključujemo da je učitelj matematike suočen sličnim problemima u Hrvatskoj kao i u zemljama okruženja. Postojećim sveučilišnim programima nije u dovoljnoj mjeri stručno osposobljen. Slabo poznaje školsku matematiku. Zbog skromnih materijalnih uvjeta na fakultetu i u školama nema priliku u dovoljnoj mjeri ovladati informacijskim tehnologijama i inteligentnim sustavima učenja. Uglavnom ostaje nespreman za stručni prihvati učenika s posebnim potrebama. Zazire od ozbiljnih problema djece i mladih kao što su droga, nasilje, alkohol. Nedostatne su mu kompetencije za suradnju s roditeljima i timski rad.

Vlade deklarativno propagiraju protok ljudi i važnost stručnog i znanstvenoga napredovanja. Postoje pomaci u uvođenju reda u obrazovne sustave. Većina učitelja bez pogovora podržava sve takve prijedloge, čak i kada su proturječni.

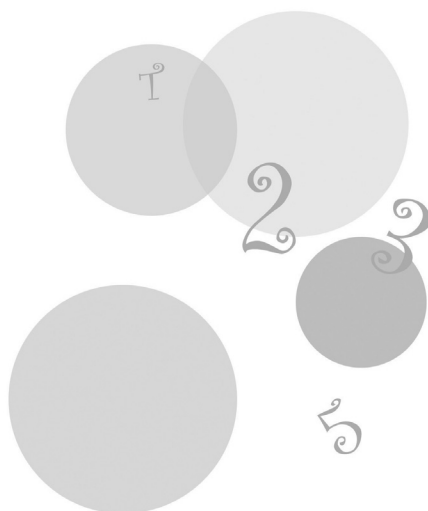
Ostaje nam nadati se da će akademska zajednica u Hrvatskoj, ali i u okruženju iznaći snage i umješnosti za donošenje i provođenje dobrih i važnih odluka u znanosti i obrazovanju, te da će u novim okolnostima učiteljeva izobrazba i cjeloživotno napredovanje biti slobodnije i domišljenije. Učenicima i njihovim učiteljima/cama nije mjesto na marginama društva. Učitelj ne smije biti potplaćen, jer njegov je "krajnji proizvod" oduvijek bio i ostaje – budućnost društva.

Margita Pavleковиć

² Nacionalna strategija izjednačavanja mogućnosti za osobe s invaliditetom od 2007. do 2015., Vlada RH, Zagreb, 2007.

1.

Novi pristupi u izobrazbi učitelja matematike



U ovom poglavlju autori u svojim istraživanjima odgovaraju na pitanje zašto je učitelj matematike deficitarno zanimanje. Analizira se zastupljenost i položaj matematike u programima studija kojima se osposobljavaju odgojitelji i učitelji matematike za djecu predškolske dobi, kao i nastavnici za osnovnu i srednje škole. Naime, 2005. godine većina studenata prvi puta je upisana po novoodobrenim tzv. bolonjskim programima. Pet godina kasnije, sredinom 2010. godine pripadnici te iste generacije polažu diplomske ispite na integriranim, petogodišnjim sveučilišnim učiteljskim studijima te studijima matematike nastavnoga smjera i u Hrvatskoj stječu akademski naziv *magistar primarnoga obrazovanja* odnosno *magistar matematike*. Uočavaju se nedostaci i propusti u programima učiteljskih studija. Predlažu se novi sadržaji i originalni postupci kojima bi se, prema mišljenju autora, mogla postići kvalitetnija izobrazba učitelja.

Zašto nedostaje profesora matematike? Stavovi gimnazijalaca o zanimanju profesor matematike

Sanja Rukavina, Dina Kovačević i Milan Bakić

Odjel za matematiku, Sveučilište u Rijeci, Hrvatska

Sažetak. Nedostatak je profesora matematike na svim razinama obrazovanja u Republici Hrvatskoj problem o kojemu se često govori u matematičkoj javnosti. Iako je ovaj nedostatak evidentan i široj javnosti kako kroz podatke Hrvatskoga zavoda za zapošljavanje, tako i kroz česte natječaje na kojima se traže profesori matematike i stipendije za deficitarna zanimanja na lokalnoj i državnoj razini koje se, između ostaloga, raspisuju i za zanimanje profesor matematike, činjenica je da svake godine ostaje mjesta na onim studijima matematike koji obrazuju profesore matematike.

Kako bismo pokušali odgovoriti na pitanje zašto nedostaje profesora matematike, proveli smo istraživanje o stavovima gimnazijalaca prirodoslovno-matematičkoga i općega smjera o zanimanju profesor matematike. Cilj je istraživanja ispitati stavove gimnazijalaca o zanimanju profesor matematike, faktore koji utječu na dotični stav te u kojoj su mjeri na isti utjecali, odnosno utječu njihovi dosadašnji profesori matematike. Također, cilj je i ispitati informiranost gimnazijalaca o međusobnom odnosu nekolicine zanimanja, među kojima je i profesor matematike, s obzirom na potražnju na tržištu rada i odnosu prosječnih plaća pojedinih zanimanja, kao i stavove o međusobnom odnosu odabranih zanimanja po kriteriju njihove atraktivnosti.

Ključne riječi: profesor matematike, stavovi gimnazijalaca

Uvod

Posljednjih se nekoliko godina uočava nedostatak profesora matematike na svim razinama obrazovanja. U razdoblju od siječnja do studenoga 2010. godine Hrvatskom je zavodu za zapošljavanje (HZZ) prijavljeno čak 400 slobodnih radnih mjesta za učitelje matematike i 553 slobodnih radnih mjesta za profesore matematike. S druge strane, u Hrvatskoj trenutno postoje četiri visokoškolske ustanove na kojima se obrazuju ovi kadrovi i prema podacima o njihovim upisnim kvotama svake bi godine moglo diplomirati 250 novih profesora matematike. Međutim, taj je

broj nerijetko i dvostruko manji. Nedostatak stručnoga kadra rezultira nestručnim zamjenama iz matematike koje često nisu u stanju na kvalitetan način prenijeti potrebna znanja iz matematike ni u učenike usaditi interes i ljubav prema matematici, koji su značajni čimbenici u odabiru zanimanja profesor matematike. Tako dolazimo do zatvorenoga kruga iz kojega se izlaz ne nazire.

Kako bismo pokušali odgovoriti na pitanje zašto nedostaje profesora matematike, proveli smo tijekom studenog 2010. godine istraživanje o stavovima gimnazijalaca prirodoslovno-matematičkoga i općega smjera o zanimanju profesor matematike. Cilj je istraživanja bio ispitati faktore koji utječu na dotični stav te u kojoj su mjeri na isti utjecali, odnosno utječu njihovi dosadašnji profesori matematike. Nadalje, zbog pretpostavke da stavovi srednjoškolaca o zanimanju profesor matematike, između ostaloga, imaju uporište i u saznanjima o visini primanja, zaposlivosti, cjelokupnoj situaciji hrvatske prosvjete i slično, istraživanjem smo obuhvatili i njihova mišljenja o zaposlivosti i visini plaće nekolicine zanimanja, među kojima je i zanimanje profesor matematike.

Metodologija istraživanja i rezultati

Istraživanje je provedeno anketiranjem učenika u četiri srednje škole. Anketirano je ukupno 412 učenika prirodoslovno-matematičkoga i općega gimnazijskog smjera. U obradu su podataka ušli odgovori 396 ispitanika. Nisu obrađivani nepotpuno ispunjeni upitnici. Uzorak su činili učenici svih razreda srednje škole, dakle, radi se o ispitanicima čija se dob kreće od 14 do 18 godina. Istraživanjem je obuhvaćen 91 učenik prvoga razreda, 86 učenika drugoga razreda, 83 učenika trećega razreda i 136 učenika četvrtoga razreda. Razlog zašto su istraživanjem obuhvaćeni učenici navedenih gimnazijskih smjerova jest očekivanje da će upravo oni činiti većinu budućih studenata matematike.

Raspodjela ispitanika po smjeru koji pohađaju i spolu prikazana je u Tablici 1.

Ispitanici	Opći smjer	Prirodoslovno-matematički smjer	Ukupno
Učenici	83	82	165
Učenice	192	39	231
Ukupno	275	121	396

Tablica 1. Broj ispitanika prema spolu i gimnazijskom usmjerenju.

“Zanimanje profesor matematike smatram atraktivnim zanimanjem”

Učinjena je analiza učestalosti pojedinih odgovora na prvo anketno pitanje “Zanimanje profesor matematike smatram atraktivnim zanimanjem” s obzirom na spol (Tablica 2.), gimnazijsko usmjerenje (Tablica 3.) i razred (Tablica 4.) ispitanika. Da bi se olakšala interpretacija rezultata, u pripremi su podataka združeni odgovori “u potpunosti se odnosi na mene” i “uglavnom se odnosi na mene” u kategoriju “odnosi se na mene”, te odgovori “uglavnom se ne odnosi na mene” i “u potpunosti se ne odnosi na mene” u kategoriju “ne odnosi se na mene”.

“Zanimanje profesor matematike smatram atraktivnim zanimanjem.”		
Spol ispitanika	Odnosi se na mene	Ne odnosi se na mene
Učenici	25, 45%	74, 55%
Učenice	17, 31%	82, 69%
Ukupno	20, 7%	79, 3%

Tablica 2. Učestalost pojedinih kategorija odgovora na anketno pitanje “Zanimanje profesor matematike smatram atraktivnim zanimanjem” s obzirom na spol ispitanika.

Svega 20, 7% ispitanika zanimanje profesor matematike smatra atraktivnim, dok čak 79, 3% ispitanika smatra da se radi o neatraktivnom zanimanju. Iako podaci HZZ-a upućuju na veliku zastupljenost žena među profesorima matematike u Republici Hrvatskoj, manji postotak ispitanih učenica smatra da se radi o atraktivnom zanimanju u odnosu na svoje muške kolege.

“Zanimanje profesor matematike smatram atraktivnim zanimanjem.”		
Usmjerenje ispitanika	Odnosi se na mene	Ne odnosi se na mene
Opći smjer	17, 09%	82, 91%
Prirodoslovno-matematički smjer	28, 91 + %	71, 09%
Ukupno	20, 7%	79, 3%

Tablica 3. Učestalost pojedinih kategorija odgovora na anketno pitanje “Zanimanje profesor matematike smatram atraktivnim zanimanjem” s obzirom na usmjerenje ispitanika.

Dobiveni rezultati potvrđuju da učenici prirodoslovno-matematičkog usmjerenja imaju nešto pozitivniji stav prema zanimanju profesor matematike od učenika općega usmjerenja. Takvi su rezultati u skladu s izborom gimnazijskoga usmjerenja koji učenici pohađaju. Istraživanja također pokazuju da učenici koji pohađaju prirodoslovno-matematički smjer imaju pozitivniji stav o matematici kao predmetu (Arambašić, Vlahović-Štetić i Severinac, 2005) pa je ovaj rezultat i u tom smislu očekivan.

“Zanimanje profesor matematike smatram atraktivnim zanimanjem.”		
Razred ispitanika	Odnosi se na mene	Ne odnosi se na mene
1. razred	30, 76%	69, 24%
2. razred	18, 59%	81, 41%
3. razred	25, 29%	74, 71%
4. razred	12, 49%	87, 51%
Ukupno	20, 7%	79, 3%

Tablica 4. Učestalost pojedinih kategorija odgovora na anketno pitanje “Zanimanje profesor matematike smatram atraktivnim zanimanjem” s obzirom na razred ispitanika.

U Tablici 4. se vidi da značajan pad od “boljeg” prema “lošijem” stavu o atraktivnosti zanimanja profesor matematike nastupa kod gimnazijalaca četvrtog razreda. Moguće je da to dijelom ima veze s činjenicom da se radi o maturantima koji su ozbiljno počeli promišljati o svom budućem pozivu, kao i o pozitivnim i negativnim stranama potencijalnih zanimanja.

“O zanimanju profesor matematike razmišljam kao o potencijalnom budućem zanimanju”

Analiza odgovora na ovo anketno pitanje provedena je s obzirom na spol (Tablica 5.), gimnazijsko usmjerenje (Tablica 6.) i razred (Tablica 7.) ispitanika. Kao i u prethodnom pitanju združeni su odgovori “u potpunosti se odnosi na mene” i “uglavnom se odnosi na mene” u kategoriju “odnosi se na mene”, te odgovori “uglavnom se ne odnosi na mene” i “u potpunosti se ne odnosi na mene” u kategoriju “ne odnosi se na mene”.

“O zanimanju profesor matematike razmišljam kao o potencijalnom budućem zanimanju.”		
Spol ispitanika	Odnosi se na mene	Ne odnosi se na mene
Učenici	12, 12%	87, 88%
Učenice	8, 65%	91, 35%
Ukupno	10, 09%	89, 91%

Tablica 5. Učestalost pojedinih kategorija odgovora na anketno pitanje “O zanimanju profesor matematike razmišljam kao o potencijalnom budućem zanimanju” s obzirom na spol ispitanika.

S obzirom na iskazane stavove o atraktivnosti zanimanja profesor matematike dobiveni su rezultati “lošiji” od očekivanoga. Naime, tek 10, 09% ispitanika razmišlja o zanimanju profesor matematike kao o potencijalnom budućem zanimanju, a čak 89, 91% ne vidi to zanimanje kao svoje potencijalno buduće zanimanje. Kao i kod prethodnog pitanja učenice su iskazale manje interesa za ovo zanimanje od svojih muških vršnjaka.

“O zanimanju profesor matematike razmišljam kao o potencijalnom budućem zanimanju.”		
Usmjerenje ispitanika	Odnosi se na mene	Ne odnosi se na mene
Opći smjer	6, 17%	93, 83%
Prirodoslovno-matematički smjer	19%	81%
Ukupno	10, 09%	89, 91%

Tablica 6. Učestalost pojedinih kategorija odgovora na anketno pitanje “O zanimanju profesor matematike razmišljam kao o potencijalnom budućem zanimanju” s obzirom na usmjerenje ispitanika.

Tek jedna trećina gimnazijalaca općega smjera koji su zanimanje profesor matematike okarakterizirali atraktivnim razmišlja o tom zanimanju kao o potencijalnom budućem zanimanju dok to vrijedi za približno dvije trećine učenika s istim stavom koji pohađaju prirodoslovno-matematički smjer. Moguće je da na ovakvo razmišljanje utječe i uvjerenje o vlastitoj pripremljenosti za uspješno stjecanje potrebnoga matematičkog obrazovanja s obzirom na fond sati predmeta Matematika u promatranim usmjerenjima.

“O zanimanju profesor matematike razmišljam kao o potencijalnom budućem zanimanju.”		
Razred ispitanika	Odnosi se na mene	Ne odnosi se na mene
1. razred	23, 06%	76, 94%
2. razred	5, 8%	94, 2%
3. razred	10, 83%	89, 17%
4. razred	3, 67%	96, 33%
Ukupno	10, 09%	18, 43%

Tablica 7. Učestalost pojedinih kategorija odgovora na anketno pitanje “O zanimanju profesor matematike razmišljam kao o potencijalnom budućem zanimanju.” s obzirom na razred ispitanika.

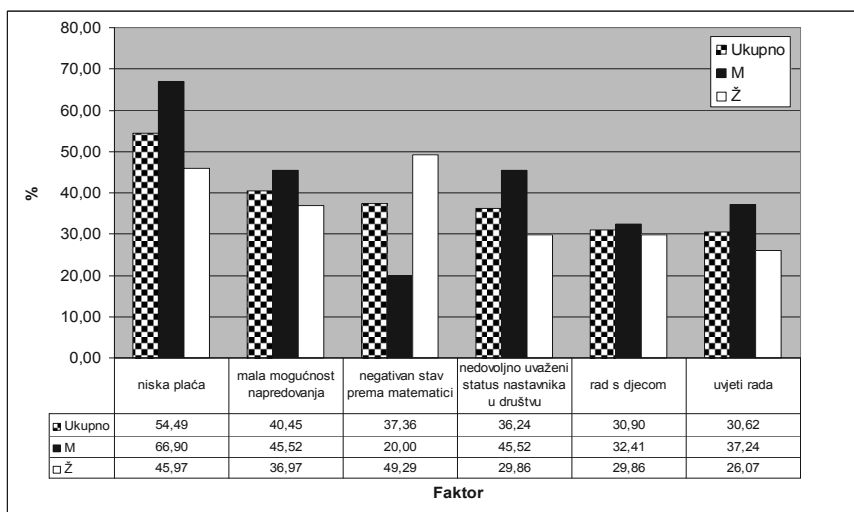
O zanimanju profesor matematike kao o svom budućem zanimanju razmišlja 75% učenika prvih razreda koji su to zanimanje ocijenili kao atraktivno zanimanje dok svega 29, 41% učenika četvrtog razreda koji to zanimanje smatraju atraktivnim razmišljaju o njemu kao o potencijalno budućem zanimanju.

Faktori koji utječu na stav ispitanika prema zanimanju profesor matematike

Trećim anketnim pitanjem ispitivani su faktori koji utječu na stav ispitanika o zanimanju profesor matematike. Ispitanici su trebali označiti jedan ili više faktora od trinaest ponuđenih za koje smatraju da su u najvećoj mjeri utjecali na njihove odgovore na prva dva pitanja. Ponuđeni su sljedeći odgovori: sigurna državna plaća, velika vjerojatnost zaposlenja, dugačak godišnji odmor, radno vrijeme, pozitivan stav prema matematici, uvažen status nastavnika u društvu, velika društvena odgovornost zanimanja, niska plaća, mala mogućnost napredovanja, uvjeti rada, rad s djecom, negativan stav prema matematici i nedovoljno uvažen status nastavnika u društvu. Također, ispitanici su mogli upisati faktor koji nije bio ponuđen upitnikom, a koji je utjecao na njihov stav. S obzirom da je primarni cilj ovoga istraživanja bio ispitati zašto učenici ne odabiru zanimanje profesor matematike, rezultati su ovoga pitanja analizirani s obzirom na odgovor ispitanika na drugo pitanje “O zanimanju profesor matematike razmišljam kao o potencijalnom budućem zanimanju”. Utvrđeni su najčešći faktori u odgovorima ispitanika koji ne razmišljaju o zanimanju profesor matematike kao potencijalno budućem, kao i kod onih ispitanika koji o istome zanimanju razmišljaju kao o potencijalnom budućem. Prvih je šest najčešće odabranih faktora od strane ispitanika koji o zanimanju profesor matematike ne razmišljaju kao o potencijalnom budućem zanimanju s obzirom na spol prikazano na Slici 1.

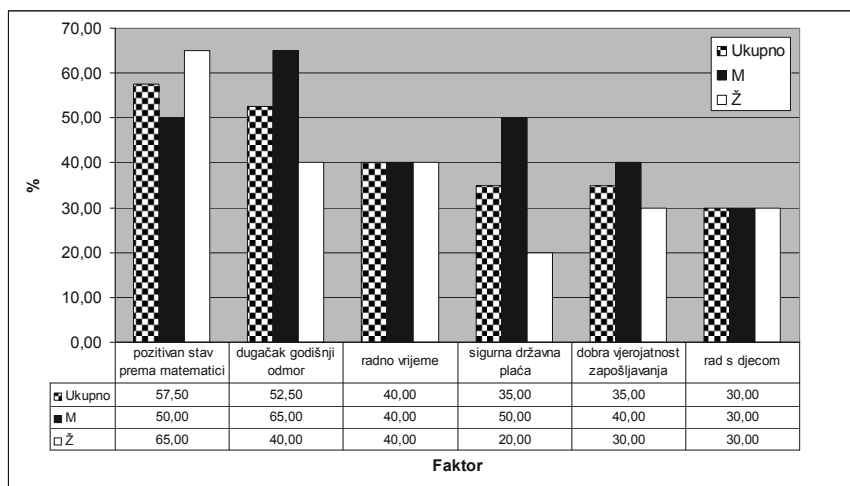
Među ispitanicima koji o zanimanju profesor matematike ne razmišljaju kao o potencijalnom budućem zanimanju njih 54, 49% kao faktor koji utječe na taj stav navodi nisku plaću, pri čemu ovaj faktor kao jedan od razloga nezainteresiranosti za zanimanje profesor matematike češće navode muški ispitanici. Taj se rezultat može dijelom objasniti razlikama u spolnim ulogama, odnosno ustaljenim vjerovanjem da muškarac treba prehraniti obitelj. Učenici također češće od učenica navode kao razlog za svoj stav malu mogućnost napredovanja i nedovoljno uvažen status nastavnika u društvu. Negativan stav prema samom predmetu Matematika je u slučaju 37, 36% ispitanika jedan od razloga nezainteresiranosti za poziv profesor

matematike, pri čemu ovaj faktor mnogo češće ističu učenice (njih 49,29%) u odnosu na učenike (20%). Rad s djecom i uvjete rada češće kao razlog negativnog stava navode muški ispitanici.



Slika 1. Prvih šest najčešće odabiranih faktora od strane ispitanika koji o zanimanju profesor matematike ne razmišljaju kao o potencijalnom budućem zanimanju s obzirom na spol.

Prvih šest najčešće odabiranih faktora od strane ispitanika koji o zanimanju profesor matematike razmišljaju kao o potencijalnom budućem zanimanju s obzirom na spol prikazano je na Slici 2.



Slika 2. Prvih šest najčešće odabiranih faktora od strane ispitanika koji o zanimanju profesor matematike razmišljaju kao o potencijalnom budućem zanimanju s obzirom na spol.

Ispitanici koji razmišljaju o zanimanju profesor matematike kao o potencijalnom budućem, najčešće kao faktor koji utječe na njihov stav ističu pozitivan stav prema predmetu Matematika, što navodi 57,5% ispitanika iz ove kategorije. Idući najučestaliji faktor jest dugačak godišnji odmor (52,5%). Uzme li se k tome u obzir i činjenica da je faktor radnoga vremena odabralo 40% ispitanika ove kategorije, možemo zaključiti da u mnogih učenika koji razmišljaju o zanimanju profesor matematike kao potencijalno budućem zanimanju postoji pogrešna percepcija ovog zanimanja te da ga smatraju nezahtjevnim poslom koji ostavlja mnogo slobodnoga vremena. Ostali često odabirani faktori su i sigurna državna plaća (35%), velika vjerojatnost zaposlenja (35%) i rad s djecom (30%).

“Na moj sveukupan stav prema zanimanju profesor matematike utjecali su moji dosadašnji profesori matematike”

Pri analizi učestalosti odgovora na ovo anketno pitanje ponovno su združeni odgovori “u potpunosti se odnosi na mene” i “uglavnom se odnosi na mene” u kategoriju “odnosi se na mene”, te odgovori “uglavnom se ne odnosi na mene” i “u potpunosti se ne odnosi na mene” u kategoriju “ne odnosi se na mene”. Analiza je provedena s obzirom na stav ispitanika o zanimanju profesor matematike, odnosno s obzirom na to razmišljaju li o zanimanju profesor matematike kao o potencijalnom budućem zanimanju. U Tablici 8. su ispitanici koji o zanimanju profesor matematike razmišljaju kao o potencijalnom budućem zanimanju označeni s “pozitivan stav”, dok su ispitanici koji o istom zanimanju ne razmišljaju kao o potencijalnom budućem zanimanju označeni s “negativan stav”.

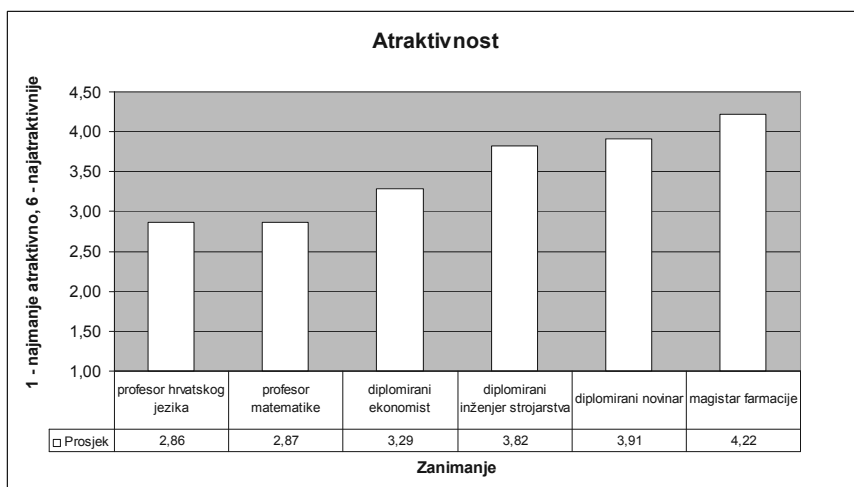
“Na moj sveukupan stav prema zanimanju profesor matematike utjecali su moji dosadašnji profesori matematike.”		
Stav prema zanimanju	Odnosi se na mene	Ne odnosi se na mene
Pozitivan stav	55%	45%
Negativan stav	36,51%	63,49%
Ukupno	38,37%	61,63%

Tablica 8. Učestalost pojedinih kategorija odgovora na anketno pitanje “Na moj sveukupan stav prema zanimanju profesor matematike utjecali su moji dosadašnji profesori matematike” s obzirom na stav ispitanika prema zanimanju profesor matematike.

Dosadašnji profesori matematike utjecali su na 38,37% ispitanika, pri čemu je njihov utjecaj značajniji kod ispitanika s pozitivnim stavom, što znači da se radilo o profesorima koji su na dio svojih učenika uspjeli prenijeti interes i ljubav prema predmetu Matematika.

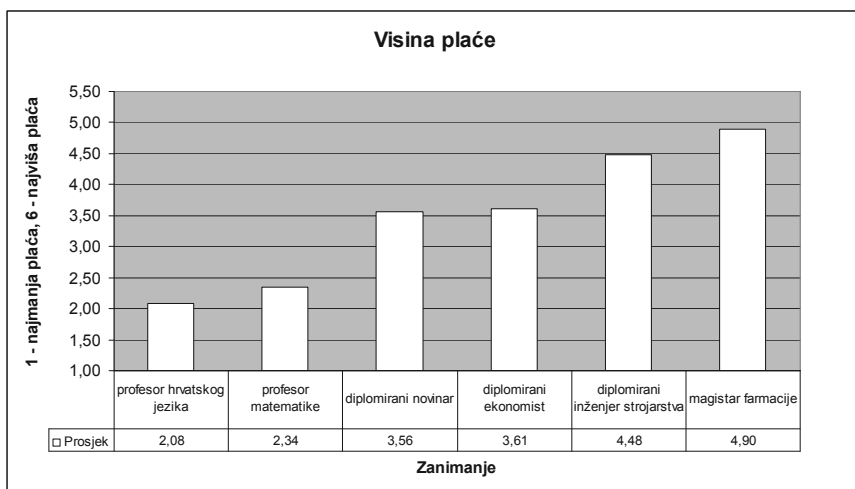
Usporedba s drugim zanimanjima

Od ispitanika se tražilo da prema vlastitu mišljenju rangiraju šest ponuđenih zanimanja (profesor matematike, profesor hrvatskog jezika i književnosti, diplomirani ekonomist, diplomirani inženjer strojarstva, magistar farmacije, diplomirani novinar), i to po njihovoj atraktivnosti, visini plaće i zaposlivosti. Analiza odgovora prikazana je na Slici 3. (atraktivnost zanimanja), Slici 4. (visina plaće) i Slici 5. (zaposlivost).



Slika 3. Srednje ocjene zanimanja s obzirom na njihovu atraktivnost.

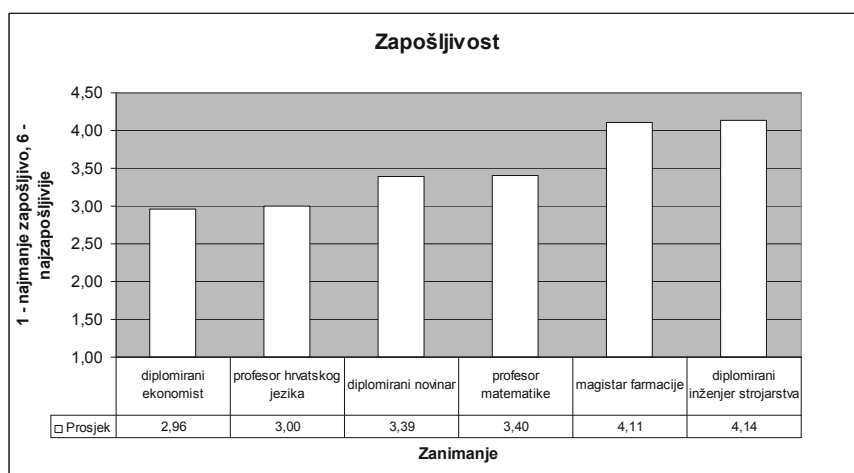
Među ponuđenim zanimanjima ispitanici smatraju najmanje atraktivnima (u gotovo jednakoj mjeri) zanimanja profesor hrvatskog jezika i profesor matematike. Iz takvih se rezultata može zaključiti da je ispitanicima neatraktivan profesorski poziv općenito, a ne isključivo zanimanje profesor matematike.



Slika 4. Srednje ocjene zanimanja s obzirom na visine plaća.

Procjena ispitanika međusobnog odnosa visina plaća šest ponuđenih zanimanja je prilično realna (prema podacima Državnoga zavoda za statistiku), iako plaće diplomiranoga novinara i diplomiranoga ekonomista mogu varirati unutar širokoga ranga. Iz dobivenih se rezultata može zaključiti da postoji ovisnost između visine očekivane plaće za pojedino zanimanje i njegove atraktivnosti, odnosno da

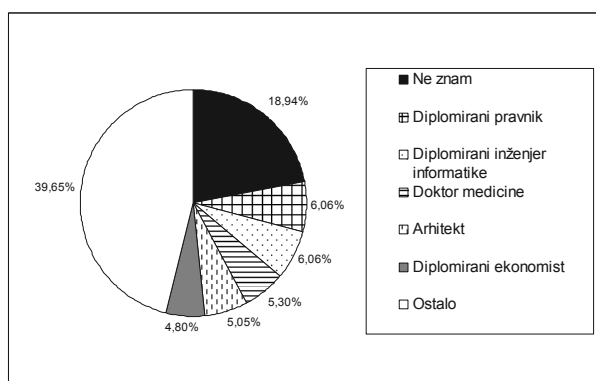
kod prosječnog ispitanika očekivana visina plaće pojedinog zanimanja utječe na njegovu atraktivnost.



Slika 5. Srednje ocjene zanimanja s obzirom na zaposljivost zanimanja.

Kao što smo već naveli, u Republici Hrvatskoj je u razdoblju od siječnja do studenoga 2010. godine prijavljeno čak 553 slobodnih radnih mjesta za profesore matematike i 400 slobodnih radnih mjesta za učitelje matematike. Nadalje, u istom je periodu prijavljeno 172 slobodna mjesta za magistre farmacije i 342 slobodna radna mjesta za diplomirane inženjere strojarstva. S druge strane, diplomirani ekonomist, profesor hrvatskog jezika i književnosti te diplomirani novinar se nalaze među deset najbrojnijih zanimanja s obzirom na broj nezaposlenih osoba. Prema dobivenim rezultatima može se zaključiti da ispitanici nisu dovoljno upoznati s navedenim informacijama.

Posljednje je anketno pitanje bilo slobodnog tipa. Ispitanici su trebali upisati zanimanje kojim bi se željeli baviti u budućnosti. Rezultati su prikazani na Slici 6. na kojoj je istaknuto pet najatraktivnijih zanimanja.



Slika 6. Zanimanja kojim se ispitanici žele baviti u budućnosti.

Od 369 je ispitanika njih petero na posljednje pitanje odgovorilo da želi biti profesor matematike, što je 1,35% svih ispitanika.

Diskusija i zaključak

Rezultati su provedenog istraživanja u skladu s prisutnim nedostatkom profesora matematike. Naime, gotovo 80% ispitanika ne smatra ovo zanimanje atraktivnim, a tek 10% razmišlja o njemu kao o potencijalno budućem zanimanju. Pomalo iznenađuje da iako su žene znatno zastupljenije u samom zanimanju, anketirane učenice ovo zanimanje smatraju manje atraktivnim u odnosu na svoje vršnjake i rjeđe o njemu razmišljaju kao o svom budućem zanimanju. Moguće je da učenice imaju negativniji stav prema samom predmetu Matematika od učenika te da taj stav prenose i na zanimanje profesor matematike. Provedena su brojna istraživanja o utjecaju spola na stav prema predmetu Matematika, od kojih neka potvrđuju da je on značajan, te da se kreće od manjeg na mlađem uzrastu ispitanika prema snažnijem na uzorcima gimnazijalaca i studenata. S druge strane, postoje istraživanja koja pokazuju da gimnazijalci prema predmetu Matematika imaju neutralan stav i ne slažu se s pretpostavkom da je matematika muška domena (Arambašić, Vlahović-Štetić i Severinac, 2005).

Za profesore je matematike u srednjoj školi posebno zanimljiv podatak da o zanimanju profesor matematike kao o svom budućem zanimanju razmišlja 75% učenika prvih razreda koji su to zanimanje ocijenili kao atraktivno zanimanje dok svega 29,41% učenika četvrtog razreda koji to zanimanje smatraju atraktivnim razmišlja o njemu kao o potencijalno budućem zanimanju. Uzme li se u obzir i činjenica da je postotak učenika koji navedeno zanimanje smatraju atraktivnim značajno manji u četvrtom nego u prvom razredu srednje škole, jasno da je da treba učiniti dodatan napor kako bi se upravo tijekom srednjoškolskog obrazovanja smanjilo opadanje interesa za ovo zanimanje.

Većina ispitanika kao razlog svoga negativnog stava ističe visinu primanja, što bi svakako trebalo imati na umu želi li se izbjeći daljnji pad interesa za ovo zanimanje. Loši rezultati i za drugo ponuđeno profesorsko zanimanje (profesor hrvatskog jezika i književnosti) upućuju na neatraktivnost rada u prosvjeti općenito. Također, ispitanici uglavnom nisu svjesni činjenice da je od šest ponuđenih upravo profesor matematike zanimanje koje omogućuje najbrže zapošljavanje. Učenici su to zanimanje s obzirom na zaposlivost ocijenili srednjom ocjenom 3,4, slično kao i zanimanje diplomirani novinar (3,39), gdje postoji značajan broj nezaposlenih osoba s visokom stručnom spremom. Pitanje koje se postavlja jest bi li rezultati istraživanja o stavovima gimnazijalaca o atraktivnosti zanimanja profesor matematike bili drukčiji da su ispitanici jače svjesni podataka o zaposlivosti toga zanimanja.

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Matematika u programima profesionalne edukacije odgojitelja

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Sažetak. Matematika je sastavni dio kulture (posebice u arhitekturi i umjetnosti) te svakodnevnome životu. Iako rezultati nekih istraživanja ukazuju na postojanje neutralnih ili negativnih stavova prema matematici, ljudi ju svakodnevno rabe. Telefoniranje, plaćanje, mjerenje i sl. neke su od svakodnevnih situacija za koja su potrebna matematička znanja. Upravo se aktivnostima funkcionalne uporabe matematike u ranom odgoju i obrazovanju daje velika pozornost. U Nacionalnom se okviru kurikulumu za predškolski odgoj i obrazovanje te opće obvezno i srednjoškolsko obrazovanje navodi nužnost stjecanja matematičkih kompetencija potrebnih za rješavanje svakodnevnih problemskih situacija. U slučaju predškolskog odgoja i obrazovanja nositelj (točnije dizajner i organizator) tih aktivnosti je odgojitelj. Polazeći od pretpostavki da je profesionalna edukacija odgojitelja karakterizirana pragmatičnošću, da su matematička znanja neophodna za svakodnevno funkcioniranje te da je rano djetinjstvo razdoblje u kojemu djeca intenzivno uče o svome okruženju i stječu oruđa kulture (jezik, simbole...), postavlja se pitanje prisutnosti matematike u profesionalnoj edukaciji odgojitelja. Kako bi se dobio uvid u zastupljenost matematike u programima profesionalne edukacije odgojitelja, analizirani su programi trogodišnjih stručnih studija za odgojitelje predškolske djece sa 5 sveučilišta u Republici Hrvatskoj. Dobiveni rezultati ukazuju na nisku zastupljenost matematike u navedenim programima te na reduciranje matematike u smjeru profesionalnoga alata. Obzirom da su samo dva sveučilišta u Republici Hrvatskoj pokrenula sveučilišni studij za odgojitelje, ovi rezultati mogu poslužiti kao okvir daljnjem propitivanju važnosti matematike u profesionalnoj izobrazbi odgojitelja i njezinom mjestu u institucionalnom predškolskom odgoju.

Ključne riječi: matematika, programi profesionalne edukacije, odgojitelji, pragmatičnost, funkcionalnost

Uvodna razmatranja

Matematika je sastavni dio kulture (posebice u arhitekturi i umjetnosti) te svakodnevnome životu. Iako rezultati nekih istraživanja ukazuju na postojanje neutralnih ili negativnih stavova prema matematici (Wittmann, 2006) ljudi ju ipak

svakodnevno rabe. Takva, funkcionalna uporaba brojeva i matematike prisutna je već u ranom djetinjstvu. Važnost matematike vidljiva je i u dokumentima Europske Unije i Republike Hrvatske. Tako Europska Komisija (2004) navodi ključne kompetencije za cjeloživotno učenje na slijedećim područjima: komunikacija na materinjem i stranom jeziku, informacijsko-komunikacijska tehnologija (ICT), matematička pismenost i kompetencije vezane uz znanost i tehnologiju, poduzetničke kompetencije, osposobljenost za cjeloživotno učenje, interpersonalne i građanske kompetencije te kulturalna ekspresija. U Nacionalnom se okvirnom kurikulumu za predškolski odgoj i obrazovanje te opće obvezno i srednjoškolsko obrazovanje u Republici Hrvatskoj (MZOS, 2010) navodi nužnost stjecanja matematičkih kompetencija potrebnih za rješavanje svakodnevnih problemskih situacija. U slučaju predškolskog odgoja i obrazovanja nositelj (točnije dizajner i organizator) tih aktivnosti je odgojitelj. Kako bi se osigurala uspješna (u smislu učinkovitosti) pedagoška djelatnost, budući se odgojitelji u Republici Hrvatskoj profesionalno educiraju na stručnim i sveučilišnim studijima. Svrha stručnih studija jest "opremiti" studenta primjerenim praktičnim kompetencijama, koje će kao radnik moći svakodnevno aplicirati u svojoj profesionalnoj djelatnosti. Tijekom svoje profesionalne edukacije odgojitelji "izgrađuju sustav osobnih vrijednosti, temeljen na individualnim iskustvima socijalizacije, na iskustvima osobne participacije u edukacijskom procesu te na spoznajama iz područja 'struke'" (Irović, Romstein, 2007, 229). Propitujući učinkovitost pedagoške djelatnosti odgojitelja Colker (2008) navodi kako je ona (učinkovitost) pod izravnim utjecajem odgojiteljevih znanja, vještina i karakteristika ličnosti. Kao važan čimbenik odgojiteljeve profesionalne uspješnosti, studenti – budući odgojitelji navode upravo profesionalnu edukaciju (Irović, Romstein, 2007). Obzirom da su u dokumentima na međunarodnoj razini, a koje prihvaća i Republika Hrvatska, navedene kompetencije potrebne za samostalnu i učinkovitu profesionalnu djelatnost odgojitelja, Lepičnik-Vodopivec (2007) nakon provedenog istraživanja iznosi slijedeće rezultate: na prvome mjestu po važnosti (iz perspektive studenata – budućih odgojitelja) nalaze se znanja i razumijevanje razvojnih karakteristika djece predškolske dobi, njihovih individualnih razlika i potreba, poznavanje kognitivno-razvojnih teorija iz kojih slijede kompetencije sa područja metodologije izvedbe. Kako je već ranije rečeno, matematika je već od rane dobi svakodnevno prisutna. Implementacijom Bolonjskoga procesa hrvatska sveučilišta i pripadajuće sastavnice dobivaju priliku za koncipiranje studija u odnosu na zahtjeve odgojiteljske profesije. Kako bi se stekao uvid u postojanje matematike i matematičkih sadržaja analizirano je 5 programa stručnih (trogodišnjih) studija za odgojitelje predškolske djece: Odjel za izobrazbu učitelja i odgajatelja predškolske djece Sveučilišta u Zadru (stručni studijski program za odgajatelje predškolske djece), Učiteljski fakultet u Osijeku (trogodišnji stručni studij za odgajatelja predškolske djece), Učiteljski fakultet u Rijeci (stručni studij predškolskog odgoja), Filozofski fakultet u Splitu (stručni studijski program odgojitelj djece predškolske dobi), te Učiteljski fakultet u Zagrebu (prijedlog preddiplomskog stručnog studijskog programa za odgojitelje predškolske djece). Svi su programi iz 2004./2005. godine kada se krenulo sa studijem prema Bolonjskom procesu. Analiza kolegija učinjena je prema slijedećim kriterijima: postojanje kolegija, cilj/evi kolegija, sadržaji kolegija i očekivane kompetencije.

Analizom kolegija utvrđeno je nepostojanje kolegija Matematika te niska zastupljenost matematičkih sadržaja u navedenim studijskim programima, odnosno

njezino reduciranje u smjeru profesionalnoga alata, što je posebice vidljivo u okvirima kolegija Metodologija pedagoških istraživanja, Statistika i Informatika.

Postojanje kolegija Matematika na studijima predškolskog odgoja

Učenje matematike moguće je već u ranom djetinjstvu. Rezultati istraživanja o učenju matematike u ranoj dobi ukazuju slijedeće: učenje matematike s jednostavnim računskim operacijama poput klasifikacije i pridruživanja izravno utječe na uspješnost učenja matematike tijekom prvog razreda osnovne škole (Bodrovski i Farkas, 2007), djeca predškolske dobi, vrlo rano, čak prije 5. g. života, imaju sposobnosti razumijevanja prostornih odnosa, brojeva, količine i geometrije što se može dodatno poticati i razvijati kroz igre konstrukcije, posebice građenja kocaka (Sarama i Clements, 2004). Obzirom na mogućnosti učenja matematike u ranoj dobi, njezinim vrijednostima obuhvaćenim u aktualnim odgojno-obrazovnim dokumentima na međunarodnoj razini i razini Republike Hrvatske te važnosti kompetencija (iz percepcije samih studenata) sa području metodologije izvedbe, vrijedno je propitati aktualnu "ponudu" matematičkih sadržaja u programima profesionalne edukacije odgojitelja.

Od 5 analiziranih studijskih programa niti jedan nije imao kolegij Matematika. No, postoje kolegiji srodni matematici, odnosno oni koji sadrže elemente matematike. Tako na Učiteljskom fakultetu u Zagrebu postoje kolegiji Metodika predškolskog odgoja I i II; na Učiteljskom fakultetu u Osijeku studenti se susreću s matematikom u okviru kolegija Tablični kalkulator, Metodika predškolskog odgoja IV i Metodologija pedagoških istraživanja; na Učiteljskom fakultetu u Rijeci u okvirima kolegija Osnove pedagoške metodologije i Osnove statistike; na Odsjeku za izobrazbu učitelja i odgojitelja Sveučilišta u Zadru unutar kolegija Metodika odgojno-obrazovnoga rada III i Informatika II; te na Filozofskom fakultetu u Splitu u kolegijima Osnove informatike, Metodike predškolskog odgoja II i Osnove metodologije pedagoških istraživanja.

Analiza kolegija s ponuđenim matematičkim sadržajima

Obzirom da niti u jednom programu profesionalne edukacije nije ponuđen kolegij matematika, pozornost smo usmjerili na analizu kolegija u kojima se pojavljuju matematički sadržaji. Kolegiji koji svojim sadržajima studentima nude, između ostalih, i matematiku su: Metodika predškolskog odgoja (poznavanje okoline) 1 i 2 (Učiteljski fakultet u Zagrebu); Tablični kalkulator, Metodika predškolskog odgoja 4 i Metodologija pedagoških istraživanja (Učiteljski fakultet u Osijeku); Osnove pedagoške metodologije i Osnove statistike (Učiteljski fakultet u Rijeci); Metodika odgojno-obrazovnog rada 3 i Informatika 2 (Odsjek za izobrazbu učitelja i odgojitelja Sveučilišta u Zadru); Osnove informatike, Metodika predškolskog odgoja 2 i Osnove metodologije pedagoških istraživanja (Filozofski fakultet u Splitu).

Analizom sadržaja utvrđeno je da ponuđeni matematički sadržaji unutar studijskih programa nisu u funkciji pedagoškog rada s djecom. Matematički sadržaji koji se mogu naći u navedenim kolegijima uglavnom su usmjereni na istraživačke kompetencije budućih odgojitelja, što je samo jedan aspekt profesionalne djelatnosti odgojitelja. Takav, redukcionistički pristup matematici ukazuje na nedovoljno razumijevanje njezine svakodnevne prisutnosti i važnosti. S druge pak strane, kolegiji koji se odnose na statistiku pretpostavljaju visoku razinu matematičkih znanja studenata pa se u sadržajima pojavljuju primjerice hikvadrat test, kovarijansa, korelacija i sl., što u pedagoškoj praksi odgojitelji uglavnom ne rabe. Iako podizanje kvalitete odgojno-obrazovnog rada pretpostavlja posjedovanje “znanstvenih kompetencija” sa područja metodologije, statistike i pedagoških istraživanja (Matićević, 2007, 304) koje facilitiraju dizajniranje razvojno primjerene prakse, preduvjet razumijevanju složenijih statističkih operacija je razumijevanje elementarnih matematičkih pojmova, što je u ovom slučaju izostalo. Iz navedenog je vidljiva paralelna prisutnost redukcionizma i visoke razine zahtijeva spram studenta. Stoga je potrebno usuglasiti razine zahtijeva spram studenata te propitati relevantnost sadržaja koji im se nude u smislu njihove primjenjivosti u kasnijoj profesionalnoj djelatnosti.

Gledajući pojedine sastavnice programa, također se može uočiti neusuglašenost i nedosljednost. Tako ciljevi (koji se tumače kao polazišne točke u kreiranju kolegija) u pojedinim programima kolegija (primjerice Metodika predškolskog odgoja 1 i 2, Metodika odgojno – obrazovnog rada 3, Informatika 2) nisu niti navedeni pa se nameće pitanje vrednovanja ishoda u kolegija kojemu cilj nije poznat. Što se tiče kompetencija, dva fakulteta uopće nemaju navedene kompetencije, te je nejasno što studenti “dobivaju” nakon položenog ispita. Naime, kompetencije su na međunarodnoj razini svojevrsna zajednička načela u profesionalnoj edukaciji odgojitelja. One su u funkciji “rješavanja aktualnih i budućih globalnih zahtijeva i očekivanja” (Babić, 2007, 33) što je neizbježan preduvjet studiranju “po Bologni”.

No, određeni kolegiji (primjerice Metodika odgojno-obrazovnog rada 3) idu i korak dalje te studentima nude sadržaje i kompetencije vezane uz igru, koju tumače kao poligon za testiranje vlastitih matematičkih kompetencija (npr. “Igra kao temeljna metoda u području razvijanja početnih matematičkih pojmova”) za što je potrebna metakompetentnost iz navedenog područja.

Iz navedenog se može zaključiti kako ponuđeni matematički sadržaji nisu usklađeni te studentima ne nude dovoljno informacija o načinima implementacije matematike u svakodnevni pedagoški rad. Uz to, matematika se vidi kao alat za uspješnu istraživačku djelatnost što je, zapravo, samo jedan aspekt profesionalne djelatnosti odgojitelja. Obzirom da je matematika prisutna u svakodnevnom životu a njezina pragmatična vrijednost je jedna od najčešće spominjanih (Wittmann, 2006) važno je promisliti o mogućnostima njezine implementacije u nove, sveučilišne, programe profesionalne edukacije odgojitelja.

Analizom programa profesionalne edukacije odgojitelja utvrđeno je nepostojanje kolegija Matematika. Ono što postoji su matematički sadržaji unutar kolegija

Statistika, Metodologija, Informatika te u nešto manjoj mjeri u metodikama što ukazuje na izostanak viđenja vrijednosti matematike u izravnom pedagoškom radu s djecom predškolske dobi.

Analizom kolegija utvrđena je niska zastupljenost matematike i njezino reduciranje u smjeru profesionalnoga alata, posebice u okvirima kolegija Metodologija pedagoških istraživanja, Statistika i Informatika. Iako su kompetencije samostalnog znanstveno-istraživačkoga rada važan aspekt odgojiteljeve profesionalne djelatnosti one su, zapravo, potporanj u građenju razvojno primjerene prakse. Mnogi autori (Liebeck, 1994; Wittmann, 2006; Bodrovski i Farkas, 2007) podsjećaju kako se u radu s djecom predškolske dobi matematika, točnije matematička znanja, uzimaju kao "mjera" procjene djetetove spremnosti za polazak u osnovnu školu, no zapravo je praktična uporaba matematike njezina vrijednost koja se očituje već tijekom ranog djetinjstva.

Trenutno se u Republici Hrvatskoj programi profesionalne edukacije odgojitelja usmjeravaju sa stručnih na sveučilišne dodiplomske i diplomске studije, što je jedinstvena prilika za propitivanje aktualnih programa i kompetencija. Prema dostupnim podacima, jedino je Učiteljski fakultet u Osijeku studentima ponudio matematičke kolegije – Matematičku kulturu i komunikaciju te Matematiku u igri i rasonodi. Time se studentima omogućio širi izbor matematičkih sadržaja i stjecanje relevantnih kompetencija potrebnih za uspješnu profesionalnu djelatnost.

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Savremeni pristupi učenja matematike na učiteljskim fakultetima u Srbiji

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Rezime. U ovom radu daćemo kratak pregled jednog načina ostvarivanja matematičkog obrazovanja studenata učiteljskih fakulteta u Srbiji, sa posebnim osvrtom na načine razvoja kreativnosti i planiranja nastave matematike kao preduslova njene uspješne realizacije.

Ključne reči: nastava, matematika, aktivno učenje

Uvod

Od nastanka organizovanih oblika podučavanja, pa sve do savremenih obrazovnih sistema, stalno se teži ka osmišljavanju novih i sve efikasnijih postupaka i tehnika učenja. Shodno tome uvodi se i nastava kojom bi se, uz maksimalnu aktivizaciju neposrednih učesnika u nastavnom procesu, obezbedio kvalitet prenešenog znanja. Na taj način znanja koja primaju učenici ne smeju ostati pasivna činjenica bez mogućnosti primene. Stalni naponi unapređivanja podučavanja i nastave doveli su do nastanka velikog broja tipova nastave i metoda kojima se ona izvodi. Te metode su vremenom, u većoj ili manjoj meri, zauzimale značajno mesto u nastavnom procesu ili su nestajale.

Pokušaj postavljanja temelja znanja i njegovog izučavanja budućim generacijama, u oblasti vaspitanja i obrazovanja, u vreme dubokih i brzih društvenih, državnih, informaciono-tehnoloških i naučnih postignuća, znači precizno, promišljeno i višestrano postavljanje puteva i načina ličnog i društvenog razvoja dece i učenika od predškolskog uzrasta do ulaska u visokoškolski sistem.

Nastava sa kojom se možemo susresti u svakodnevnoj školskoj praksi, ne samo matematike, u mnogo čemu ne može ispuniti zahteve koje savremeno društvo od nje zahteva. Trenutni nastavni proces u velikoj meri je skoncentrisan na predavača koji je u fokusu i koji učenicima pruža činjenične informacije zasnovane na knjiškom znanju. Akcenat je na onoj vrsti učenja na koje će se nadalje pozivati, a ne primenjivati, i kojim se stimuliše korišćenje malog broja čula. Ovakvim autoritativnim

istupanjem u izlaganju učenici ne dobijaju potrebnu motivaciju za dubljim otkrivanjem izloženih znanja, osnove koje usvajaju su slabo upotrebljive, a spoznaja činjenica je praćena nezadovoljstvom, prisilom, teranjem na rad i gubljenjem želje za nova otkrića.

Sasvim je jasno da nastava matematike, kao bazični predmet svake reforme, zahteva sveobuhvatno obrazovanje onih koji je izvode, a samim tim i njihovu maksimalnu osposobljenost i pripremljenost u planiranju i izvođenju.

Pokušavajući da održi korak sa savremenim trendovima i pruži maksimalno obrazovanje budućim aktivnim učesnicima u obrazovnom sistemu, na Učiteljskim, Pedagoškim, ali i drugim nastavničkim fakultetima u Srbiji, sve više počinje da se primenjuje neki od mnogobrojnih pedagoških pravaca koji su često nazivani jednim imenom - aktivno učenje. U nastavku rada prikazaćemo jedan od novijih pravaca aktivnog učenja koji se sve više primenjuje u metodičkom obrazovanju budućih učitelja i pripremi za njihovo samostalno izvođenje časova, a koji je zauzeo značano mesto na Pedagoškom fakultetu u Jagodini.

Aktivno učenje/nastava (AUN) je projekat započet 1994. godine od strane tima saradnika Instituta za psihologiju Filozofskog fakulteta u Beogradu, na čelu sa rukovodiocem projekta prof. dr Ivanom Ivićem, a u saradnji sa Kancelarijom UNICEF-a u Beogradu. U okviru ovog projekta razvijen je specifičan način pripreme i analize časova, na osnovu kojih budući učitelji spremaju svoje pisane pripreme i vrše procenu izvedenih časova u okviru predmeta Metodika nastave matematike. U samom radu nećemo se osvrnati na metode¹ AUN-a već na postupak nastanka scenarija.

Proces izrade *scenarija* za AUN časove je ključna faza pripreme studenata u sprovođenju ideja o aktivnom učenju u nastavnoj praksi. Termin *scenario* namerno je pozajmljen iz filmske terminologije jer je to segment vremena u kome se na učioničkoj „sceni“ odigravaju krupni i bitni procesi unutar svake jedinice i koje treba na neki način režirati i o njima napraviti pisani trag.

Na samom početku priče o stvaranju scenarija navedimo najvažnije razlike između pisanja pripreme za tradicionalni, kod nas najzastupljeniji, oblik nastave i pisanja scenarija (tabela 1):

¹ Detaljna razrada metoda AUN-a može se naći u priručniku [1]

Klasična pisana priprema za čas:	AUN scenario:
Planira se za izvođenje jednog časa od 45 minuta	Planira se za izvođenje jednog časa, ali i celine manje (deo časa) ili veće od jednog časa (dvočas, blok časova, ciklus većeg broja časova itd.).
U njoj se specifikuje šta radi nastavnik , kako će izvesti čas.	Akcentat je na tome šta rade učenici , specifikuju se nastavne situacije (koje bi trebalo da izazovu različite aktivnosti relevantne za dati sadržaj) i aktivnosti učenika (koje će biti izazvane u toj situaciji).
Specifikuje se sadržaj (ŠTA se radi, raspored gradiva i vremenska artikulacija uz navođenje nastavnih metoda i didaktičkih sredstava koja će se koristiti).	Dominira KAKO će se izazvati određene aktivnosti učenika i KOJE aktivnosti će biti izazvane, pedagoška interakcija među učenicima i između učitelja i učenika. A to znači da je naglasak na metodama nastave kroz učenj i to specifične za gradivo.
Težište je na izvođenju , realizaciji časa.	Težište je na pripremi pre časa, dizajniranju nastavne zamisli koja će na najbolji način realizovati određene celjeve (i vremenska priprema uzima više vremena od samog izvođenja časa – od dolaženja do ideje, pa do kreiranja nastavne situacije).
Pisana priprema ima skoro uniformnu strukturu : uvod, tok, zaključak (pa otud preti opasnost šablonizacije, mehaničkog prepisivanja priprema za nastavu, svojih i tuđih).	Ne može da ima fiksnu, uniformnu strukturu (to ne znači da ne treba da ima neke osnovne podatke o nastavnom sadržaju, ciljevima, uzrastu i td.) već vrlo različite strukture scenarija zavisno od autorove ideje. Ova karakteristika proističe iz činjenice da se isto gradivo može učiti na više različitih načina.
Dominira predavačka uloga nastavnika i, delom, organizatorska.	Dominiraju: organizatorska (režiserska, dizajnerska), motivaciona uloga učitelja, uloga učitelja kao partnera u pedagoškoj interakciji .

Tabela 1. Razlika između pisane pripreme za čas i scenarija za AUN nastavu.

Razlike su jasne i dovoljno velike da možemo reći da one nisu tehničke prirode već da se kroz njih praktično vidi suština različitih shvatanja prirode nastavnih procesa. Dakle, dok je kod klasične pripreme za čas naglasak na školskim programima i ulozi predavača (procesu podučavanja), kod scenarija je naglasak na procesu učenja u pedagoški precizno definisanim uslovima i na ishodima tog procesa.

Pre početka pisanja svakog konkretnog scenarija za čas matematike, č moraju imati u vidu sve specifične okolnosti pod kojima se on izvodi:

- specifični ciljevi određene teme gradiva;
- specifičnost prirode nastavnog sadržaja;
- specifičan odabir metoda nastave/učenja koji moraju biti usaglašeni sa planiranim ciljevima s jedne, i prirodom konkretnih sadržaja koji se izučavaju s druge strane;

- konkretne okolnosti u kojima se odvija proces učenja;
- povezanost sa drugim matematičkim sadržajima koje su učenici usvojili;
- nivo predznanja učenika, a u vezi je sa specifičnostima društvene zajednice u kojoj žive;
- specifične ideje studenta o tome kako da se izvede nastava u planiranim i zamišljenim okolnostima.

Priprema studenata za pisanje scenarija poželjno je da se odvija u dve etape. Prvu etapu čini izrada idejne skice za čas koja sadrži prve ideje o tome kako bi trebao čas da izgleda i ovo predstavlja prvi susret sa idejama studenata o konkretnom času. Od scenarija se razlikuje po stepenu razrađenosti ideja i zadataka koji će se obrađivati. Ona treba da posluži studentu kao polazna odrednica za diskusiju, razradu i planiranje samog scenarija. Nakon dorade i doterivanja idejne skice dobija se prva razrađena verzija. U toj verziji već se jasno definišu osnovni parametri planirane nastave:

- koji i koliki deo gradiva se obrađuje, šta čini nastavni sadržaj;
- koji ciljevi se žele ostvariti; koji ciljevi se samim časom mogu realizovati, a koji su ciljevi dugoročni;
- koje metode će se upotrebiti;
- koji deo nastave se planira (jedan čas, manje ili više od jednog časa);
- koja motivaciona sredstva će se koristiti;
- koja je osnovna zamisao nastavne situacije kojom se čvrsto povezuju ciljevi, priroda gradiva i metode i kojom se u velikoj meri garantuje uvlačenje velikog broja učenika u relevantne aktivnosti koje vode ka ostvarenju obrazovnih efekata.

Ovako urađena prva verzija scenarija nije i njen konačni oblik. U izradi pripreme obično se posebna pažnja obrati na jedan njegov segment, dok neki drugi segmenti mogu, iako značajni, ostati u njegovoj senci. Dakle, osnovni problem, kako klasičnih priprema tako i scenarija, predstavlja neadekvatan uvid u celovitost pripreme, odnosno scenarija za jednu nastavnu celinu koja se obrađuje. Iz tog razloga autorski tim AUN-a osmislio je i autorizovao psihološko-didaktički postupak ocene pisanog scenarija pod nazivom konstruktivna kritička analiza scenarija, a koja je primenljiva i na klasične pisane pripreme za nastavni čas.

Rad na kritičkoj konstruktivnoj analizi scenarija je finalni kontakt sa studentom pre izvođenja časa. Ona počinje određivanjem osnovnih ciljeva časa koji mogu biti kratkoročni (vezani za taj čas) ili dugoročni. Označimo ih redom sa $A_1, A_2, A_3 \dots, A_n$. Posmatrano iz ugla učenika i onoga što oni konkretno rade, svaki čas možemo predstaviti kao listu učeničkih aktivnosti i zadataka koje rade i to onim redom kojim se javljaju u scenariju (označimo ih kao $act_1, act_2, act_3 \dots, act_m$). U tom cilju svaki student vrši formiranje sledeće tabele 2 za svoj konkretni scenario:

	A_1	A_2	A_3	...	A_n	Relevantnost aktivnosti
act_1						
act_2						
act_3						
...						
act_m						

Tabela 2. Primer tabele za konstruktivnu kritičku analizu.

Za svaku od aktivnosti datih u tabeli studenti određuju stepen njene relevantnosti u odnosu na prirodu i vrstu svojih sadržaja (najjednostavniji oblik označavanja je sa + i –). Pored toga za svaku od datih aktivnosti određuju i koji od osnovnih ciljeva časa se njome ostvaruju (npr. označavanjem sa + cilja koji se njome ostvaruje).

Noseća ideja časa predstavlja jasno definisanu zamisao šta se želi konkretnim nizom aktivnosti postići i koje promene izazvati u znanjima i umenjima učenika. Jasan uvid u uspešnost ove ideje na ostvarivanje planiranih ciljeva časa, može se steći kroz analizu aktivnosti koje kod dece izazivaju planirane nastavne situacije, a na osnovu podataka koje dobiju kao povratnu informaciju ovom tabelom:

- Ukoliko je u tabeli dosta aktivnosti označeno kao ne relevantne potrebno je zameniti te aktivnosti novim tako da se celina zamisli časa ne promeni ili se vratiti na pravljenje nove skice časa;
- Ukoliko neka aktivnost ne ispunjava niti jedan cilj časa takvu aktivnost je potrebno prilagoditi ciljevima ili je zameniti novom;
- Ukoliko u tabeli neki cilj je ispunjen zanemarljivo malim brojem aktivnosti potrebno je ili osmisliti nove aktivnosti u cilju njegovog ispunjenja ili ga izbaciti sa liste planiranih ciljeva za taj čas.

Osvrnimo se ukratko i na sam način pisanja scenarija. Naslov svakog scenarija je naziv nastavne jedinice ili jedinica koje se njime realizuju. Scenario počinje opštim podacima o autoru i samom času (tip časa, planirani ciljevi, materijali potrebni za realizaciju), nakon čega sledi niz koraka kojima se opisuju planirane aktivnosti. Korak u scenariju je sekvenca koja predstavlja najmanju smislenu jedinicu samog časa. To je zapravo deo časa, ali deo u kome su još uvek prisutna i očigledna svojstva časa kao celine. Istraživanja pokazuju da ovakve sekvence časa imaju najoptimalniji nivo opštosti. Ukoliko bi se u opisu časa koristili sitniji segmenti, na primer, pojedinačni iskazi učenika ili opisi svake pojedinačne spolja vidljive aktivnosti, izgubila bi se smislenost časa, što bi zahtevalo dodatne postupke da bi se ti segmenti ponovo povezali i dali smisao času. Granice koraka u scenariju se prepoznaju po promeni aktivnosti učenika. Svaki korak se opisuje iz ugla aktivnosti učenika, osim koraka u kojima glavnu ulogu ima student, ali se i tada naglašava šta rade učenici za to vreme (da li samo slušaju ili imaju neki zadatak). Sam način pisanja koraka u scenariju može da varira. Oni mogu biti najpre samo imenovani, a tek onda detaljno opisani, mada je uobičajnije da se u onom

trenutku kada se i navode u scenariju i detaljno opišu. Instrukcije u okviru svakog koraka mogu biti zadane na dva načina. Jedan je u upravnom govoru, tj. u onom obliku kako bi na samom času trebalo da se prezentuje učenicima, dok je drugi u obliku zadavanja ključnog cilja, a predavaču je ostavljeno da samostalno formuliše instrukcije. Analogno je i sa zadacima koje učenici treba da urade. Zadaci mogu biti precizno definisani, zadati u okviru samog koraka ili na kraju u vidu priloga, a mogu biti zadati i u vidu sadržaja koji treba da budu zastupljeni ali sa jasnim uputstvom za njihovo sastavljanje.

Studente je potrebno osposobiti da racionalno i nepristrasno mogu sagledati obrazovne efekte koje su postigli na času i da na pravi način urade samoevaluaciju svog časa. Autorski tim AUN-a osmislio je metodu psihološko-didaktičke analize održanog časa pod nazivom *sekvencijalna analiza* (u daljem tekstu SA). Jasno je da skoncentrisanost studenta na svim delovima časa ne može biti ista. Zbog toga se preporučuje, a u cilju sveobuhvatnog sagledavanja svih delova časa, video dokumentovanje časa za koje se radi SA.

SA se sprovodi tako što se najpre raščlanjuje održana celovita nastavna aktivnost na sekvence (sekvence definišemo analogno kao kod konstruktivne kritičke analize). U tom cilju može nam poslužiti tabela 3.

Redni broj sekvence	Ime sekvence
1	
2	
...	

Tabela 3. Nazivi sekvence na održanoj nastavnoj aktivnosti.

Nakon toga vrši se analiza svake određene sekvence ponaosob. Analiza sekvence se vrši prema sledećem nizu objektivnih parametara: aktivnosti učenika, intervencije studenta; relevantnost aktivnosti učenika u odnosu na: a) funkciju date sekvence; b) cilj časa; v) prirodu nastavnog sadržaja. Za analizu svake sekvence može se koristiti tabela 4:

Naziv sekvence					
Vreme trajanja: ___ min.					
Aktivnosti učenika		Intervencije studenta	Relevantnost aktivnosti u odnosu na		
Broj i vrsta aktivnosti	Broj aktivnih učenika		funkciju sekvence	cilj časa	prirodu oblasti
			+ / -	+ / -	+ / -
			+ / -	+ / -	+ / -

Tabela 4. Prikaz analize sekvence.

Kod aktivnosti učenika u tabeli navode se sve one aktivnosti koje su bile zastupljene u okviru sekvence i ako su se ponavljale broj njihovih ponavljanja. Intervencija studenta podrazumeva navođenje svih aktivnosti koje je imao, bez obzira da li se odnose na direktnu interakciju sa učenicima ili na samostalno prezentovanje delova sekvence. Aktivnost u okviru sekvence je relevantna (i označavamo je sa + u tabeli) sa aspekta:

- funkcije sekvence ako ispunjava specifične ciljeve ostvarenja sekvence;
- cilja časa ako predstavlja aktivnost kojom se ispunjava predviđeni cilj časa;
- prirode oblasti ako obuhvata nastavne sadržaje, procedure i načine mišljenja koji su specifični za nastavnu oblast na koju se scenario odnosi.

Pored kvalitativnog aspekta, SA poseduje i kvantitativni aspekt jer u sebi sadrži broj aktivnih učenika u svakoj sekvenci (prosto i tačno prebrojavanje svih učenika koji su bili uključeni u svaku od aktivnosti), kao i vremenski okvir trajanja svake aktivnosti. U zajednici, kvalitativni i kvantitativni aspekti čine jedinstvenu funkcionalnu celinu. Na osnovu ova dva aspekta možemo doneti valjanu ocenu didaktičke efikasnosti svakog segmenta časa i časa u celini.

Obaveza učitelja je uvući svakog učenika u proces usvajanja znanja. Najvažniji preduslov u ovom procesu je izbegnuti pružanje gotovih, čistih i konciznih novih znanja u prvom izlaganju jer takva predavanja, za učenike, mogu biti dosadna i veoma često zamaskirati potrebu samostalnog učenja i nadogradnje van redovne nastave. Upravo zbog toga potrebno je i neophodno da se još iz baze nastanka budućih učitelja počne sa njihovom detaljnom pripremom za budući poziv što se sa izloženim načinom primene AUN-a u dobroj meri i postiže.

U nastavku dajemo primer jednog scenarija za časove aktivnog učenja

CRTANJE NORMALNIH PRAVIH

Razred: III

Tip časa: Obrada

Trajanje: 45 minuta

CILJEVI:

- 1) Ponavljanje znanja o pravom uglu.
- 2) Usvajanje tehnike crtanja normalnih pravih.
- 3) Primena stečenih znanja o crtanju normalnih pravih.
- 4) Osposobljavanje za logičko rasuđivanje i zaključivanje.

MATERIJAL:

- * dva školska lenjira (jedan trougaoni lenjir sa pravim uglom)
- * dva lenjira (jedan trougaoni lenjir sa pravim uglom)
- * radni list

Napomena: Scenario je napisan za izvođenje diferencirane nastave sa zadacima na tri nivoa složenosti.

TOK ČASA

KORAK 1. Obnavljanje znanja o pravom uglu

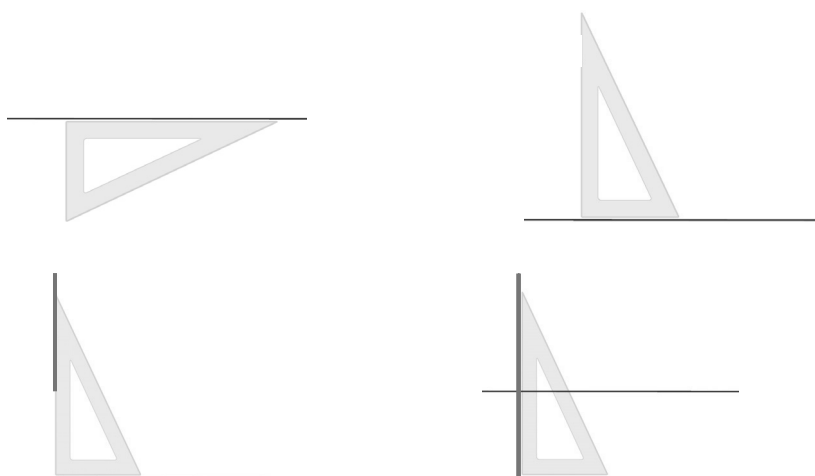
Učitelj izvodi jednog učenika koji pomoću pribora na tabli crta jedan prav ugao i daje isstrukciju ostalim učenicima da i oni samostalno u svojim sveskama crtaju ono što radi učenik ispred table. Učitelj vodi diskusiju sa učenicima i podstiče ih da samostalno odgovore na pitanja: šta dobijamo ako krake ugla produžimo preko temena pravog ugla; koliko pravih mogu uočiti na nacrtanoj slici; koliko još uglova, pored jednog pravog, mogu da uoče između ove dve prave; kakvi su to uglovi. Uporedo sa odgovaranjem na postavljena pitanja neko od učenika upotpunjuje započetu sliku na tabli, a ostali u svojim sveskama, a u skladu sa pitanjima koje učitelj postavlja.

KORAK 2. Usvajanje tehnike crtanja normalnih pravih

Učitelj pokazuje još jedanput sliku dve normalne prave koje su na tabli i prislanja prav ugao sa trougaonog lenjira uz prav ugao između normalnih pravih. Učitelj ih navodi da još jednom ponove da kada želimo da vidimo da li je ugao prav jednu stranicu lenjira poklapamo sa jednim krakom ugla, a drugu stranicu sa drugim krakom.

Učitelj briše jednu od dve nacrtane prave i navode učenike da zaključe kako bi mogli ponovo da nacrtamo pravu koja je normalna na pravu koju nismo izbrisali.

Kada učenici zaključe način crtanja normalne prave učitelj ponavlja još jedanput postupak crtanja normalnih pravih koristeći sledeće slike koje nakon objašnjavanja lepi na vidno mesto u učionici.



Učitelj zadaje svim učenicima da nacrtaju jedan par normalnih pravih u svojim sveskama ali tako da se prave koje crtaju ne poklapaju sa pravim linijama koje već imaju u svojim sveskama.

KORAK 3. Crtanje normalnih pravih sa zadatim uslovom

Učenici rade samostalno zadatak koji im zadaje učitelj. Učitelj napominje da ukoliko imaju problema pri izradi mogu zatražiti pomoć od učenika koji radi isti zadatak. Zavisno od nivoa učenici rade sledeće zadatke:

Nivo 1

Nacrtaj pravu a i na njoj tačku A . Nacrtaj pravu b koja je normalna na pravu a i sadrži tačku A .

Nivo 2 i nivo 3

Nacrtaj pravu a i van nje tačku A . Nacrtaj pravu b koja je normalna na pravu a i sadrži tačku A .

Nakon samostalne izrade zadatka učitelj govori učenicima da sa drugom koji je radio isti zadatak diskutuju o tome kako su uradili zadatak i da jedni drugima objasne postupak konstrukcije. Nakon ovoga jedan učenik prvog i jedan učenik drugog nivoa rade zadatak na tabli (ne istovremeno) ponavljajući postupak crtanja.

KORAK 4. Uvežbavanje crtanja normalnih pravih

Učenici dobijaju od učitelja papir sa zadacima koji rade u paru sa još jednim učenikom istog nivoa (drug iz klupe). Učitelj ih upućuje da najpre prodiskutuju o zadacima zajedno, a zatim da samostalno, svako na papiru koji je dobio ili u svesci, urade zadatke.

Nivo 1

Na slici je data mapa piratskog ostrva.



Pirat Ndžagumi je na ostrvu zakopao blago. Kada se vratio kući, u državu Gernzi, njegov sin Mbrljaru je našao mapu na čijoj poleđini je pisalo: „Nacrtaj pravu kroz tačke V i M . Nacrtaj pravu koja je normalna na pravu koju si već nacrtao i sadrži tačku K . Blago je u preseku dve prave koje si nacrtao.“ Pomozi Ndžaguminom sinu Mbrljaru da pronađe blago.

Nivo 2.

1. (Slika ista kao u zadatku za nivo 1) Pirat Ndžagumi je na ostrvu zakopao blago. Kada se vratio kući, u državu Gernzi, njegov sin Mbrljaru je našao mapu na čijoj poleđini je pisalo: „Nacrtao sam pravu kroz tačke V i M . Onda sam nacrtao pravu koja je normalna na nju i koja sadrži tačku K . Blago je u preseku ovih pravih.“ Pomozi Ndžaguminom sinu Mbrljaru da pronađe blago.

2. Nacrtaj dve tačke A i V . Nacrtaj pravu a koja prolazi kroz tačke A i V . Nacrtaj dve prave b i c koje su normalne na pravu a tako da jedna prolazi kroz tačku A , a druga kroz tačku V .

Nivo 3

1. (Slika ista kao u zadatku za nivo 1) Pirat Ndžagumi je na ostrvu zakopao blago. Kada se vratio kući, u državu Gernzi, njegov sin Mbrljaru je našao mapu na čijoj poleđini je pisalo: „Krenuo sam iz podnožja Vulkanskih planina (koje sam na karti označio tačkom V) i išao samo pravo do majmunskog igrališta (koje sam na karti označio tačkom M). Moj papagaj Kukutiru je ostao u šatoru gde smo spavali (šator sam označio sa K na karti) ali je posle nekog vremena krenuo da me traži. Išao je po pravoj liniji koja je normalna na put kojim sam ja išao. Blago sam sakrio tamo gde su se putevi kojima smo išli presekli.“ Pomozi Ndžaguminom sinu Mbrljaru da pronađe blago.

2. Nacrtaj dve prave a i b u svesci i jednu tačku A koja nije ni na pravoj a ni na pravoj b . Nacrtaj dve prave tako da je jedna normalna na pravu a , a druga na pravu b , a obe prolaze kroz tačku A .

KORAK 5. Kolektivna provera zadataka

Učitelj poziva da se svi učenici koji su radili zadatak istog nivoa okupe oko jednog stola i da zajedno prodiskutuju o postupku rešavanja i načinima izrade zadatka.

KORAK 6. Zadavanje domaćeg zadatka

Učitelj saopštava učenicima da za domaći zadatak imaju da svako nacrtaju mapu jednog ostrva i da osmisle zadatak sličan onom koji su radili na času gde će zakopano blago tražiti tako što će na neki način crtati normalne prave.

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Obrazovanje budućih nastavnika matematike: ponavljanje matematičkih sadržaja iz osnovne i srednje škole

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Sažetak. Priprema studenata za buduće nastavničko zanimanje složen je i težak zadatak. Ta priprema uključuje tri komponente: matematičko znanje, didaktiku nastave matematike i psihologiju učenja (Hodgson, 2001). Matematičko znanje poznato je kao jedna od ključnih osobina nastavnika matematike, stoga su i mnoge studije istraživale matematičko znanje nastavnika matematike i budućih nastavnika matematike, otkrivajući brojne nedostatke (Ponte & Chapman, 2010). Mnoge od tih studija bile su motivirane Kleinovom paradigmom “Doppelte Diskontinuität” (dvostruki prekid). Godine 1924. Felix Klein (2004ab) koristi izraz “Doppelte Diskontinuität” kako bi opisao dva jaza u obrazovanju nastavnika matematike. Klein je bio motiviran jazom između školske i sveučilišne matematike: prvo, visokoškolska matematika ima vrlo malo dodirnih točaka sa srednjoškolskom i osnovnoškolskom matematikom; drugo diplomirani studenti koji su se vraćali u škole, u svojoj praksi koristili su vrlo malo onoga što su iskusili tijekom svog studiranja. Prošlo je gotovo stoljeće od trenutka kada je Klein fomulirao svoju paradigmu, a dvostruki prekid i dalje postoji (Barton, 2008).

U ovom radu ćemo izložiti rezultate ankete provedene među budućim nastavnicima matematike koji su odgovarali na pitanja iz osnovnoškolske matematike. Istraživanje je bilo motivirano sljedećim pitanjima: Što očekujemo da naši studenti matematike znaju iz osnovnoškolske matematike? Što podrazumijevamo pod “znati”?

Ključne riječi: budućí nastavnici matematike, matematičko znanje

Uvod

Priprema studenata za buduće nastavničko zanimanje složen je i težak postupak. Osnovne komponente ovog iznimno važnog pripremnog razdoblja su matematičko znanje, didaktika nastave matematike i psihologija učenja (Hodgson, 2001). Matematičko znanje se ističe kao jedna od ključnih osobina nas-

tavnika matematike, stoga su i mnoge studije istraživale matematičko znanje nastavnika matematike i studenata nastavnčkog studija matematike, otkrivajući brojne nedostatke (Ponte & Chapman, 2010). Mnoge od tih studija bile su motivirane Kleinovom paradigmom poznatom pod nazivom "Doppelte Diskontinuität" (dvostruki prekid). Godine 1924. Felix Klein koristi izraz "Doppelte Diskontinuität" kako bi opisao dva jaza u obrazovanju nastavnika matematike: prvo, visokoškolska matematika ima vrlo malo dodirnih točaka sa srednjoškolskom i osnovnoškolskom matematikom; drugo, diplomirani studenti koji se vraćaju u škole u svojoj praksi koriste vrlo malo znanja prikupljenog tijekom studiranja. Prošlo je gotovo stoljeće od trenutka kada je Klein formulirao svoju paradigmu, a dvostruki prekid je i dalje prisutan (Barton, 2008). Studije i dalje svjedoče o postojanju „začaranog kruga“: budući nastavnici upisuju se na sveučilišne studije bez dovoljnog razumijevanja školske matematike, tijekom studija vrlo malo su u doticaju s matematikom koju će poučavati i gotovo nepripremljeni ulaze u učionice kako bi obrazovali nove generacije (e.g. Chick, 2004; Ponte & Chapman, 2008).

Teoretski okvir

Mnoge studije pokazale su kako je matematičko znanje nastavnika neadekvatno, pogotovo kada promatramo što nastavnici znaju i kako zadržavaju znanje o matematičkim konceptima i procesima (Ponte i Chapman, 2008). Pokazalo se kako nastavnici ne posjeduju duboko, temeljito i široko znanje sadržaja koje podučavaju. Steinberg i sur. (1985) uočavaju kako se nastavnici-pripravnici u srednjim školama, u situacijama kada ne znaju matematički sadržaj koji bi trebali predavati, koriste raznim strategijama koje najčešće uključuju oslanjanje na udžbenike kao prigodan izvor informacija. Pristup školskoj matematici često je karakteriziran stavom da srednjoškolski nastavnici već znaju matematiku koju trebaju podučavati, a nastavnici u osnovnoj školi trebaju vrlo malo matematičkog znanja (Rowland, 2006). No, pokazatelji iz Velike Britanije, Sjedinjenih Američkih Država, Australije i brojnih zemalja opovrgavaju oba dijela te izjave (npr. Alexander i sur. 1992.; Ma, 1999, Chick, 2002).

Schulman (1981) prepoznaje sedam kategorija znanja koje posjeduje nastavnik (učitelj): predmetno znanje, pedagoško-predmetno znanje, znanje o tome kako učenici razmišljaju, kurikularno znanje, opće pedagoško znanje, znanje o obrazovnim kontekstima i znanje o odgojno-obrazovnim ciljevima i vrijednostima. Tri od navedenih kategorija su eksplicitno usredotočene na poznavanje sadržaja predmeta koji nastavnik podučava: predmetno znanje, pedagoško-predmetno znanje i kurikularno znanje. Predmetno znanje je znanje o sadržaju discipline (Shulman, 1986, str 9.), koje se sastoji od ključnih činjenica, pojmova, načela i objašnjenja u okvirima discipline. Kurikularno znanje uključuje opseg i redoslijed nastavnih programa i materijala koji se u njima koriste. Pedagoško-predmetno znanje je vezano uz upotrebu predmetnog znanja u nastavi, što podrazumijeva poznavanje načina objašnjavanja i predstavljanja koncepata.

Rowland (2006) je proučavao predmetno znanje studenata učiteljskih studija u Velikoj Britaniji, s naglaskom na povezanost tog znanja s izvođenjem nastave u

razredu. Pokazalo se da je učinkovito podučavanje bilo povezano s čvrstim predmetnim znanjem, te da su mnogi studenti imali značajnih problema s generalizacijom i dokazima. Wilson i sur. (2009) uspoređivali su matematičko znanje budućih nastavnika matematike i nastavnika u službi, te otkrili značajne razlike u detaljima, vokabularu i dubini objašnjenja, međutim kvaliteta znanja o matematičkim pojmovima bila je slična. Southwell i Penglase (2005) utvrdili su brojne nedostatke u matematičkom znanju budućih nastavnika matematike, koji su se odnosili na operacije s razlomcima, na množenje decimalnih razlomaka te na mjerenje. Chick (2004) je u svojem radu opisala poteškoće koje su imali budućni nastavnici kad su trebali objasniti istovjetnost $\frac{3}{8}$ i 37,5%, te obrazložiti postupak dodavanja nule cijelom broju pri množenju s 10. Kada su ispitivali znanje budućih srednjoškolskih nastavnika, Wilburne i Long (2010) pronašli su značajne manjkavosti u područjima kao što su analiza podataka, algebra i u osnovama matematičke analize.

Iako su posljednjih desetljeća provedena brojna istraživanja na temu predmetnog i pedagoško-predmetnog znanja budućih nastavnika i nastavnika iz prakse, još uvijek postoje mnoga otvorena pitanja koja zahtijevaju daljnja istraživanja (Chick, 2003). Vođeni ovom problematikom, ispitali smo znanje studenata matematike s posebnim osvrtom na osnovnoškolsku matematiku. Ovo istraživanje je motivirano sljedećim pitanjima: Koja znanja iz osnovnoškolske matematike očekujemo da naši studenti matematike posjeduju? Što podrazumijevamo pod pojmom “znati”?

Metodologija

Dvije grupe studenata matematike sudjelovale su u ovom istraživanju. Prva skupina sastojala se od studenata prve godine koji su se upisali na sveučilišni petogodišnji studij matematike, s namjerom da postanu nastavnici matematike za osnovnu ili srednju školu. Druga grupa studenata sastojala se od studenata četvrte godine koji su studirali visokoškolsku matematiku te se nakon treće godine opredijelili za nastavničko zanimanje. Naš uzorak sastoji se od 71 studenta: 40 studenata prve godine i 31 studenta četvrte godine. Sudjelovanje u istraživanju je bilo dobrovoljno, pa su neki studenti odbili odgovoriti na dana pitanja. Ti studenti nisu uključeni u uzorački broj. Isti upitnik je primijenjen na obje skupine ispitanika na početku akademske godine 2010./2011., za vrijeme predavanja nekog matematičkog kolegija. Također, vrijeme za ispunjavanje upitnika nije bilo ograničeno. Upitnik se sastojao od deset pitanja koja su obuhvaćala osnovnoškolsku matematiku, u rasponu od petog do osmog razreda. Pitanja su kreirana tako da se utvrdi posjeduju li studenti potrebna matematička znanja o temama koje će podučavati u svom nastavničkom radu. U nekim pitanjima studenti su zamoljeni da objasne određene matematičke pojmove, a neka pitanja zahtijevala su rješenje matematičkog problema. Odgovori su svrstani u kategorije po broju izrazito različitih objašnjenja/rješenja te ovisno o tome jesu li objašnjenja/rješenja ispravna. Ovdje, radi nedostatka prostora, nećemo proučavati sve kategorije za svako pojedino pitanje, ali ćemo proučiti posebno zanimljive rezultate koji su se pojavili među odgovorima studenata. Sva pitanja se mogu naći u dodatku.

Rezultati i diskusija

Rezultati provedenog istraživanja pokazuju kako na niti jedno od ponuđenih pitanja niti jedna skupina ispitanih studenata nije u potpunosti točno odgovorila. Štoviše, nitko od ispitanika nije točno riješio zadatak broj 7. Također je primjetan i vrlo mali udio točnih odgovora i na druga pitanja, posebno na peto i šesto. Zadaci s najvećim brojem točnih odgovora bili su 4.b i 6.b. Broj točnih odgovora za ostala pitanja može se vidjeti u sljedećoj tablici.

Pitanja	Prva godina N = 40	Četvrta godina N = 31
Zadatak 1.	10	7
Zadatak 2.	25	17
Zadatak 3.	10	1
Zadatak 4.a	23	25
Zadatak 4.b	28	22
Zadatak 5.	3	2
Zadatak 6.a	7	5
Zadatak 6.b	25	25
Zadatak 7.	0	0
Zadatak 8.	17	15
Zadatak 9.	3	4
Zadatak 10.	21	15

Tablica 1. Broj točnih odgovora dviju grupa studenata.

Kako smo već i naglasili, nije nam namjera diskutirati zasebno rezultate postignute na svakom od postignutih pitanja, već se orijentirati na najinteresantije rezultate i zaključke provedenog ispitivanja.

Zadatak 1. U prvom zadatku se od studenata tražilo da pojasne postupak određivanja najvećeg zajedničkog djelitelja dvaju ili više prirodnih brojeva. Samo četvrtina ispitanih studenata ponudila je točan odgovor (Tablica 2). Ključni termini poput prostih djelitelja u studentskim pojašnjenjima su se pojavljivali izuzetno rijetko. Neki od ispitanika su korektno proveli metodu određivanja najvećeg zajedničkog djelitelja na konkretnom primjeru, no bez dodatnog pojašnjenja te su njihovi odgovori klasificirani kao netočni. Istaknimo kako se neočekivani ishod ovdje ističe u broju studenata koji su ponudili potpuno besmislen odgovor navodeći kako se najveći zajednički djelitelj dobiva množenjem danih prirodnih brojeva.

Kategorije	Prva godina N = 40	Četvrta godina N = 31
Točan odgovor	10	7
Bez odgovora	14	9
Besmislen odgovor	12	11
Naveden primjer bez pojašnjenja	4	4

Tablica 2. Kategorije studentskih odgovora na **Zadatak 1.**

Zadatak 5. Obje grupe ispitanika su imale velikih poteškoća kada se od njih tražilo da pojasne na koji način se zbrajaju cijeli brojevi, što se može vidjeti iz Tablice 3. Posebno je zapanjujuće kako čak trećina studenata prve i trećina studenata četvrte godine studija nije niti pokušala odgovoriti na postavljeno pitanje.

Kategorije	Prva godina N = 40	Četvrta godina N = 31
Točan odgovor	3	2
Bez odgovora	13	11
Netočan odgovor u slučaju brojeva različitog predznaka	6	4
Besmislen odgovor	4	14
Naveden primjer bez pojašnjenja	4	0

Tablica 3. Kategorije studentskih odgovora na Zadatak 5.

Napomenimo kako su studenti u pojašnjenjima rijetko koristili koncept oduzimanja manje apsolutne vrijednosti od veće. Odgovori studenata koji su postupak zbrajanja cijelih brojeva pojašnjavali na primjerima smatrani su netočnima, jer je u zadatku od njih zatraženo da taj postupak opišu riječima, tj. da pojasne o kakvom je računu zaista formalno riječ. Sve u svemu, nameće se zaključak kako je studentima bio problem pronaći odgovarajući način kako opisati sam računski postupak. Ishod ovog zadatka ne implicira da studenti nisu u stanju zbrajati cijele brojeve, ali pokazuju jasan nedostatak formalnog matematičkog izražavanja i konceptualnog razumijevanja odgovarajućeg postupka. Slični rezultati se pojavljuju i u radovima Thompson i Thompson (1996) te Chick (2003), u kojima su ispitani budući nastavnici u Sjedinjenim Američkim Državama te u Australiji. Stoga rezultati ovog istraživanja nisu izoliran slučaj edukacije u Hrvatskoj, već idu ruku pod ruku s globalnim problemom u edukaciji budućih nastavnika.

Još jedan očiti problem s kojim se susrećemo leži u razumijevanju matematičkih izraza. Iako se matematički jezik smatra univerzalnim jezikom, pretpostavka da će studenti koji se svakodnevno susreću s matematičkim pojmovima i izrazima moći razumjeti terminologiju i sintaksu danog problema pokazuje se netočnom. Postoje brojna istraživanja koja opovrgavaju ovu pretpostavku (npr. Chick, 2001; Southwell i Penglase, 2005). U ovom istraživanju navedeni se problem može jasno vidjeti pri odgovorima na zadatak 6.a, gdje su studenti računali točan broj učenika koji igra nogomet, iako se od njih tražilo da odrede udio koji ti učenici čine obzirom na ukupan broj učenika. Slično tome, u zadatku 7. su brojni studenti samo naveli vrste četverokuta koje poznaju, bez pokušaja da ih dodatno klasificiraju.

Pogreške u računanju proporcionalnosti mogu se vidjeti u zadatku 9. Studenti su primijenili algoritamski postupak za računanje s razmjerima bez zdravorazumskog razmišljanja o postavkama tog algoritma. Slični slučajevi u kojima ispitanici algoritamski računaju razmjere bez prethodnog promišljanja o danom problem se mogu naći u radovima Cramer i sur. (1993) i Lim (2008). Rezultati zadatka 9. precizno oslikavaju i ukupne rezultate provedenog istraživanja. Studenti tijekom rješavanja danih zadataka izrazito teže računanju, što pokazuje snažnu proceduralnu orijentiranost u rješavanju matematičkih problema, pri čemu studenti povezuju svoje

matematičko znanje isključivo s računskim postupcima. Ovaj problem je primjetan već i u zadacima 1. i 5. u kojima se tražilo pojašnjenje riječima. Proceduralno znanje se ističe također i u zadatku 8., gdje su studenti uglavnom dobili netočna rješenja. Studenti su postavili sustav jednažbi, odredili brojeve koji bi trebali predstavljati bratove i sestrine godine, no propustili su provjeriti zadovoljavaju li dobivena rješenja uvjete zadatka. Prema Davis i McGowen (2001), ovakav tip strogo proceduralne orijentiranosti u matematici zahtjeva značajnu prebacivanja težišta na konceptualizaciju matematike kao discipline bogate odnosima, a ne discipline koja se sastoji samo od formula te točnih ili netočnih odgovora.

Rezultati provedenog istraživanja jasno pokazuju kako su studenti zaboravili znatan dio osnovnoškolske matematike (Tablica 1). Naravno, prirodno je zapitati se je li pravedno postaviti takvu tvrdnju, jer su u istraživanje bile uključene dvije skupine studenata različitih predznanja. No, studenti prve godine su polagali državnu mature, pri čemu se od njih zahtjevala viša razina kako bi mogli upisati studij matematike. S druge strane, studenti četvrte godine su barem tri prethodne godine proveli na studiju matematike, što dovoljno govori samo za sebe. Prema tome, za očekivati je da obje grupe ispitanika posjeduju znanje potrebno za rješavanje svih postavljenih zadataka ili barem većine njih. Ipak, usporedimo li rezultate provedenog istraživanja s rezultatima novijih istraživanja slične tematike, ispostavlja se da su oni daleko od iznenađujućih (npr. Southwell i Penglase, 2005; Wilburne i Long, 2010).

Zaključak

Postoje značajni pokazatelji da završetak studija matematike nije dovoljan za razvoj matematičkih znanja koja su potrebna za podučavanje matematike (Monk, 1994). U posljednjem desetljeću pojavile su se mnoge studije koje su proučavale matematičko znanje budućih nastavnika i koje su istaknule da je bez dobrog matematičkog znanja, pedagoško znanje od male koristi (npr. Davis i McGowen, 2001, Chick, 2001; Southwell i Penglase, 2005). Te studije također ukazuju na potrebu za izradom matematičkih obrazovnih programa koji će povezati sveučilišne matematičke kolegije, kolegije vezane uz metodiku nastave matematike za osnovnu i srednju školu, te školske matematičke sadržaje kroz čitav kurikulum. Ti kolegiji pripremili bi buduće nastavnike za podučavanje školske matematike; kolegiji bi posebnu pažnju posvetili temeljnim matematičkim pojmovima, pomogli bi budućim nastavnicima da svladaju procedure i koncepte, te bi im omogućili da razviju sposobnost izražavanja.

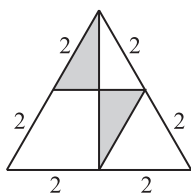
Na Odjelu za matematiku Sveučilišta u Osijeku, školska matematika ponavlja se u okviru kolegija Metodika nastave matematike 1 kroz nekoliko kolokvija. Neka pitanja iz upitnika pojavila su se u prvom kolokviju, što nam je omogućilo usporedbu rezultata iz upitnika i kolokvija. Rezultati su se značajno poboljšali za sva pitanja osim 6.a, gdje su studenti još uvijek izračunavali eksplicitan broj učenika koji su igrali nogomet. Međutim, količina vremena povećana školskoj matematici još uvijek je nedovoljna, pogotovo što očekujemo da studenti samostalno ponove gradivo školske matematike za kolokvije. Stoga podržavamo ideju kolegija posvećenog samo školskoj matematici, koji bi ponovio "zaboravljene" matematičke pojmove i

angažirao studenata u drugačijem kontekstu; ne više u kontekstu gdje su oni učenici već nastavnici.

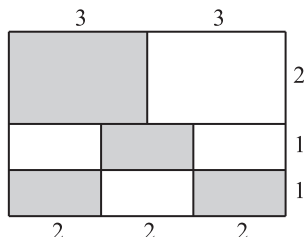
Dodatak

1. Objasnite postupak određivanja najvećeg zajedničkog djelitelja dva ili više brojeva.
2. Četiri jedrenjaka susreću se na Otoku vode. Prvi dolazi svakih 16 dana, drugi svakih 8 dana, treći svakih 12 dana a četvrti svakih 6 dana. Kada će se jedrenjaci ponovno susresti?
3. Nacrtajte sve osi simetrije sljedećih likova: pravokutnik, kvadrat, romb, jednakokračni trokut i jednakostranični trokut.
4. Razlomkom izrazi dio lika koji je obojen:

a)



b)



5. Objasnite kako se zbrajaju cijeli brojevi.
6. Dvije petine učenika neke škole bavi se sportom. Od tih koji se bave sportom njih tri sedmine igraju nogomet.
 - a) Koliki dio učenika te škole igra nogomet?
 - b) Ako u školi ima 805 učenika, koliko njih se bavi sportom?
7. Kako dijelimo četverokute s obzirom na paralelnost njihovih stranica?
8. Sestra je starija 8 godina od brata. Za 5 godina bit će dvostruko starija. Koliko godina ima sestra, a koliko brat?
9. 42 radnika mogu podići nasip za 25 dana. Nakon 5 dana s posla je izostalo 7 radnika. Za koliko će dana nasip biti podignut?
10. Ako je a realan broj, čemu je jednak $\sqrt{a^2}$?

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Konceptualne mape u obrazovanju učitelja matematike

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Sažetak. Zahtjevi za novim pristupima učenju i obrazovanju učitelja matematike postaju sve jači. Sve se više govori o različitim pristupima reprezentaciji znanja, a zatim i ocjenjivanju i evaluaciji **u procesima učenja i nastave matematike**. Konceptualne mape, mentalne mape, mape znanja kao i drugi slični pristupi daju dobru osnovu za svrhovito i trajno učenje kako učitelja tako i učenika s ciljem poboljšanja znanja vještina i sposobnosti.

U ovom se radu raspravlja o utjecaju, strategiji i trendovima upotrebe konceptualnih mapa u obrazovanju.

Ključne riječi: konceptualne mape, mentalne mape, mape znanja, obrazovanje

Uvod

U matematičkoj literaturi često se navode načela matematičkog obrazovanja: (Gollub, J. P., Bertenthal, M. W., Labov, J. B. and Curtis, P. C., Editors (2002), *Learning and Understanding: Improving Advanced Study of Mathematics and Science in U.S. High Schools* National Academies Press). To je 7 načela za učenje i razumijevanje:

1. Konceptualno znanje

Učenje s razumijevanjem olakšano je kad se novo i postojeće znanje strukturira oko glavnih koncepata i načela određene discipline.

2. Prethodno znanje

Učenici se koriste postojećim znanjima da izgrade novo s razumijevanjem.

3. Metakognicija

Učenje se bitno olakšava korištenjem metakognitivnih strategija koje identificiraju, prate i reguliraju kognitivne procese.

4. Razlike između učenika

Učenici imaju različite strategije, pristupe, vrste sposobnosti i stilove učenja koji su funkcija interakcija između njihova nasljeđa i prethodnog iskustva.

5. Motivacija

Motivacija učenika na učenje i njegova samosvijest utječu na ono što će biti učeno, koliko će biti učeno i koliko će se napora uložiti u proces učenja.

6. Situira učenje

Čine ga praksa i aktivnosti u kojima učenici, dok uče, oblikuju ono što je naučeno.

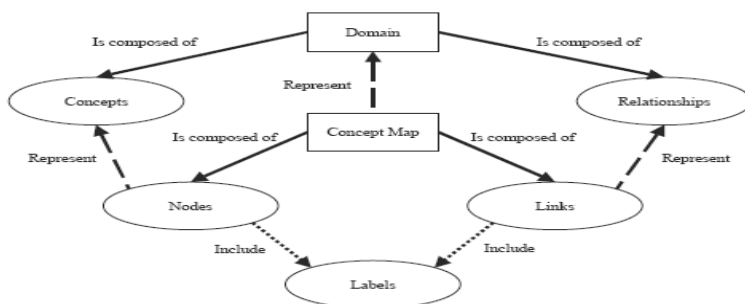
7. Zajednice učenja

Učenje se obogaćuje kroz procese socijalno podržanih interakcija.

Sva ova načela, a posebno načela 1., 2. i 4. usko su povezana s **konceptualnim mapama** koje se opisuju u nastavku rada.

Konceptualne mape, mentalne mape, mape znanja

Konceptualne mape (pojmovne mape) grafički su alati za organizaciju i prezentiranje znanja. One uključuju **pojmove** (obično uokvirene) i **veze** između **pojmovi** koje su prikazane **crta**ma. Riječi na crti, tzv. *povezujuće riječi ili povezujuće fraze*, određuju vezu između dva pojma. **Pojam** definiramo kao osjetnu pravilnost u događajima i stvarima ili kao evidenciju događaja ili stvari, imenovane etiketom. Etiketa za većinu pojmova jest riječ, iako se ponekad koristimo simbolima kao što su + ili %, a ponekad rabimo i više od jedne riječi. **Propozicije** sadrže dva ili više pojma povezana *povezujućom riječi ili frazom* tako da oblikuju suvislu tvrdnju. Slike 1. i 2. prikazuju konceptualne mape koje opisuju strukturu mape i ilustriraju gore navedene karakteristike. (Novak, J. D. & A. J. Cañas (2008.)).

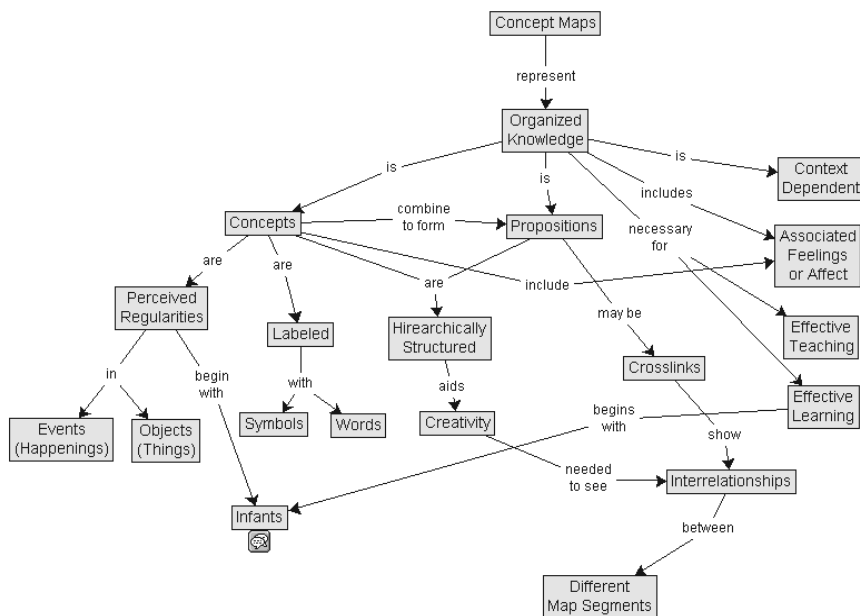


Slika 1. Jednostavna konceptualna mapa konceptualne mape.

Važna karakteristika pojmovnih mapa jest u tome da su pojmovi predstavljeni **hijerarhijski** s najviše uključenim i najviše općim pojmovima na vrhu mape i sa specifičnijim, manje općim pojmovima hijerarhijski niže. **Hijerarhijska struktura** za određeno područje znanja također ovisi o sadržaju u kojem se to znanje upotrebljava ili se uzima u obzir. Stoga, najbolje bi bilo konstruirati pojmovnu mapu s naglaskom na neka određena pitanja na koja želimo odgovoriti. Ta pitanja zovemo **fokusna pitanja**.

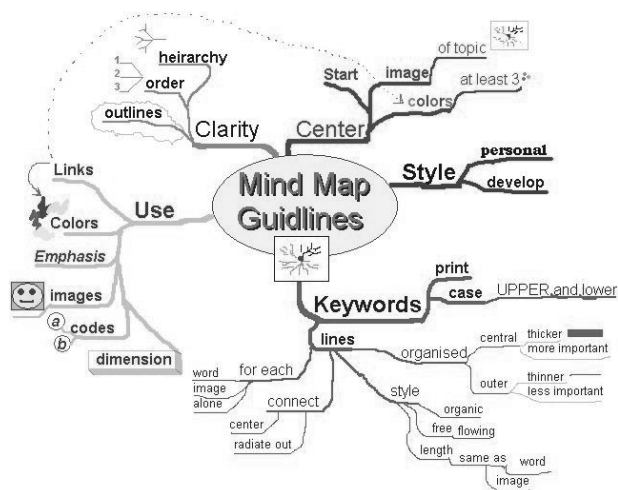
Sljedeća je važna karakteristika pojmovnih mapa uključivanje **poveznica koje se križaju** (crosslinks). To su veze između pojmova jednog segmenta ili domene

s drugim segmentima ili domenama pojmovnih mapa. Ukrštene poveznice pomažu da shvatimo kako su pojmovi predstavljeni na mapi u jednoj domeni znanja povezani s pojmovima u drugoj domeni. U stvaranju novog znanja, *poveznice koje se križaju* često predstavljaju *kreativne skokove* u ulozi proizvođača znanja.



Slika 2. Složenija konceptualna mapa konceptualne mape.

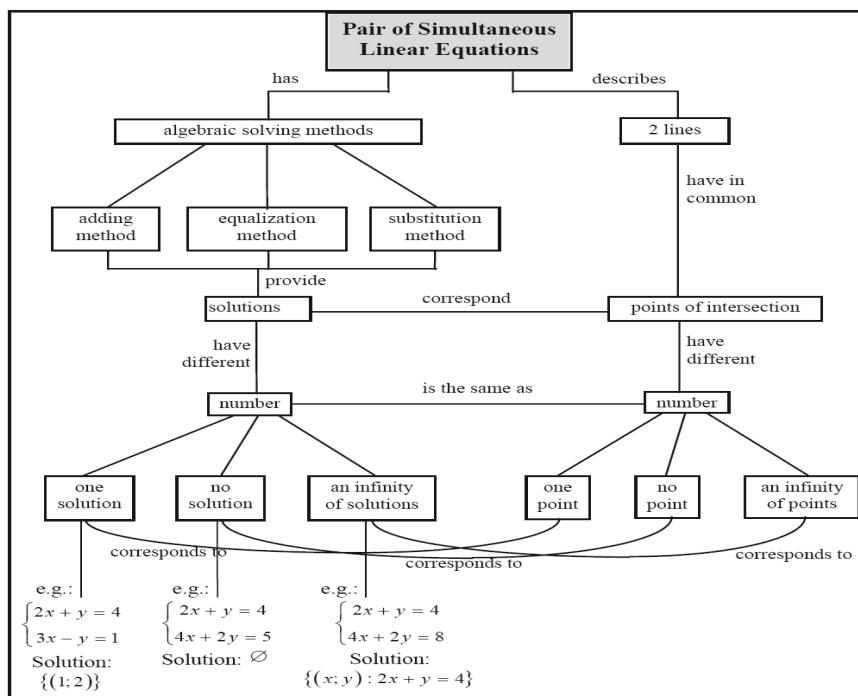
Na sljedećim slikama prikazane su najprije ilustracije definicija konceptualnih mapa i mentalnih mapa, a zatim dva primjera iz matematike i to za sustav *linearnih* *jednadžbi* i za *kružnicu*.



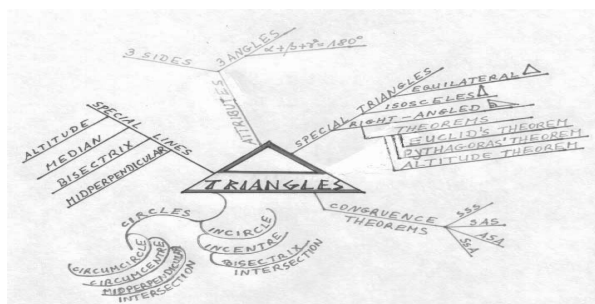
Slika 3. Mentalna mapa ili umna mapa.

Mentalne mape ili umne mape vrsta su konceptualnih mapa sa sljedećim karakteristikama:

1. **glavni pojam, ideja ili fokus** prikazan je u središtu slike
2. glavne teme izlaze i granaju se kao ogranci iz središta
3. ogranci sadrže ključnu sliku ili ključnu riječ koja je napisana na pridruženoj liniji
4. teme od manje važnosti reprezentiraju se kao izdanci važnijih grana
5. grane oblikuju povezanu strukturu stabla

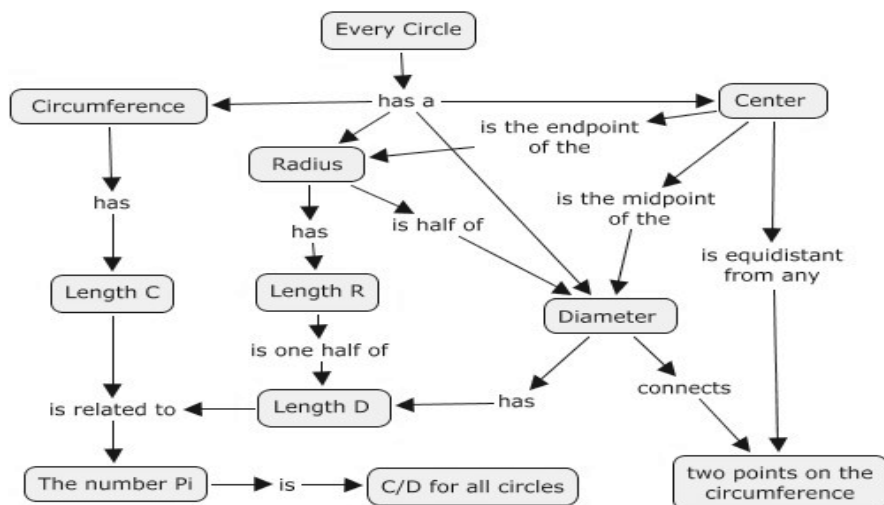


Slika 4. Konceptualna mapa za sustav linearnih jednadžbi.



Slika 5. Mentalna mapa trokuta.

Izvor za slike 4. i 5. Brinkmann, A. (2003.), *Graphical Knowledge Display – Mind Mapping and Concept Mapping as Efficient Tools in Mathematics Education*, Mathematics Education Review, The Journal of Association of Mathematics Education Teachers, Number 16, April 2003, pp. 39–48.

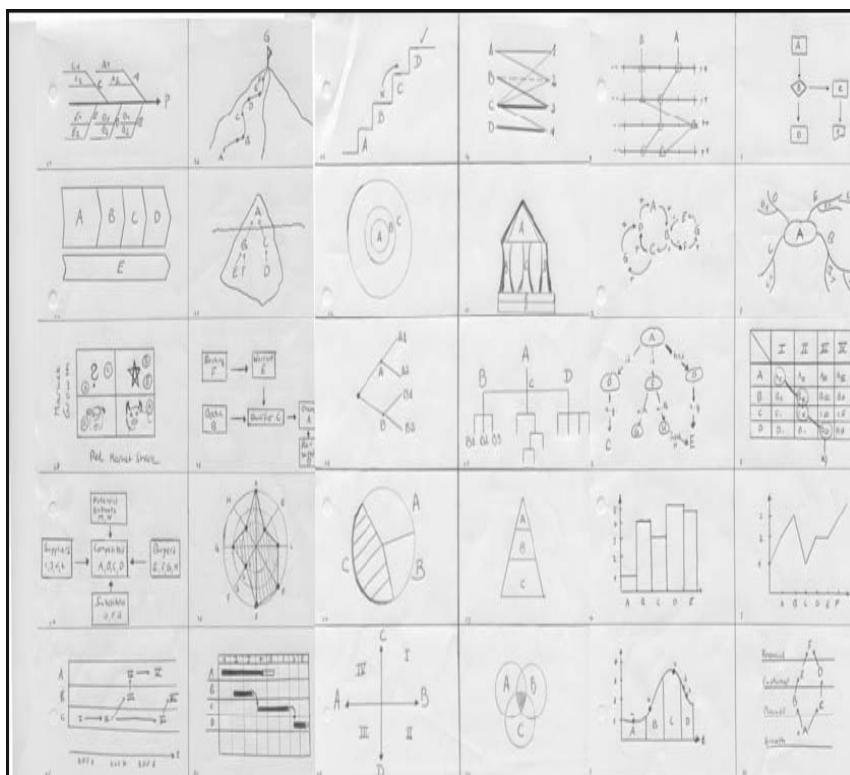


Slika 6. Konceptualna mapa kružnice.

Izvor: Caldwell, W. H., Al-Rubae, F., Caldwell, L. F., Campese, M. (2006.), *Developing a Concept Mapping approach to mathematics achievement*, Concept Maps: Theory, Methodology, Technology, Proc. of the Second Int. Conference on Concept Mapping San José, Costa Rica, 2006., A. J. Cañas, J. D. Novak, Eds.

Vizualizacija podataka, informacija i znanja

Konceptualne mape samo su vrsta vizualizacije, odnosno grafičke reprezentacije u nekom području znanja. Vizualizacija podataka, informacija i znanja kao način reprezentacije služi kako za kreiranje, tako i za transfer, komunikaciju i korištenje znanja i veoma je važna u području obrazovanja (Chen Min i dr. (2007.), *Data, Information and Knowledge in Visualization*). Postoje brojni tipovi različitih grafičkih i drugih prikaza kao i strogih formalnih metoda i jezika za reprezentaciju znanja. U svom radu Eppler, M. J., (2007.), navodi 30-tak različitih prikaza (vidi sliku 7.) i vrši njihovu taksonomiju.



1. Fishbone diagram	11. Steps diagram	21. Profile chart
2. Porter's value chain	12. Concentric circles diagram	22. Loop (or system) diagram
3. The BCG portfolio matrix	13. Decision tree	23. Concept map
4. Porter's five forces diagram	14. Pie chart	24. Bar chart
5. Technology roadmap	15. Cartesian coordinates	25. Life cycle diagram
6. Trail diagram	16. Connectance diagram	26. Flow chart
7. Iceberg diagram	17. Temple diagram	27. Mind map
8. Toulmin diagram	18. Org chart / tree	28. Morphological box
9. Radar chart	19. Pyramid	29. Line chart
10. Gantt chart	20. Venn diagram	30. Strategy map (BSC)

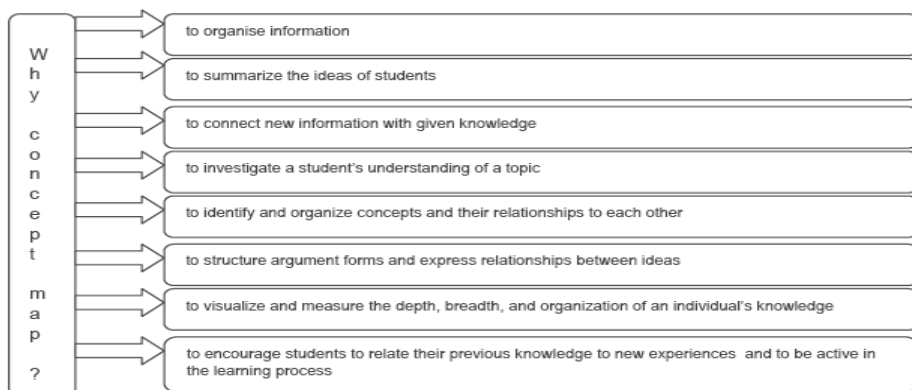
Slika 7. Selekcija 30 raznih grafičkih prikaza.

Izvor: Eppler M. J., Platts K. (2007.), *An Empirical Classification of Visual Methods for Management: Results of Picture Sorting Experiments with Managers and Students*. IV'07:335–341.

Upotreba konceptulnih mapa

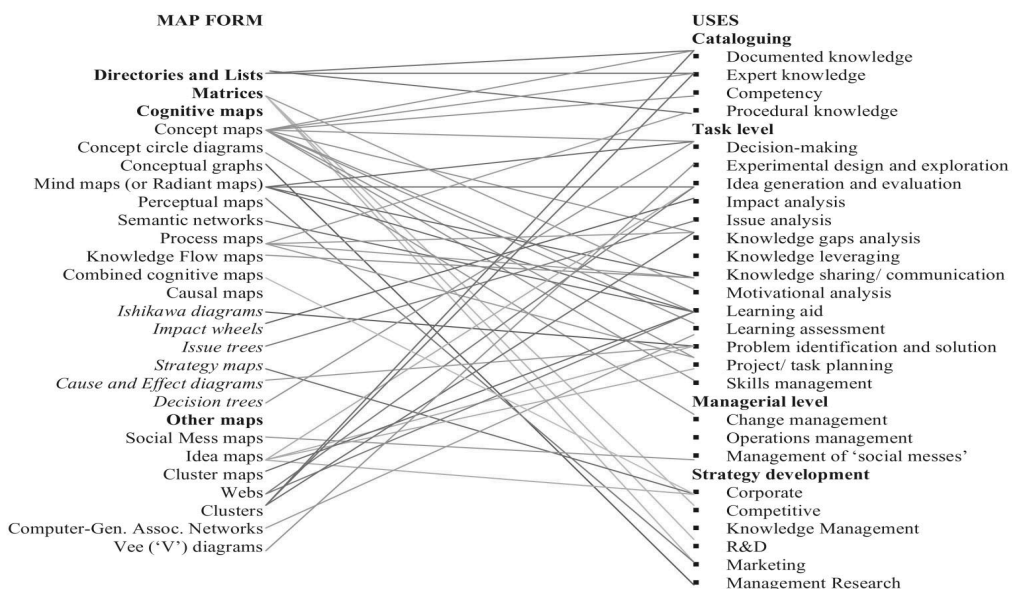
Konceptualne mape koriste se za brojne primjene u edukaciji, e-učenju (vidi **CARNET** http://elacd.carnet.hr/index.php/Course_Design_2009-2010/KONCEPTUALNE_MAPE) i nizu drugih područja. U učenju se konceptualne mape koriste za usvajanje novih pojmova, povezivanju starog i novog znanja, ocjenjivanju i organiziranju znanja, boljem prisjećanju, brzini i kvaliteti učenja itd.

(vidi sliku 8.). Folkes C. R. i dr. (2004.) (vidi sliku 9.) opisuju postupke za mapiranje znanja, njihovu korist i način upotrebe.



Slika 8. Upotreba konceptualnih mapa.

Izvor: Çakmak M. (2010.), *An examination of concept maps created by prospective teachers on teacher roles*, Procedia Social and Behavioral Sciences 2 (2010.) 2464–2468.



Slika 9. Vrste mapa i njihova upotreba.

Izvor: Folkes, C. R., P. Quintas, A. Demaid (2004), *Knowledge Mapping: Map Types, Contexts and Uses*. KM-SUE 4, The Open University: Milton Keynes.

Izgradnja i softver za konceptualne mape

Detaljni prikaz izgradnje konceptualnih mapa kao i njihovo korištenje dan je u radu Novaka J. i dr. (2008.).

U posljednje vrijeme razvijaju se i brojni drugi pristupi za konstrukciju konceptualnih mapa (vidi, npr., <http://www.cs.joensuu.fi/pages/whamalai/sciwri/belinda.pdf>) i rad autora Tseng S. S. i dr. (2007.). Na stranicama WikiIT nalazimo listu i adrese softvera kojim se možemo koristiti za izradu konceptualnih i drugih mapa. Ovaj popis obuhvaća gotovo sve softverske proizvode koji su “free” i koji su svima dostupni.

Primjeri izgradnje i korištenja konceptualnih mapa

Učenici

Na primjerima učenika iz Finske i Italije ilustriramo uporabu konceptualnih mapa.

Učenici iz Finske mapirali su prednosti i nedostatke korištenja interneta.

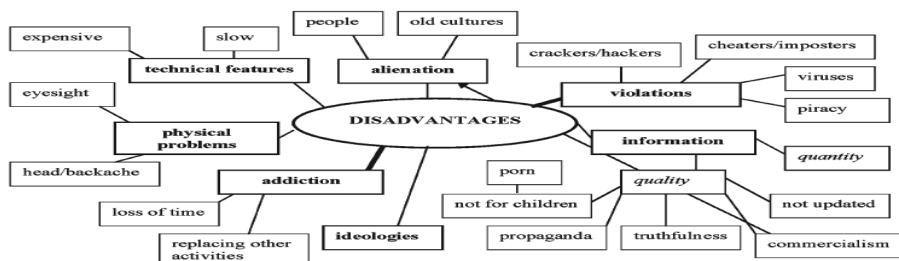
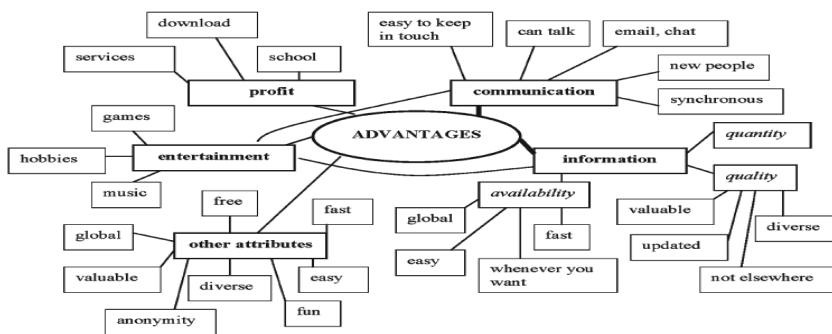


Figure 2. Mapping the disadvantages of the Internet as reported by Finnish 15-year-olds



Slika 10. Prednosti i nedostaci interneta.

Izvor: Leino, K. (2006.), Reading the Web-Students' Perceptions about the Internet, *Scandinavian Journal of Educational Research*, 50(5), str. 541–557.

Učenici iz Italije starosti 9–10 godina nakon korištenja konceptualnih mapa odgovarali su na sljedeća pitanja:

What's a map?	It's a scheme where I study a topic and think about it. It's a useful scheme to study in a logical way. It's a synthetic scheme organized like our thought, where every topic is linked to another. It's a logical method of studying following a scheme, so that I can speak fluently about a topic. It's a scheme showing functions and detail of... something. It's a little scheme where I put the main things about a topic. It's like a diagram explaining a key word. It's the summary of a page of a book. It's a scheme where I write the most important points. It's a written and well arranged scheme. It's a scheme which I can consult to see the main things.
What do you do when building a map?	I look for words and information. I give a meaning, a word, to each point. I put words in order. What do you mean by "putting in order"?: • I put them in their right place. • I classify them. • I put at the top the most important. I build my map with my pencil or with my post-it. I put the words in relationship with arrows and linking phrases. I change the position of words by rearranging arrows and linking phrases.
What's its use for?	It helps me to remember what I have studied: The scheme fixes in my mind. It helps me to understand better a concept and to tell a topic. It helps me to study, to think, to understand. It helps me to have a well arranged, logical visual scheme. It helps me to study better and quickly. It helps me to review what I do not remember and to link different topics. It helps me to know a lot of things simply by reading summaries.

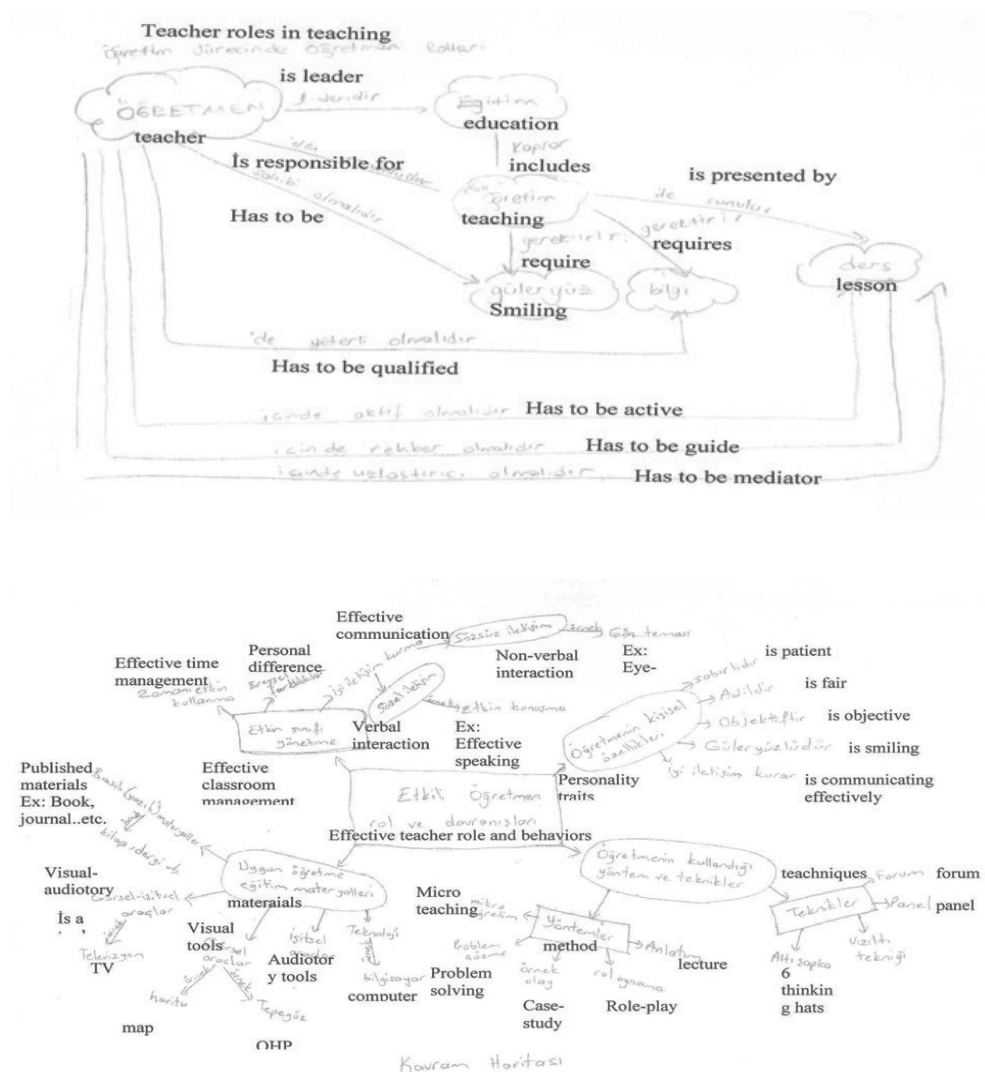
Tablica 1. Što djeca kažu o konceptualnim mapama?

Izvor: Berionni A. & Baldoni M. O. (2004.), The Words of Science: The Construction of Science Knowledge using Concept Maps in Italian Primary School, Proc. of the First Int. Conference on Concept Mapping, A. J. Cañas, J. D. Novak, F. M. González, Eds. Pamplona, Spain

Odgovori na prethodna pitanja pokazuju da učenici pri sustavnom korištenju konceptualnih mapa mogu ne samo razviti kognitivne kompetencije nego i metakognitivne kompetencije, tj. mišljenje o svojem mišljenju te biti svjesna svojeg procesa učenja i testirati stupanj svog znanja. Kognitivne i metakognitivne kompetencije usvojene u motivirajućem kontekstu učenja polazna su točka razvoja najznačajnije kompetencije, a to je znati "učiti kako se uči".

Nastavnici

U nastavku prikazujemo neke rezultate istraživanja provedenih 2009. godine na studentima – budućim učiteljima na temu “Uloga budućih učitelja u učenju” i “Konseptualne mape u formativnom ocjenjivanju”.



Slika 11. Primjeri konseptualnih mapa koje su izradili budućı učitelji u Turskoj.

Izvor: Çakmak M., 2010., An examination of concept maps created by prospective teachers on teacher roles, Procedia Social and Behavioral Sciences 2

Table 1. Student teachers' Perceived Usefulness of Concept Mapping as a Formative Assessment

Perceptions	NA	VL	SW	TGE
1. Concept mapping helped me to understand the key concepts of the subject I studied in the class.	2.4	9.0	33.7	54.8
2. Concept mapping helped me to improve my learning of the course content.	2.4	7.8	48.8	41.0
3. Concept mapping increased my motivation in learning the course content.	3.6	7.8	47.6	41.0
4. Concept mapping increased my involvement in the class.	.6	2.4	30.7	66.3
5. Concept mapping helped me to communicate my learning to others in the class.	0	4.8	54.8	40.4
6. Concept mapping stimulated me to think independently.	0	5.4	55.4	39.2
7. Concept mapping helped me to learn cooperatively with my class colleagues.	.6	10.8	84.9	3.6
8. Making connections among concepts in concept mappings tasks challenged my thinking.	0	1.2	59.6	39.2
9. Concept mapping helped me to clarify the interrelationships among course content.	1.2	6.6	48.2	44.0
10. Concept mapping helped me to see the missing components in my learning of the course content.	0	5.4	36.1	58.4
11. Visualizing my learning through concept mapping tasks reduced the ambiguity of the concepts expressed verbally by the instructor.	1.2	7.8	69.3	21.7
12. Concept mapping helped me to gain a better understanding of my learning processes in the class.	2.4	6.6	33.7	57.2

Note: 1 = Not at all, 2 = Very little, 3 = Somewhat, 4 = To a great extend

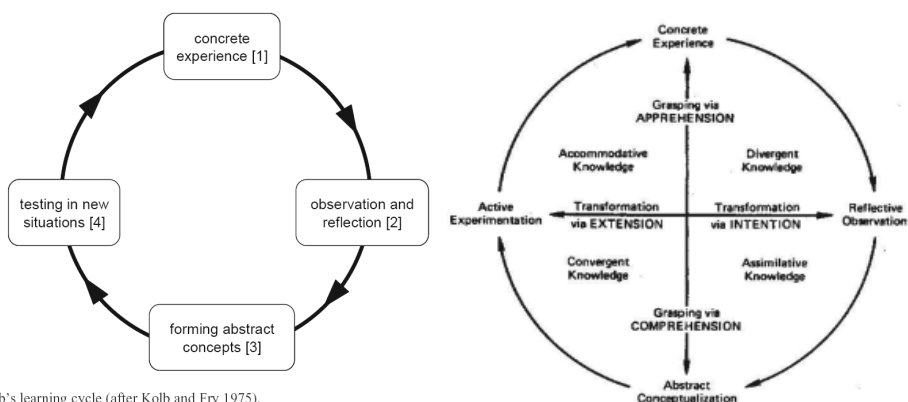
Tablica 2. Kako studenti – budući učitelji vide korist konceptualnih mapa kod formativnog ocjenjivanja.

Izvor: Buldu M., Buldu N., (2010.), Concept mapping as a formative assessment in college classrooms: Measuring usefulness and student satisfaction Procedia Social and Behavioral Sciences 2 2099–2104

Kvaliteta učenja, stilovi učenja i konceptualne mape

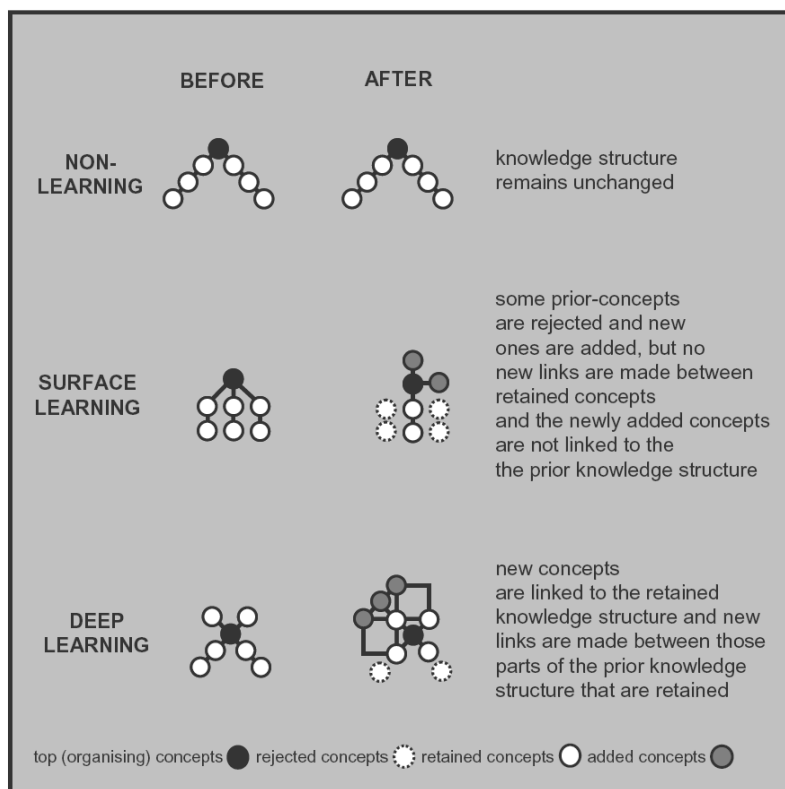
Za mjerenje kvalitete učenja često se služimo različitim modelima. Neki autori koriste se

pritom brojnim stilovima učenja kojih ima preko 70-tak. Jedan od najčešće korištenih jest KOLB-ov model učenja i stilova učenja koji je prikazan na slici 12. Na temelju stanja prethodnog i novostečenog znanja mjeri se kvaliteta učenja kao što to prikazuje slika 13. Hay (2007.) koristi se konceptulnim mapama da komparira strukture znanja *prije i poslije* učenja (slika 13.). Također se ovdje navode metode i različite strukture kojima se opisuju prethodno znanje i učenje. Konceptualne strukture prikazane su na slici 13. *prije i poslije* učenja. Pojmovi su prikazani kao krugovi, a veze kao crte. *Ne-učenje* (na vrhu slike) sastoji se od jednostavnog ponavljanja prijašnjeg znanja i predstavlja samo staro znanje bez promjena. *Površinsko učenje* (u sredini slike) sastoji se u dodavanju ili brisanju pojmova bez ikakve integracije s drugim dijelovima prethodne strukture znanja. *Dubinsko učenje* sastoji se od integracije prethodnog znanja i novog znanja na način koji značajno povećava razumijevanje sadržaja.



Kolb's learning cycle (after Kolb and Fry 1975).

Slika 12. Kolbov ciklus učenja i stilovi učenja.



Slika 13. Kvaliteta učenja.

Izvor: Hay, D. B. (2007.). Using concept maps to measure deep, surface and non-learning outcomes, *Studies in Higher Education*, 32, 1, 39–57.

Adrese

CARNET http://elacd.carnet.hr/index.php/Course_Design_2009-2010/KONCEPTUALNE_MAPE

IHMS ConceptMapsTools <http://cmap.ihmc.us/conceptmap.html>

EDUTECHWIKI http://edutechwiki.unige.ch/en/Concept_map

PSYCHOLOGY WIKI http://psychology.wikia.com/index.php?title=Concept_map

WIKIT 1 http://www.informationtamers.com/WikIT/index.php?title=Mind_maps

WIKIT 2. http://www.informationtamers.com/WikIT/index.php?title=Concept_maps_or_mind_maps%3Fthe_choice

WIKI http://en.wikipedia.org/wiki/Concept_mapping

CAST http://www.cast.org/publications/ncac/ncac_goudl.html

GRAPHIC ORGANIZER <http://www.graphic.org/index.html>

IQ MATRIX <http://iqmatrix.com/>

KMWIKI <http://kmwiki.wikispaces.com/>

Zaključak

Zahtjevi za novim pristupima učenju i obrazovanju učitelja matematike postaju sve jači i izazovniji. Sve više se nalažavaju različiti pristupi za reprezentaciju i transfer znanja, a zatim i za ocjenjivanje i evaluaciju ***u procesima učenja i nastave matematike***. Konceptualne mape, mentalne mape i mape znanja daju dobru osnovu za svrhovito i trajno učenje kako učitelja tako i učenika s ciljem poboljšanja znanja vještina i sposobnosti. Pri sustavnom korištenju konceptualnih mapa mogu se ne samo razviti kognitivne kompetencije pojedinaca nego i metakognitivne kompetencije, tj. mišljenje o svojem mišljenju te biti svjestan svojeg procesa učenja i testirati stupanj svog znanja. Kognitivne i metakognitivne kompetencije usvojene u značajnom i motivirajućem kontekstu učenja polazna su točka razvoja najznačajnije kompetencije, a to je znati “učiti kako se uči”.

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Brojeva crta kao aritmetički stroj

Miljenko Stanić

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Sažetak. U članku će se pokušati predstaviti aritmetičke operacije na brojevnoj crti kao aritmetički stroj, koristeći se samo “mehaničkim” zapovijedima, kao što su dužina koraka, smjer udesno ili ulijevo, odnosno u smjeru prema/ili od 0 i slično. Operacije će biti predstavljene uobičajenim vizualnim objektima koji se koriste u školskoj nastavi, kao što su strelice, dužine, brojke i slično, stoga se stroj može shvatiti kao vizualna sintaksa u smislu dijagramske logike. Mehanički rezultat je vizualna figura na brojevnoj crti kao stop-akcija programa na stroju koji korespondira s računskim operacijama u aritmetičkoj strukturi, u smislu logičke ispravnosti i potpunosti. Računske operacije, s osobitim naglaskom na dijeljenje i 0, mogu se simulirati kao program na brojevnoj crti. S druge strane, stroj se može koristiti u nastavi matematike kao očigledno sredstvo za učenje apstraktnih aritmetičkih operacija, temeljenih na neposrednoj intuiciji učenika u razrednoj nastavi. Obrada operacija na brojevnoj crti, u njenom općem obliku, prikazana je dinamički s malim računalnim programom.

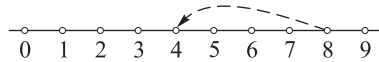
Cljučne riječi: aritmetičke operacije, brojeva crta, dijagramska sintaksa, računalni program

U ovom članku želi se predstaviti vizualizaciju aritmetičkih operacija na brojevnoj crti. Model brojeve crte praktički je prisutan u svakoj školskoj učionici razredne nastave. Udžbenici matematike za drugi i treći razred u Hrvatskoj predočuju računske operacije pomoću “skokova” na brojevnoj crti. Upravo taj pristup uzet je kao temelj za prikaz aritmetičkog stroja koji bi dobro i u cijelosti objasnio aritmetičke operacije. Stroj je prikazan najčešće kao dijadičko stablo s komandama koje su u suštini iste kao i kod računalnog programiranja, kao COPY, DELETE, GOTO i sl. Potpuno vizualno prikazivanje apstraktnih sadržaja u matematici je na tragu ideja izloženih u članku [2] i knjigama [1] i [4].

Metodički pristup nastavi o brojevnoj crti obogaćen je malim računalnim programom kojem je cilj prezentirati teorijske strojeve generirane u članku.

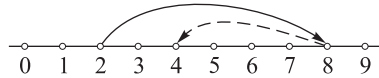
1. *nosač* $\eta = (\text{————})$ je jednodimenzionalni kontinuirani objekt koji može biti otvorena ili zatvorena crta. $P \subset \eta$ je podskup *referentnih* točaka na nosaču η .

2. Definirajmo strukturu *brojevne crte* $\mathbf{b} = (t_0, f \circ I)$. t_0 je istaknuta točka, a $f \circ I$ je funkcija koja svakom grafičkom objektu na crti pridružuje njegovo ime. $I : P \rightarrow D$ je funkcija *usmjerenja*. Za skup $D \subseteq \mathbf{N} \times \mathbf{N}$ vrijedi da je standardno uređen i da je
 1. $\{(n, 0) | n \in \mathbf{N}\} \subset D$.
 2. Izaberimo točku $t \in P$, tako da je $I(t) = (0, 0)$, $f(I(t)) = 0$, odnosno kraće ćemo je označiti kao t_0 .
 3. Za točku $v \in P$ reći ćemo da je v *vizualno* između t_0 i w akko je $(I(w) \neq I(v))$ i $(I(w) < I(v))$. $f : I(P) \rightarrow T$ je funkcija *označavanja*. T je skup *imena* za grafičke objekte ($\mathbf{N} \subseteq T$). Ako je $n, m \in I(P)$ ($n < m$), onda je $(f(n) < f(m))$.
3. *Točke kružići*: $t_n = (\circ)_{f(n)}$, ($n \in \mathbf{N} \times \mathbf{N}$) su osnovni objekti na brojevnoj crti.
4. *Skokovi* $l_{ch}(\gamma, \tau) = (\overset{\curvearrowright}{\gamma} \overset{\curvearrowright}{\tau})(\gamma, \tau)$ i $l_{ph}(\gamma, \tau) = (\overset{\curvearrowleft}{\gamma} \overset{\curvearrowleft}{\tau})(\gamma, \tau)$. γ je *rep skoka* a τ je *glava skoka*, $h \in \mathbf{N}$ je *ime skoka*.
5. Generiraj *skok na brojevnoj crti* je proširenje $\mathbf{b}$ sa skokom:
 $l_{ih}(n_{\gamma,i,h}, n_{\tau,i,h}) (n_{\gamma,i,h}, n_{\tau,i,h} \in T) (f \circ I)(\gamma) = n_{\gamma,i,h}, ((f \circ I)(\tau) = n_{\tau,i,h})$ u brojevnju crtu $\mathbf{b} \cup l_{ih}$.
6. *Točka t_n sadržana je u skoku l_{ih}* : $(\forall m, k \in \mathbf{N} \times \mathbf{N}) (f(m) = n_{\gamma,i,h} \wedge f(k) = n_{\tau,i,h}) \implies (m \leq n \leq k) \vee (k \leq n \leq m)$



t_6 je sadržan u l_c .

7. *Preklapanje* $\supset$ je relacija: $l_{i,h}(\gamma, \tau) \supset l_{j,k}(\gamma, \tau)$ $i, j \in \{c, p\}$ akko $(\forall m, n \in \mathbf{N} \times \mathbf{N}), f(m) = n_{\gamma,j,k}, f(n) = n_{\tau,j,k} \implies (t_n \text{ i } t_m \text{ sadržani u skoku } l_{ih})$.



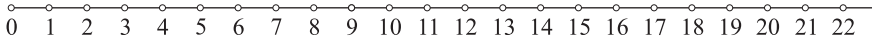
8. *Kopiranje COPY operacija*: $\text{COPY}_n l_{ih}(\gamma, \tau) = l_{ih}(\gamma, \tau) (n_{\gamma,i,h} = f(n))$
9. *Uklanjanje DELETE operacija*: $\text{DELETE } l_{ih}(\gamma, \tau)$ je brojevnja crta $\mathbf{b}/l_{ih}(\gamma, \tau)$
10. *Udvostručenje* $l_{jd}(\gamma, \tau) (j \in \{c, p\})$ je: $l_{jm}(\gamma, \tau)$ takav $l_{jd+1}(\gamma, \tau) = \text{COPY}_n l_{jd}(\gamma, \tau) (n_{\gamma,j,m} = n_{\gamma,j,d}, n_{\tau,j,m} = n_{\tau,j,d+1})$
11. *Polovište točaka* t_α i t_β je točka t_x takva da je $l_{jd}(\gamma, \tau) (n_{\gamma,ji} = f(\alpha) \text{ i } n_{\tau,jd} = f(\beta))$ *udvostručenje* $l_{jh}(\gamma, \tau) (n_{\gamma,hi} = f(\alpha) \text{ i } n_{\tau,hd} = f(x))$.
12. *Okretanje (invertiranje) $\mathcal{I}$ operacija* na skokovima koja zamjenjuje γ i τ .

$$\mathcal{I}l_{ch}(\gamma, \tau) = \mathcal{I}(\overset{\curvearrowright}{\gamma} \overset{\curvearrowright}{\tau}) = l'_{ch}(\overset{\curvearrowleft}{\tau} \overset{\curvearrowleft}{\gamma})(\tau, \gamma)$$

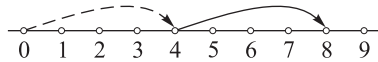
Strojevi za računske operacija u N

Točke na brojevnoj crti imenujemo prirodnim brojevima $f : D = \{(n, 0), n \in \mathbb{N}\} \rightarrow n, (T = N)$, grafičke objekte (kružići na crti) izjednačujemo s njihovim imenima, odnosno: $t_n = n$.

Definirajmo računske operacije na brojevnom polupravcu. $t_0 = 0$ je početna točka polupravca.

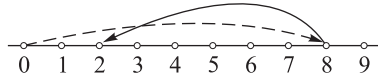


1. Zbrajanje $a + b$ je:



1. START: $l_c(\gamma, \tau), n_{\gamma,c} = 0$ i $n_{\tau,c} = a$ skok od 0 do lijevog pribrojnika;
2. Generiraj skok $l_{p1}(\gamma, \tau) n_{\gamma,p1}=0, n_{\tau,p1}=b, l_p(\gamma, \tau)=\text{COPY} n_{\tau,c} \mathcal{J} l_{p1}(\gamma, \tau)$;
3. $n_{\tau,p}$ je rezultat STOP.

2. Oduzimanje $a - b$ je:



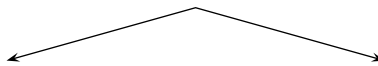
1. START: $l_c(\gamma, \tau), n_{\gamma,c} = 0$ i $n_{\tau,c} = a$ skok od 0 do umanjnika;
2. Generiraj skok $l_{p1}(\gamma, \tau) n_{\gamma,p1}=0, n_{\tau,p1}=b, l_p(\gamma, \tau)=\text{COPY} n_{\tau,c} \mathcal{J} l_{p1}(\gamma, \tau)$;
3. $n_{\tau,p}$ je rezultat STOP.

Ako je $b > a$, rezultat se ne može pokazati na brojevnoj crti.

3. Množenje: $a \times b = c$:

1. START: generiraj na brojevnoj crti $l_{pi}(\gamma, \tau), n_{\gamma,p,i} = 0$ i $n_{\tau,p,i} = 0$, $l_{p1}(\gamma, \tau), n_{\gamma,p,1} = 0$ i $n_{\tau,p,1} = a$;

2.

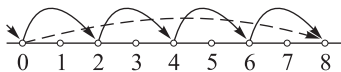


$i > b$

DELETE $l_{pi}(\gamma, \tau), c = n_{\tau,p,b}$
 $l_c(\gamma, \tau), n_{\gamma,c} = 0, n_{\tau,c} = n_{\tau,p,b}$,
 $c = n_{\tau,c,1}$, rezultat, STOP

$i \leq b$

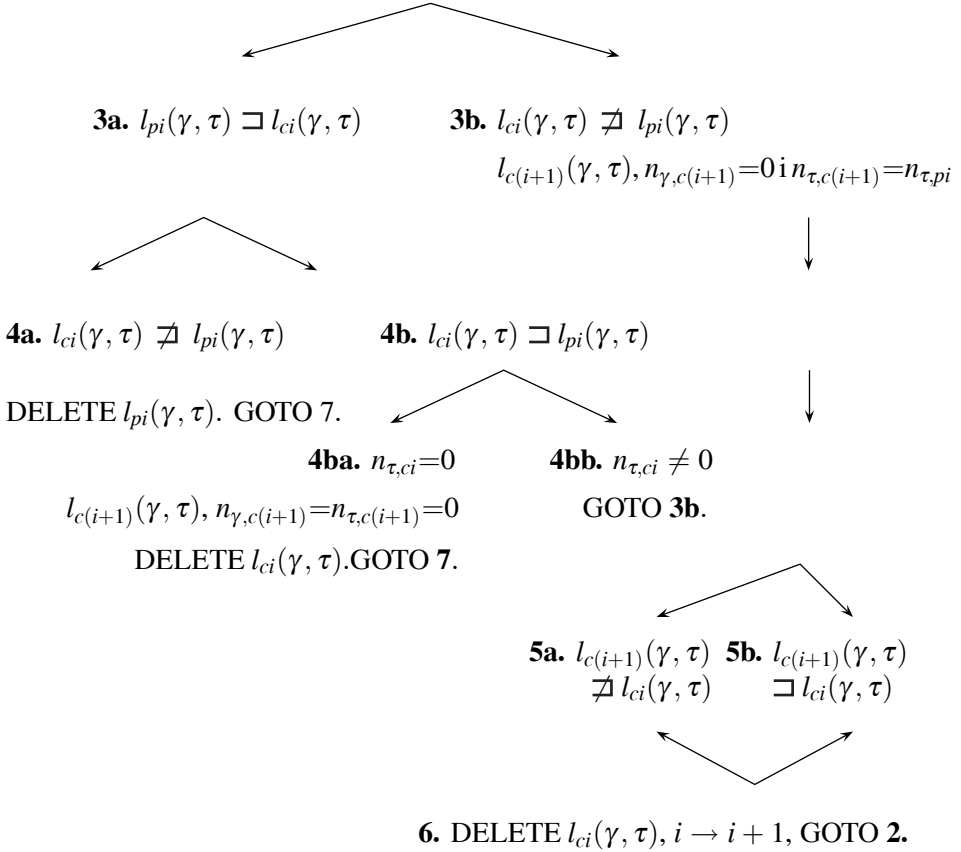
$l_{p,i+1}(\gamma, \tau) = \text{COPY } n_{\tau,p,i} l_{p,i}(\gamma, \tau)$,
 $i \rightarrow i + 1$, GOTO 2.



4. Dijeljenje $a : b$ u $\mathbf{Z}$:

1. START: $l_{ci}(\gamma, \tau), n_{\gamma, ci}=0$ i $n_{\tau, ci}=a$, (ime: $i=0$)

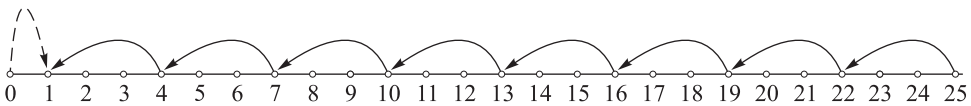
2. $l_{pi'}(\gamma, \tau), n_{\gamma, p, i'}=0$ i $n_{\tau, p, i'}=b$, $l_{pi}(\gamma, \tau)=\text{COPY}_{\tau, c, i} \mathcal{J} l_{pi'}(\gamma, \tau)$, DELETE $l_{pi'}(\gamma, \tau)$



7. Rezultat: količnik je ime posljednjeg crtkanog skoka i i ostatak $n_{\tau, c, i+1}$. STOP.

Provjerimo proceduru.

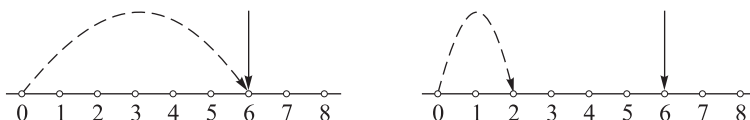
Primjer 1. $25 : 3 = 8$ i ost. 1



Možemo izabrati samo ovaj niz čvorišta u stablu stroja za dijeljenje: 1., 2., 3b., 5a., 6., 2. Ponavlja se 8 puta ista procedura. 9. put skrećemo na 3a. 4a. i dosižemo STOP.

Primjer 2. $6 : 0 =$

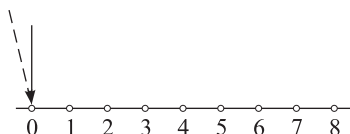
1. Po 1. START (ucrtaj puni skok) i 2. po 2. ucrtaj crtkani skok 3. 3a. odbacujemo, 3b. prihvaćamo; 4. 5b. prihvaćamo 5. 6. prihvaćamo i vraćamo se u 2.



Ciklus se ponavlja i nikad ne dosiže STOP komandu. Stroj ne može stati. Zadatak je nerješiv!

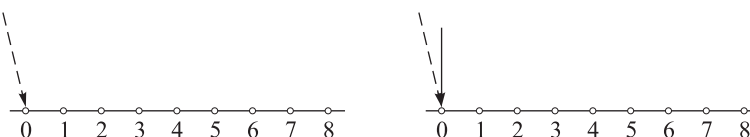
Primjer 3. $0 : 0$

1. START (ucrtaj puni skok) $i = 0$ 2. primijeni 2. ucrtaj crtkani skok 3. 3a. ili 3b. su prihvatljivi; uzmimo 3a.

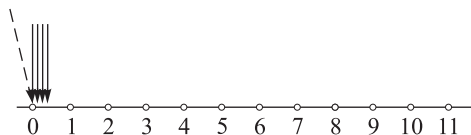


4. Možemo primijeniti samo 4b. 5. Prihvatljivo je samo 4ba. 6. Dosižemo STOP. Ili 31. Prihvatimo. 3b.

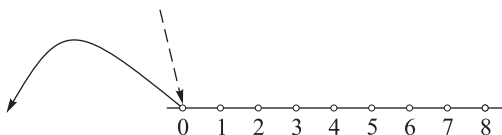
41. Primijeniti možemo samo 5b. 51. Primijenimo 6. 61. Primijenimo 2. 71. Možemo prihvatiti 3a. i doseći STOP komandu ili preko 3b. ili 4bb. Ciklus se može ponavljati po volji mnogo puta.



Izraz je neodređen.

**Primjer 4.** $0 : 6 =$

1. START (ucrtaj puni skok) $i = 0$ 2. primijeni 2. ucrtaj crtkani skok

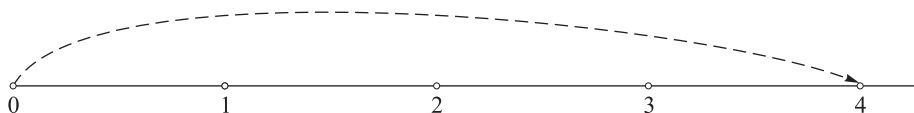


3. Možemo prihvatiti samo 3a. 4. Prihvatljivo je samo 4a. i dosižemo STOP komandu. Rezultat je 0.

Vizualizacija dijeljenja ili racionalni brojevi

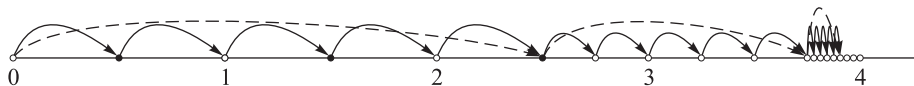
Je li moguće proširenjem brojevnice crte novim točkama u postojećem vizualnom sustavu opravdati formulu ekvivalencije: $a : b = c \iff a = b \cdot c$? Vizualni zadatak je pronaći skok (pridružen c) takve duljine da polazeći od 0 uzastopnim kopiranjem b puta istog skoka dosegamo na brojevnoj crti djeljenu.

Pogledajmo na primjer $4 : 5$.



Treba naći točku između 0 i 1 takvu da skok od 0 do te točke ponovljen 5 puta uzastopno dosegne točku 4. Skok mora biti duljine manje od 1. Je li barem $\frac{1}{2}$?

Ucrtajmo u segment od 0 do 4 polovišta jediničnih segmenata. Dakle, udvostručili smo broj točaka (zelene točke) na promatranom segmentu. Imamo li dovoljan broj točaka za barem 5 skokova? Da. Označimo dosegnutu točku crnom bojom. Ali nismo dosegli 4. Možemo li skok produžiti s još jednim polovljenjem postojećih točaka od crvene točke do točke 4. Ijubičaste točke? Ostvarili smo 5 skokova, ali još nedovoljno za doseći točku 4.



Možemo li nastaviti proces? Odnosno, u ostatku od dosegnute točke pa do djeljenu ucrtajmo nova polovišta (žute točke). Nedovoljno za nove skokove! Trebamo barem 5 točaka! Ponovimo proces s novim polovljenjem. Opet imamo nedovoljan broj točaka za 5 skokova! Ponovimo proces s novim polovljenjem. Ovom podjelom generirali smo dovoljan broj točaka za novih 5 skokova! Uočimo da su poslije posljednjeg dijeljenja ostala na raspolaganju još 3 moguća skoka, kao i kod prvog dijeljenja. Dakle, proces će se ponoviti i nastaviti do unedogled.

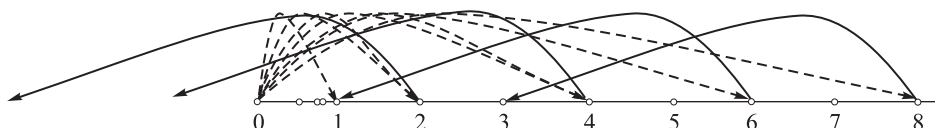


Gornji graf pokazuje duljinu traženog skoka (iscrtani skok). Da smo proces nastavili, bili bi bliže djeljenu, ali bi tada izgubili vizualnu čitljivost. Sivi iscrtani skokovi pokazuju da smo do istog rezultata mogli doći tražeći samo po jedan skok duljine djeljenu! Ispišimo zaključke koji neposredno slijede iz vizualizacije problema: 1. Polovljenjem segmenta od postignute točke do djeljenu je vizualna operacija udvostručenja broja točaka. 2. Skok je moguć ako imamo na raspolaganju

dovoljno novih točaka barem koliki je djelitelj. 3. Kod odlučivanja o skoku postoje samo dvije mogućnosti: ili je skok nemoguć ili je na raspolaganju samo jedan skok.

Na kraju možemo pojednostaviti vizualne akcije na brojevnoj crti. Po 2. slijedi da akcijom u svakom koraku procesa moramo imati dovoljan broj točaka. Svakim polovljenjem udvostručavamo broj točaka od posljednje točke pa do djeljnika.

Pogledajte gornji graf! Ako ga pažljivo slijedimo, skokovi nacrtani punom crtom su međusobno sukladni određeni djeliteljem.



Crvena isprekidana crta označava ostatak u smislu dijeljenja u skupu $\mathbf{N}$. Siva iscrtana crta je podvostručena crveno iscrtane crte. Započnimo s procesom:

1. Crveno iscrtana crta do djeljnika ne preklapa plavi skok, označimo točku 0 crvenom bojom. Ispunjena točka označava da τ krajnja točka pridružena skoku c nije bliža 0 od te točke.

2. Podvostručena iscrtana crta preklapa crveni skok, dakle dodajemo točku ispunjeno crvenom bojom.

3. Podvostručeni ostatak doseže do točke 6. On preklapa zeleni skok. Pridružimo polovište označeno zelenom točkom.

4. Podvostručeni ostatak doseže do točke 2, nedovoljno za preklapanje žutog skoka. Označimo polovište žutim rubom. (Polovište je uvijek između ispunjene točke prema 0 i najbliže neispunjene točke od 0.)

Neispunjena točka označava da τ krajnja točka pridružena skoku c nije dalje od te točke.

5. Podvostručeni ostatak doseže do točke 4, (djeljnik) i tada smo istom stanju kao u 1. Proces se ne zaustavlja. Ne možemo ostvariti crveni iscrtani skok duljine 0.

Traženi skok je veći od $\frac{3}{4}$, ali manji od $\frac{13}{16}$ itd. Sada možemo formalno ispisati vizualnu proceduru. Prvo ćemo ispisati funkciju imenovanja novih točaka dobivenih polovljenjem.

13. $g(n)$ je ime točke koje je slog sastavljen od 0 i 1, dužine $n + 2$ ovako:

a) $g(-1) = ""$ je prazan slog

b) $g(0) = 1$ i $g(1) = 10$

c) $g(n + 1) = g(n)0$ ili $g(n)1$ ($n \geq 1$)

14. Proširimo s novim grafičkim objektima točke kružići:

$$t_{b-} = t_{g(n)} = (\circ)(g(n) = g(n-1)0),$$

$$t_{c-} = t_{g(n)} = (\bullet), (g(n) = g(n-1)1), n \in \mathbb{N}.$$

Polazimo od dijeljenja u skupu $\mathbb{N}$, kojeg smo gornjem tekstu opisali. a je djeljnik, b je djelitelj, q je broj plavih skokova i r je duljina isprekidano crvenog skoka. Pretpostavimo da je $r > 0$.

1. START: $s = 0$ (0-to stanje) $f(q) = g(0) = g(-1)1$ i $f(q+1) = g(1) = g(0)0$, ucertati $t_{c,g(0)}$, $t_{b,g(1)}$ i $l_{c0}(\gamma, \tau)$, $n_{\gamma,c0} = 0$ i $n_{\tau,c0} = r$. Ispišimo niz: $v(1) = r$.

2. Neka je $s = i$. Za $k = \min_x (0 \leq x \leq s) g(s-x) = g(s-x-1)1$ (barem ako je $x = s$), postoji jedinstvena točka $t_{ck} = t_{g(s-k)}$.

Za $h = \min_z (0 \leq z \leq s) g(s-z) = g(s-z-1)1$ (barem ako je $z = s-1$), postoji jedinstvena točka $t_{bh} = t_{g(s-h)}$.

3. Udvostručimo crveni skok $l_{ci}(\gamma, \tau)$, formirajući skok $l_{cd}(\gamma, \tau)$, $v(s+1) = v(s)$, $n_{\tau,cd}$.

4. Generiraj skok $l_p(\gamma, \tau)$, $n_{\gamma,pi} = 0$, $n_{\tau,p} = b$. $l_{pi}(\gamma, \tau) = \text{COPY } n_{\tau,cd} \mathcal{J}l_p(\gamma, \tau)$. Postoje samo dvije mogućnosti.

5a. $l_{pi}(\gamma, \tau) \sqsupset l_{cd}(\gamma, \tau)$: DELETE $l_{pi}(\gamma, \tau)$.

Na polovištu, t_{ck} i t_{bk}

ucrtati $t_{g(s)0}$ odnosno

$g(s+1) = g(s)0$, $s \rightarrow s+1$. GOTO **2**.

5b. $l_{cd}(\gamma, \tau) \sqsupset l_{pi}(\gamma, \tau)$ tada

Na polovištu, t_{ck} i t_{bk}

ucrtati $t_{g(s)1}$ odnosno

$g(s+1) = g(s)1$, $s \rightarrow s+1$.

ucrtaj $l_{c(i+1)}(\gamma, \tau)$: $n_{\gamma,c(i+1)} = 0$

i $n_{\tau,c(i+1)} = n_{\tau,pi}$

$n_{\tau,c(i+1)} \neq 0$

$n_{\tau,c(i+1)} = 0$, GOTO **6**

$\exists w < s$, takav da je $v(w+1) = v(w)$, $n_{\tau,c(i+1)}$

GOTO **2**.

Tada ćemo reći: “Proces se ponavlja do unedogled. Biraj točku $t_{g(s)1}$ kao posljednju i GOTO **6** ili nastavi i GOTO **2**. sljedećim točkama tipa $t_{g(s)1}$ duljina rezultata q bit će bolja u smislu da je $b \cdot q$ bliže a .”

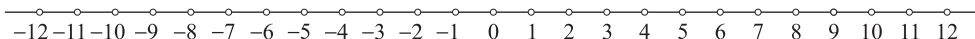
6. Nacrtaj (zeleni) $l_z(\gamma, \tau)$, takav da je $n_{\gamma,z} = 0$ i $n_{\tau,pi+1} = t_{g(s)1}$, STOP.

Ako smo točke na brojevnoj crti imenovali u binarnom sustavu, tada zeleni skok pokazuje racionalan broj napisan decimalno ovako: $q.g'(s+1)$ tako da je $g(s+1) = g(1)g'(s+1)$.

Cijeli brojevi

Polazimo od brojevne crte $\mathbf{N}$. Funkcija I (označavanja) ostaje ista. Neka se izabere *po volji veliki* n već koliko nam treba. Formirajmo novu funkciju imenovanja f ,

$$f(k) = \begin{cases} k - n, & k \geq n \\ -(n - k), & k < n \end{cases}$$



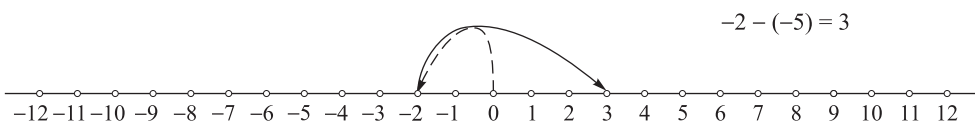
Slika prikazuje predstavu brojevne crte $\mathbf{Z}$ za $n \geq 12$. Oduzimanje: $a - b$

1. START: generiraj na brojevnoj crti $l_c(\gamma, \tau)$, $n_{\gamma,c} = 0$ i $n_{\tau,c} = a$, $l_{p1}(\gamma, \tau)$, $n_{\gamma,p,1} = 0$ i $n_{\tau,p,1} = b$

2. $l_{p2}(\gamma, \tau) = \mathcal{J}l_{p1}(\gamma, \tau)$, DELETE $l_{p1}(\gamma, \tau)$

3. $l_{p3}(\gamma, \tau) = \text{COPY } n_{\tau,c} \ l_{p2}(\gamma, \tau)$, DELETE $l_{p2}(\gamma, \tau)$

4. $n_{\tau,p,2} = c$ rezultat zbroja. STOP



Množenje: $a \times b = c$

1. START: generiraj na brojevnoj crti $l_{pi}(\gamma, \tau)$, $n_{\gamma,p,i} = 0$ i $n_{\tau,p,i} = 0$, $l_{p1}(\gamma, \tau)$, $n_{\gamma,p,1} = 0$ i $n_{\tau,p,1} = a$,

2.

$i > |b|$

$i \leq |b|$

DELETE $l_{pi}(\gamma, \tau)$, $c = n_{\tau,p,|b|}$

$l_{p,i+1}(\gamma, \tau) = \text{COPY } n_{\tau,p,i} l_{p,i}(\gamma, \tau)$,
 $i \rightarrow i + 1$, GOTO 2.

$b < 0$, $l_c(\gamma, \tau)$,

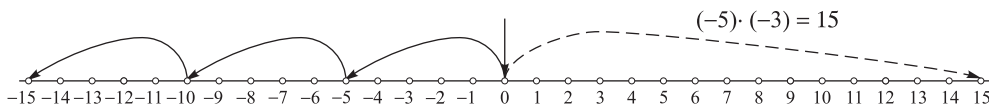
$n_{\gamma,c} = 0$, $n_{\tau,c} = n_{\tau,p,|b|}$

$l_{p1}(\gamma, \tau) = \text{COPY}_0 l_c(\gamma, \tau)$,

DELETE $l_c(\gamma, \tau)$

$c = n_{\tau,c,1}$, rezultat, STOP.

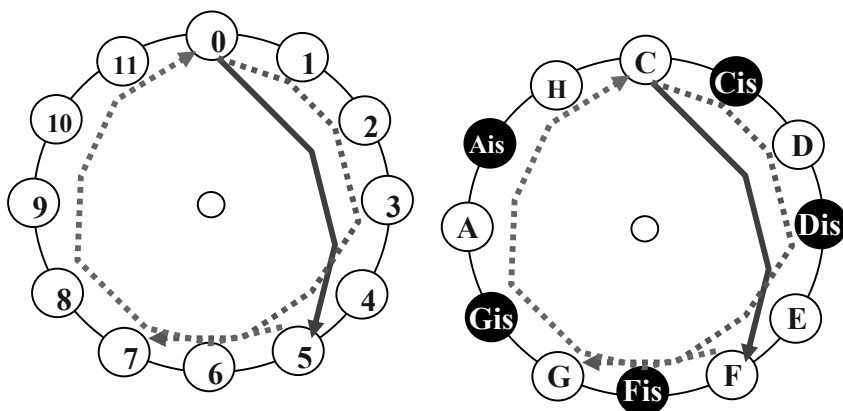
$b \geq 0$, $c = \text{rezultat}$, STOP



Brojeva crta na kružnici

Smjestimo brojnu crtu na topološki različit objekt od pravca i njegovih topološki ekvivalentnih tvorevina. Postavimo je na kružnicu. Aritmetički strojevi ostaju u principu isti, ali učenicima se može pokazati da rezultati postaju neočekivano drukčiji. Na primjer da je zbroj dva broja veći od 0 jednak 0.

($5 + 7 = 0$) kako pokazuje donja slika s 12 izabranih točaka u skupu P .



U krugove s brojevima mogu se ucrtati imena polutonova u kromatskoj C-dur ljestvici. Time se otvara mogućnost povezivanja metodike glazbene kulture i matematike. Na gornjoj slici prikazano je zbrajanje $E + \text{Cis} = F$, na C-dur ljestvici. Donja “bizarna” tablica je tablica množenja nad polutonovima s predznakom snizilicama.

X	C	Des	D	Es	E	F	Ges	G	As	A	Hes	H
C	C	C	C	C	C	C	C	C	C	C	C	C
Des	C	Des	D	Es	E	F	Ges	G	As	A	Hes	H
D	C	D	E	Ges	As	Hes	C	D	E	Ges	As	Hes
Es	C	Es	Ges	A	C	Es	Ges	A	C	Es	Ges	A
E	C	E	As	C	E	As	C	E	As	C	E	As
F	C	F	Hes	Es	As	Des	Ges	H	E	A	D	G
Ges	C	Ges	C	Ges	C	Ges	C	Ges	C	Ges	C	Ges
G	C	G	D	A	E	H	Ges	Des	As	Es	Hes	F
As	C	As	E	C	As	E	C	As	E	C	As	E
A	C	A	Ges	Es	C	A	Ges	Es	C	A	Ges	Es
Hes	C	Hes	As	Ges	E	D	C	Hes	As	Ges	E	D
H	C	H	Hes	A	As	G	Ges	F	E	Es	D	Des

Uzastupno dodavanje polutonovima C-dur ljestvice za 5 ili 7 polutonova formirat će se niz ljestvica s novim i novim predznacima (povisilicama), koje u glazbi poznajemo kao kvartni ili kvintni krug, što nije bizarno, ali je u osnovi izražena aritmetika na kružnici. Glazbeni primjeri mogli bi metodski poslužiti za proširenje pojma brojevnne crte i otvaranje novog pogleda na aritmetiku, ostvarenog pomoću manipuliranja s računalno-grafičkim objektima.

Literatura

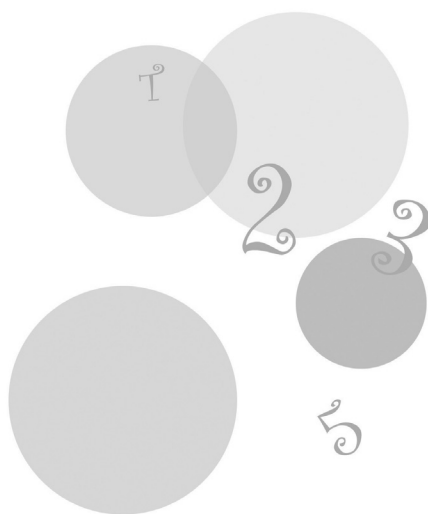
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2. Vještine, znanja i sposobnosti studenata učiteljskih studija



U ovom poglavlju autori raspravljaju o rezultatima istraživanja provedenim među studentima učiteljskih odnosno nastavničkih studija. Istraživanjima se proučavaju razine studentskih postignuća u nekim, za struku, važnim vještinama i znanjima. Osobito važnima držimo rezultate ispitivanja koja su provedena s ciljem boljeg upoznavanja studentskih strategija rješavanja problema te njihovih sposobnosti prostornoga mišljenja.

Analiza reševanja matematičnih problemov z induktivnim sklepanjem pri študentih razrednega pouka

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Povzetek. Induktivno sklepanje je del procesa raziskovanja, kjer posameznik lahko na osnovi opazovanja posameznih primerov sklepa (čeprav ne z absolutno gotovostjo) o resničnosti nekega splošnega principa. Didaktično sklepanje je po drugi strani metoda, ki jo uporabljamo, ko želimo s popolno gotovostjo demonstrirati resničnost principa. Obe metodi sta nepogrešljiva dela matematičnega sklepanja. Ko v prvih letih šolanja razvijamo matematično mišljenje, imamo najpogosteje opraviti s situacijami, kjer morajo otroci sklepati induktivno. Induktivno sklepanje se v šoli uporablja tako kot strategija pri učenju osnovnih matematičnih pojmov kot tudi pri reševanju problemov. V procesu izobraževanja bodočih učiteljev dajemo poudarek tudi na razvijanju sposobnosti za reševanje problemov prek induktivnega sklepanja. Prepričani smo, da lahko le učitelji, ki so kompetentni pri uporabi induktivnega sklepanja, pri pouku matematike organizirajo take učne situacije, ki bodo prispevale k razvoju teh kompetenc tudi pri učencih.

V prispevku so predstavljeni rezultati raziskave, ki smo jo izvedli med študenti razrednega pouka, in s katero smo preverjali njihove kompetence za induktivno sklepanje. Študenti so reševali matematični problem, ki je omogočal, da se je do rešitve in oblikovanja posplošitev prišlo z uporabo induktivnega sklepanja. Njihove rešitve so bile analizirane iz različnih zornih kotov: z vidika razumevanja problemske situacije, z vidika globine reševanja problema ter z vidika strategij, ki so bile pri tem uporabljene. Analizirali smo tudi odnos med globino in strategijo reševanja ter ugotovili, da niso vse strategije enako učinkovite pri iskanju posplošitev problema. Študenti so se pogosteje posluževali strategij, ki niso bile vezane na osnovni kontekst problema, ohranjanje konteksta pri prehajanju v višje, abstraktnejše faze reševanja se je celo izkazalo za manj učinkovito kot zgolj ukvarjanje s števili.

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Szilágyiné Szinger Ibolya

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Összefoglaló. Tanítószakos hallgatókkal végzett munkánk során többször tapasztaltuk, hogy mennyire idegenkednek a geometriától. Előfordul, hogy tanítójelöltek – akik matematikából érettségiztek – alapvető geometriai foglakkal, mint például a téglalap, négyzet, párhuzamosság, merőlegesség, tengelyes szimmetria, nincsenek teljesen tisztában. Van olyan hallgató, aki

- az általános paralelogrammát a téglalapokhoz sorolja,
- az általános paralelogramma átlóit szimmetriatengelyeknek véli,
- nem tudja, hogy a négyzet egyben téglalap is,
- a négyzet szomszédos oldalait párhuzamosnak tartja (valószínűleg a “szomszédos” szót nem értelmezi ebben az esetben megfelelően) stb.

Ezeknek a problémáknak a gyökerei feltehetőleg az alsó tagozatos tanulmányaikra nyúlnak vissza. Az előzőekben említett problémák adták az indíttatást arra, hogy ezeket a foglakat vizsgáljuk negyedik osztályos fejlesztő tanítási kísérlet keretében. Az oktatási kísérlet célja a négyzet, téglalap, párhuzamosság, merőlegesség és szimmetria foglak fejlesztése a van Hiele-modell szerinti geometriaoktatás megvalósításával, valamint a van Hiele-féle fejlődési szintek realitásának igazolása a magyar alsó tagozatos geometria oktatásában.

A kísérletet 2006 május-júniusában a bajai Eötvös József Főiskola Gyakorló Általános Iskolájának 4.c osztályában végeztük, majd ezt 2008 május-júniusában megismételtük az iskola 4.b osztályában.

A fejlesztő tanítási kísérletet mindkét alkalommal egy felmérő feladatlappal zártuk. A feladatlapot 2006-ban és 2008-ban a kísérleti osztályokon kívül a párhuzamos osztályok tanulói is kitöltötték.

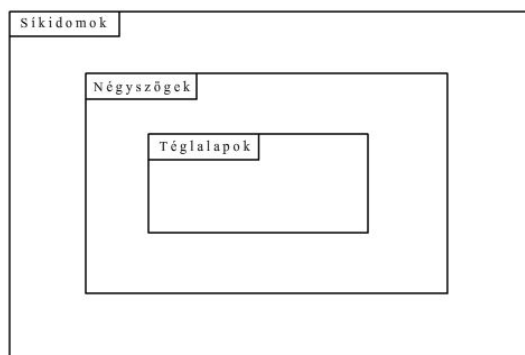
A 2008-as utóteszt feladatait 2010-ben tanító szakos hallgatókkal is megoldattuk. A 38 nappali tagozatos hallgató közül 24 negyedéves és 14 másodéves. Ismertetjük a tanítójelöltek eredményeit, melyeket összehasonlítottunk a negyedik osztályos gyerekek (56 fő) 2008-as eredményeivel.

Kulcsszavak: matematikadidaktika, geometriai foglak, van Hiele szintek

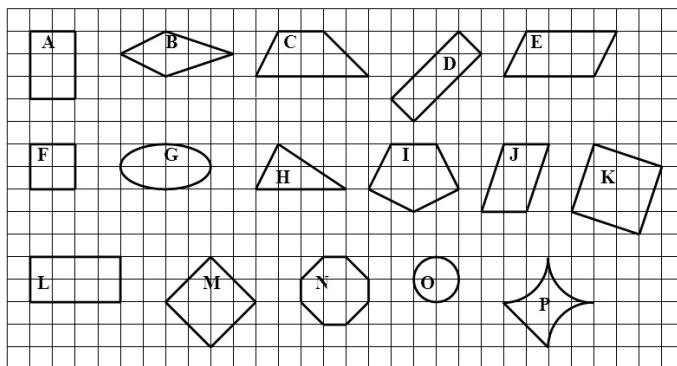
A négyzetre és téglalapra vonatkozó feladatok

Az utóteszt – négyszög, téglalap, négyzet fogalmak azonosításával kapcsolatos – 1. feladatában 16 síkidom közül kellett kiválasztani

- a négyszögeket;
- a téglalapokat;
- a négyzeteket;
- és elhelyezni a síkidomok betűjelét az ábrán a megfelelő helyre.

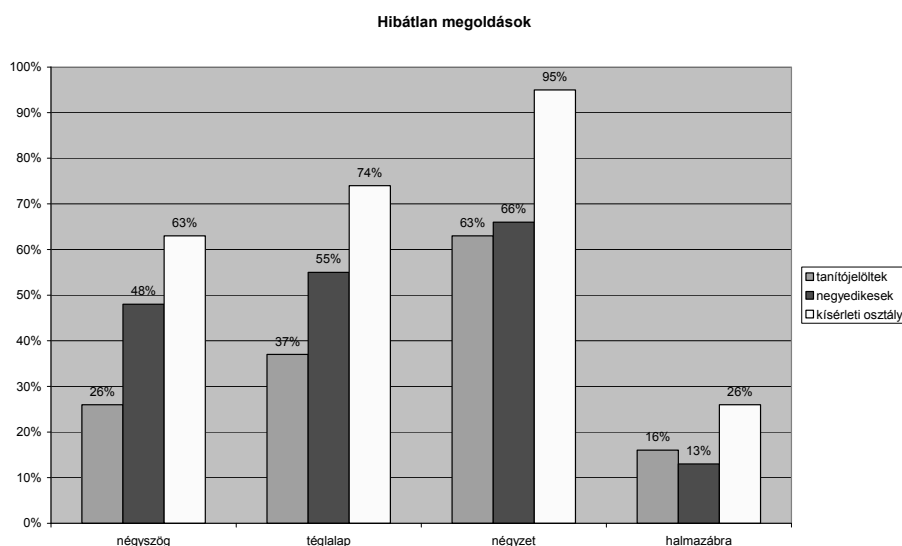


Ezt a halmazt kellett rendezniük:



A feladat megoldásának eredményességét a következő ábra szemlélteti:

A négyszögek felismerésénél a hallgatók 68%-a P jelű síkidomot helytelenül a négyszögekhez sorolta. A gyerekek 29%-a követte el ezt a hibát.



A negyedik osztályos gyerekek döntő többségének geometriai gondolkodása a *van Hiele*-féle 2. szintet éri el. Ezen a szinten még nem veszik észre az alakzatok közötti kapcsolatokat. Megállapítják ugyan az alakzatok közös tulajdonságait, például, hogy a négyzet és a téglalap is 4 csúccsal, 4 oldallal rendelkezik, szemközti oldalai párhuzamosok, szomszédos oldalai merőlegesek, minden szögük derékszög, de ebből nem következik számukra az, hogy a négyzet téglalap is egyben. A tulajdonságok megállapítása csak az alakzatok megkülönböztetéséhez szükségesek. A negyedik osztályos gyerekek esetében ezért megfelelő megoldásnak fogadtuk el azt is, ha nem sorolták a négyzeteket a téglalapokhoz, de más hibájuk nem volt. A tanító szakos hallgatóktól azonban elvártuk a négyzetek téglalapokhoz sorolását. A negyedikesek 41%-a (a kísérleti osztályban 21%-a), míg a tanítójelöltek 37%-a – tipikus hibaként – az általános paralelogrammákat a téglalapokhoz sorolta.

A négyzetek válogatásában a gyerekek 29%-ánál, valamint a hallgatók 29%-ánál egyaránt a hiba forrását az jelentette, hogy a csúcsára állított négyzet(ek)ben nem ismerték fel a négyzetet.

A halmazábra kitöltése volt a legeredménytelenebb feladatrész. Gyakori hibának számított, hogy egy betűjelzést több helyre is írtak (a gyerekek 21%-a, a hallgatók 34%-a). A többszörös osztályba sorolás a gyerekek számára nehéz, túl absztrakt feladat, amely túlmutat a *van Hiele*-féle 2. szinten, ezért várakozásunknak megfelelő a gyenge eredményük, a hallgatók esetében azonban nem.

A felmérő feladatlapnak a négyzet és a téglalap tulajdonságaira vonatkozó feladatában a megadott állítások közül azokat kellett aláhúzni, amelyek igazak

a) a négyzetre:

Szemközti oldalai párhuzamosak.

Szemközti oldalai merőlegesek.

Szomszédos oldalai párhuzamosak.

Szomszédos oldalai merőlegesek.

*Mindegyik szöge derékszög.
 Nem mindegyik szöge derékszög.
 Pontosan két tükörtengelye van.
 4 tükörtengelye van.
 8 tükörtengelye van.
 Minden oldala egyenlő hosszúságú.
 Szemközti oldalai egyenlő hosszúak.*

b) a téglalapra:

*Szemközti oldalai párhuzamosak.
 Szemközti oldalai merőlegesek.
 Szomszédos oldalai párhuzamosak.
 Szomszédos oldalai merőlegesek.
 Mindegyik szöge derékszög.
 Nem mindegyik szöge derékszög.
 4 tükörtengelye van.
 Van 2 tükörtengelye.
 Az átlók tükörtengelyek.
 Minden oldala egyenlő hosszúságú.
 Szemközti oldalai egyenlő hosszúak.*

A feladat megoldásának értékelésénél csak a hibátlan teljesítményeket emeljük ki. A négyzet tulajdonságaira vonatkozó valamennyi igaz állítást helyesen állapította meg a negyedikes tanulók 61%-a, míg a tanítójelöltek 45%-a. A téglalap esetén ezek az értékek a következőképpen alakultak: a gyerekeknél 57%, a hallgatóknál 47%. Negyedik osztályban a hibák forrását egyrészt a szemközti, illetve szomszédos szavak — még mindig — nem megfelelő értelmezésében kell keresnünk, másrészt abban, hogy a párhuzamosság és a merőlegesség fogalma még nem elég stabil. A tanítójelölteknel a legtöbb hiba a négyzet és a téglalap tükörtengelyei számának meghatározásában volt (21%, illetve 42%).

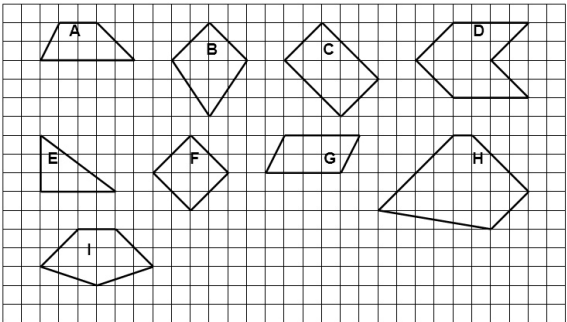
A párhuzamosságra és merőlegességre vonatkozó feladatok

A felmérő feladatlap 2. és 5. feladata kapcsolatos a párhuzamosság, valamint a merőlegesség fogalomalakulásával. Az 5. feladat a párhuzamosságot, illetve a merőlegességet a négyzet és a téglalap tulajdonságainak vizsgálata révén érinti, ezzel az előbbieken foglalkoztunk.

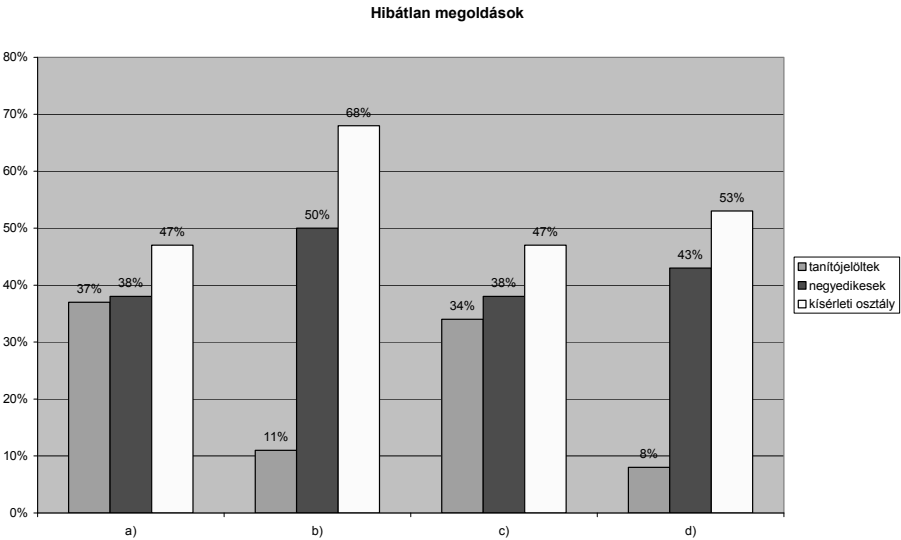
A sokszögek párhuzamos és merőleges oldalpárjainak felismerésével kapcsolatos 2. feladatban 9 alakzatot kellett vizsgálniuk, továbbá

- a) kiszínezni azonos színnel az egymással párhuzamos oldalakat;
- b) kiszínezni pirossal a derékszögeket;
- c) felsorolni a betűjelét azoknak a síkidomoknak, amelyeknek vannak párhuzamos oldalai;
- d) felsorolni a betűjelét azoknak a síkidomoknak, amelyeknek vannak merőleges oldalai.

A vizsgálat tárgyai a következő sokszögek voltak:



A feladat megoldásának eredményességét a következő ábra szemlélteti:



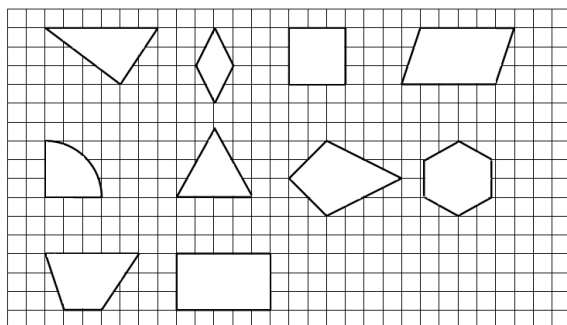
A jellemző hibák előfordulási aránya:

	tanító- jelöltek	negye- dikese	kísérleti osztály
A B jelű deltoid szemközti oldalait párhuzamosnak találta.	8%	14%	26%
A D jelű hatszögnél a ferde helyzetű egyenesek párhuzamosságát nem jelölte.	13%	34%	26%
A H jelű ötszög párhuzamos egyenespárját nem vette észre.	34%	34%	21%
A B jelű deltoidnál nem csak egy derékszöget talált.	8%	14%	11%
A D jeű hatszögnél nem jelölte a derékszöget.	82%	23%	11%
A H jelű ötszögnél nem ismerte fel a derékszöget.	45%	34%	16%

Tengelyes szimmetriával kapcsolatos feladat

A felmérő feladatlap 3. feladatában a gyerekeknek, illetőleg a hallgatóknak azt kellett eldönteniük, hogy a megadott síkidomok közül melyek tükrösek és egyidejűleg pirossal jelölniük a tükrötengelyek helyét.

A következő síkidomokat vizsgálták:



A 3. feladat megoldásának értékelése:

	tanító-jelöltek	negyedikesek	kísérleti osztály
Hibátlan megoldást adott.	5%	20%	47%
1 síkidomnál hibázott.	13%	27%	32%
A négyzetnél 2 vagy annál kevesebb szimmetriatengelyt rajzolt.	37%	18%	0%
Az általános rombusznál csak 1 szimmetriatengelyt vagy egyet sem húzott meg.	13%	7%	0%
Az általános paralelogrammánál rajzolt tükrötengelyt.	34%	7%	0%
Az egyenő oldal háromszögnél 1 szimmetriatengelyt vagy egyet sem jelölt	63%	73%	37%
A negyed körlapnál nem rajzolt tükrötengelyt.	18%	30%	11%
A szabályos hatszögnél nem találta meg az összes tükrötengelyt.	90%	55%	37%
A téglalapnál az átlókat is szimmetriatengelynek tartotta.	13%	4%	0%

A kísérleti csoportban a négyzetnél, az általános rombusznál, a paralelogrammánál és a téglalapnál mindenki hibátlanul dolgozott. A tanító szakos hallgatók követték el a legtöbb hibát.

Szomorúan kell megállapítanunk, hogy a tanítójelöltek szinte minden feladatban rosszabb teljesítményt nyújtottak, mint a negyedik osztályos tanulók. Megállapításaink csak a vizsgált – nem reprezentatív – mintákra vonatkoznak, ezért statisztikai próbákat nem végeztünk. A feladatlapon szereplő feladatok a negyedik osztályos tantervi követelményeknek felelnek meg. A tanítóknak ezt a tananyagot el kell sajátíttatniuk a gyerekekkel. Ám ha ők maguk sem sajátították el megfelelően, akkor hogyan tanítják ezt a későbbiekben? Vajon a geometriai gondolkodásuk melyik *van Hiele*-féle szintet éri el?

Úgy gondoljuk, hogy a középiskola elvégzésével reális elvárás a diákoktól a negyedik – *a teljes logikai felépítésre való törekvés, a formális dedukció* – szintjének elérése. Ezen a szinten a tanuló képes

- egyszerű tételeket megfogalmazni,
- az axiómák, definíciók, tételek lényegét és jelentőségét felfogni,
- a bizonyítások gondolatmenetét, a szükséges és elégséges feltételek szerepét megérteni.

Már a harmadik – *a lokális logikai rendezés, az informális dedukció* – szinten a többszörös osztályba sorolás nem okoz problémát. A különböző négyszögek fogalom szerinti osztályozását megértik a tanulók. Összefüggéseket látnak egy adott alakzat tulajdonságai között, illetve különböző alakzatok között. Ezen a szinten a négyzet már téglalap, sőt paralelogramma.

A tanítójelöltek 16%-a végezte el hibátlanul a többszörös osztályba sorolást.

Úgy tűnik, hogy a tanítójelöltek geometria oktatásában vissza kell mennünk sok esetben a 2. szintre. Még az eddigieknél is több konkrét tevékenység – mint a mérés, hajtogatás, nyírás, modellezés, tükröhasználat stb. – alkalmazásával kell a geometriai fogalmaikat a megfelelő szintre fejleszteni.

Irodalom

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Az elsőéves tanító szakos hallgatók matematika ismeretei

Petz Tiborné (Toth, Sz.)

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Összefoglaló.

A magyarországi 2010. májusi matematika érettségi elszomorító eredménnyel zárult (2,91 lett az országos átlag középszinten). Ez az eredmény azt mutatja, hogy a matematika okozza továbbra is a legnagyobb nehézséget az érettségiző diákok számára. Ebből kiindulva, a tanító szakra jelentkezett, első éves hallgatók matematikai ismereteit kívánom felmérni egy általam összeállított teszt segítségével. A teszt a gyakorlatorientáltság jegyében az élethez kötődő, valamint matematikai fogalmakkal megadott, standard példákból áll. A kérdések, amikre választ keresek: mekkora százalékban oldják meg a feladatokat a tanítók. Hiszen a hallgatók matematikai hozzáértése nagyon fontos a következő generációk számára. Továbbá vizsgálom, hogy mekkora a különbség a hagyományos matematika feladatot és a szövegesen megfogalmazott, más tudományterületet is érintő feladatot megoldók számában, mert a mai felgyorsult világban a matematikai életszerűség különösen fontos szerepet játszik a munkaerő piacon. Elemzem azt is, hogy vajon az érettségi jegyét befolyásolja-e, hogy a hallgató gimnáziumból, vagy szakközépiskolából érkezett. Hiszen az érettségi eredmények azt mutatják, hogy a szakközépiskolások zömében csak azokat a feladatokat tudják megoldani, amelyekhez az általános iskolában szerzett ismeretek elegendőek.

„A tanítás célja nem az adatok, hanem az értékek átadása.” (W. R. Inge)
Ezzel az idézet lehetne a dolgozatom mottója. Hiszen a magyarországi oktatás még nem ment át teljesen azon az átalakuláson, amelyet az új érettségi rendszer bevezetése elvárna. A nagyrészt frontális osztálymunka, a kevés megnyilvánulási lehetőség, a száraz tananyag elveszi a kedvüket a diákoknak a matematikától. Amíg alsó tagozaton a három legkedveltebb tantárgy között szerepel a matematika, addig az idő előrehaladtával, egyre lejjebb csúszik a ranglétrán. A mechanikusan elsajátított tudás nemcsak a kedvét veszi el a diákoknak a matematikától, hanem annak a veszélyét is hordozza, hogy olyat magolnak be, amit nem értenek.

A magyarországi 2010. májusi matematika érettségi elszomorító eredménnyel zárult (2,91 lett az országos átlag középszinten). Ez az eredmény azt mutatja, hogy

a matematika okozza továbbra is a legnagyobb nehézséget az érettségiző diákok számára. Még elszomorítóbb következménye az érettségi eredményeknek az, hogy a szakközépiskolások zömében csak azokat a feladatokat tudják megoldani, amelyekhez az általános iskolában szerzett ismeretek elegendőek.

Középszinten	Százalék átlag:	45,97
	Osztályzat átlag:	2,91
Középszinten:	Vizsgaszám:	87422



Figure 1. Országos eredmények 2010 májusában

A győri eredmények valamivel jobbakként lettek. Ez talán annak köszönhető, hogy Nyugat-Magyarország vezető régiói között tartják számon a győri régiót, és iskolák, főleg középiskolák tekintetében Győr szerencsés helyzetben van. Több olyan gimnáziuma is van, amely országos szinten az élmezőnyhöz tartozik, és szép számmal kerülnek ki belőlük matematika versenyekre, illetve jó nevű egyetemekre a diákok.

Középszinten	Százalék átlag:	54,64
	Osztályzat átlag:	3,3
Középszinten:	Vizsgaszám:	2371



Figure 2. Győri eredmények 2010 májusában

Az érettségi eredményekből kiindulva, a tanító szakra jelentkezett, első éves hallgatók matematikai ismereteit mértem fel egy általam összeállított teszt segítségével. A teszt a gyakorlatorientáltság jegyében az élethez kötődő, valamint matematikai fogalmakkal megadott, standard példákból állt. A kérdések, amikre választ kerestem: mekkora százalékban oldják meg a feladatokat a tanítók. Hiszen a hallgatók matematikai hozzáértése nagyon fontos a következő generációk számára. Továbbá megnéztem, hogy mekkora a különbség a hagyományos matematika feladatot és a szövegesen megfogalmazott, más tudományterületet is érintő feladatot megoldók számában, mert a mai felgyorsult világban a matematikai életszerűség különösen fontos szerepet játszik a munkaerő piacon. A tesztet 58 hallgató írta meg. Az érettségi jegyeik átlaga 2,89, amely közel azonos az országos átlaggal. A felmérésből számolt átlag 2,62, amely alulmúlja az érettségi átlagot.

A felmérés feladatai:

1. Melyik az a szám, amelyből kivonva a 37%-át, 2586-at kapunk?
2. Ábrázolja és jellemezze (értékkészlet, zérushely, monotonitás, szélsőérték) a következő függvényt a valós számok halmazán!

$$f(x) = (x + 2)^2 - 1$$

3. Egy háromszög három oldala 3 cm, 4 cm, és 6 cm. Mekkora a hozzá hasonló háromszög oldalai, ha a legrövidebb oldala 10,5 cm?
4. Egy 24 cm sugarú körben lévő 30 cm hosszú húrhoz mekkora középponti szög tartozik?
5. Egy bárány egy 48 m hosszú drótkerítéssel körülvett téglalap alakú telken legel. A telek egyik oldala közvetlenül a ház falához csatlakozik. Hogyan válasszuk meg a téglalap oldalainak méretét, ha szeretnénk, hogy a bárány a lehető legnagyobb területen legelhessen?
6. Egy szemüvegeket forgalmazó cég reklámja szerint a vásárló annyi százalékkal csökkentheti a megvásárolt szemüvegkeret árát, ahány éves.
 - a.) Hány százalékkal csökkentheti a kész szemüvege árát egy 17 éves vásárló, ha a kiválasztott lencse 60 000 Ft-ba, a keret pedig 25 000 Ft-ba kerül?
 - b.) Hány évesnek kell lennie annak a vásárlónak, aki a teljes szemüveg árát 17%-kal szeretné csökkenteni egy ugyanolyan szemüveg vásárlása esetén, mint az előző?
 - c.) Milyen értékű keret vásárlása esetén érhetné el a 17 éves vásárló, hogy szemüvegének ára a teljes ár 10%-ával csökkenjen, ha 60 000 Ft-os lencsére van szüksége?
7. Egy repülőgép repülési magassága 8000 méter, repülési sebessége 700 km/h. Mennyi ideig tartózkodik egy szemlélő számára a látóhatár felett, ha északról érkezik, és délre tart? (A Földet tekintsük 6378 km sugarú gömbnek.)
8. Zsuzsi gyalogtúrát tervez barátaival. Turistatérképen lemérve a tervezett útvonal hossza 57 cm. Mennyi idő alatt teszi meg a társaság a kiválasztott túraútvonalat, ha óránként átlagosan 3,8 km-t haladnak, továbbá Zsuzsi térképe 1:40 000 méretarányú?

9. Energiaszükséglet

E feladat lényege, hogy olyan ételeket kell kiválasztani, amelyek Zedország egy lakójának energiaszükségletéhez igazodnak. A következő táblázat azt mutatja, hogy különböző emberek esetében mennyi az ajánlott energiaszükséglet kilojoule-ban kifejezve (kJ).

AZ AJÁNLOTT NAPI ENERGIASZÜKSÉGLET FELNÖTTEK SZÁMÁRA			
Életkor (években)	Aktivitási szint	FÉRFIAK	NŐK
		Szükséges energiamennyiség (kJ)	Szükséges energiamennyiség (kJ)
18-től 29-ig	Könnyű	10660	8360
	Mérsékelt	11080	8780
	Nehéz	14420	9820
30-tól 59-ig	Könnyű	10450	8570
	Mérsékelt	12120	8990
	Nehéz	14210	9790
60 és afölött	Könnyű	8780	7500
	Mérsékelt	10240	7940
	Nehéz	11910	8780

EGYES FOGLALKOZÁSOKNAK MEGFELELŐ AKTIVITÁSI SZINT		
Könnyű:	Mérsékelt:	Nehéz:
Bolti eladó	Tanár	Építőmunkás
Irodai dolgozó	Üzletkötő	Fizikai munkás
Háztartásbeli	Ápoló	Sportoló

a. feladat

Molnár Károly 45 éves tanár. Mennyi az ajánlott napi energiaszükséglete kJ-ban?

b. feladat

Szalay Edit 19 éves magasugró. Egy este néhány barátja étterembe hívja Editet vacsorázni. Íme az étlap:

ETLAP		1 adag energiatartalma Edit becslése szerint (kJ)
Levesek:	Paradicsomleves	355
	GombakréMLEves	585
Főételek:	Mexikói csirke	960
	Karibi gyömbéres csirke	795
	Zsályás sertéspörkölt	920
Saláták:	Burgonyasaláta	750
	Vegyes zöldségsaláta	335
	Kuszkusz saláta	480
Desszertek:	Almás-málnás lepény	1380
	Túrótorta	1005
	Répatorta	565
Tejes turmixok:	Csokoládé	1590
	Vanília	1470

Az étteremben egységáras menü is kapható.

Menü	
ára 50 zed	
Paradicsomleves	
Karibi gyömbéres csirke	
Répatorta	

Edit minden nap feljegyzi, hogy mit evett. Az aznap vacsora előtt elfogyasztott ételek 7520 kJ energiabevitelnek felelnek meg. Edit azt szeretné elérni, hogy az összes energiabevitel 500 kJ-nál többel **ne haladja meg vagy ne maradjon alatta a számára ajánlott napi mennyiségnek**. Döntsd el, hogy vajon az egységáras menü elfogyasztásával Edit a napi ajánlott energiaszükségletéhez képest a ± 500 kJ-os határon belül marad-e!

A mérés kiértékelése:

Párba állítottam a standard feladatokat a gyakorlatorientált feladatokkal. A kiértékelés ez alapján a következő:

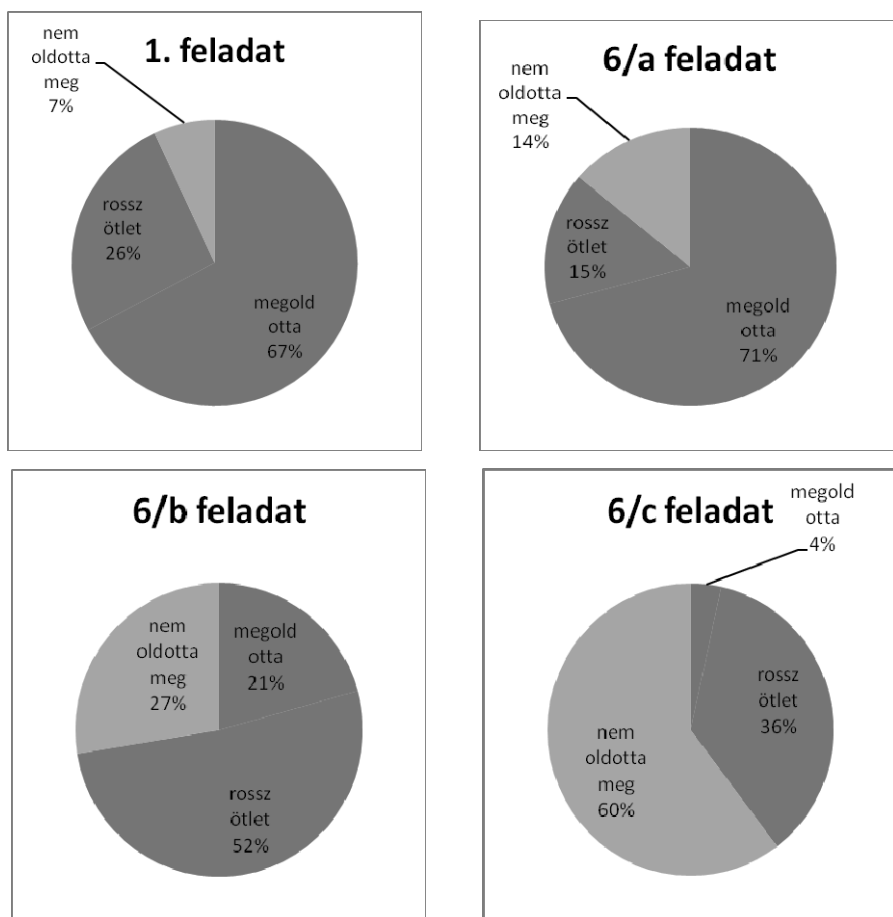


Figure 3. Az 1. és a 6. feladatok eredményessége

A százalékszámításos feladatok ahogy egyre nehezednek, annál kevésbé sikeresek a hallgatónál. Gyakran találkozunk olyan akciókkal a szemesz szaküzletekben, amely a 6. feladatban szerepel. Jó példa arra, hogy ki tudjuk-e számolni, hogy mennyi kedvezményt is kapunk valójában.

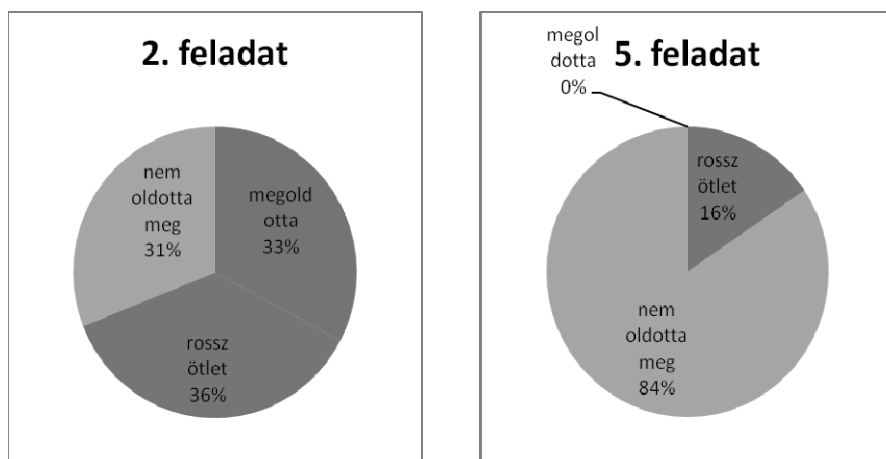


Figure 4. A 2. és a 5. feladat eredményessége

A függvények szélsőérték helyének és értékének meghatározása általában menni szokott a diákoknak, de amint egy olyan feladattal kerülnek szembe, ahol értelmezni kell a feladatot, ismét bajban vannak. Sokuknak mindegy volt, hogy adott kerület mellett mekkora az oldalak hossza, hogy maximális területet kapjunk.

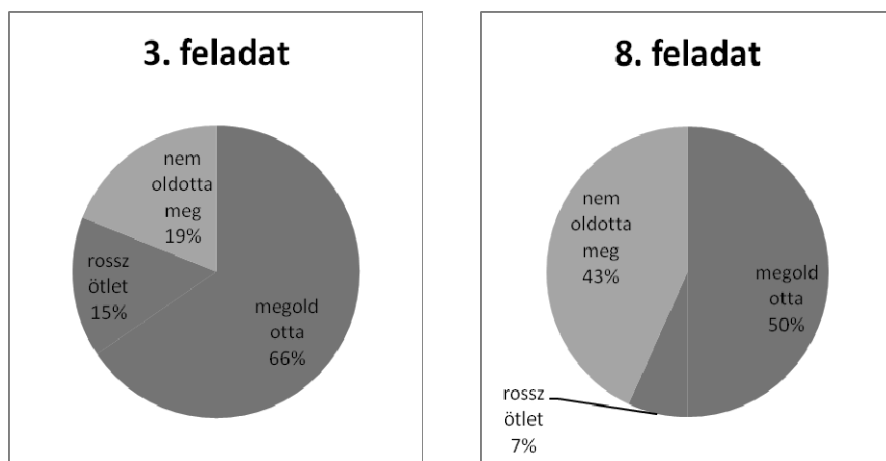


Figure 5. A 3. és a 8. feladat eredményessége

A hasonlósággal kapcsolatos könnyebb feladatok nem szoktak problémát okozni, ezt mutatja a 3. feladat megoldási százaléka is. De azon csodálkoztam, hogy a térképes feladat esetén romlott a megoldási arány, pedig már egész kisiskolás korban, környezetismeretből tanulnak a térképről olvasni.

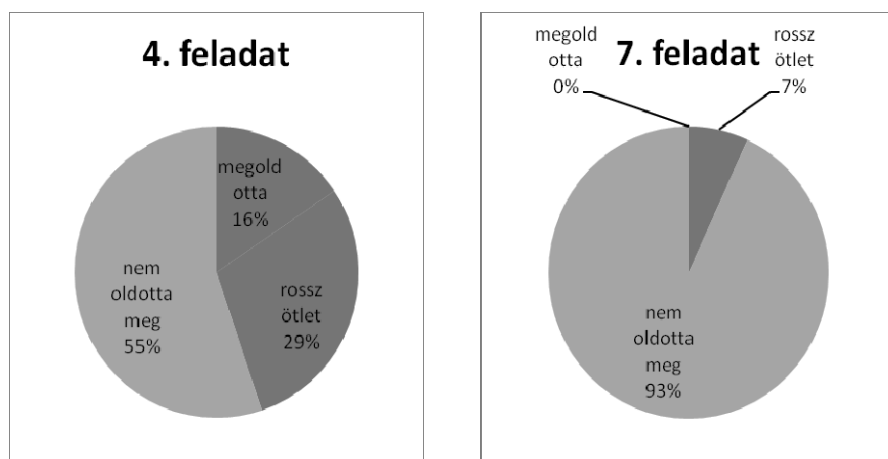


Figure 6. A 4. és a 7. feladat eredményessége

A trigonometria nem tartozik a kedvenc témakörök közé a középiskolában. A standard feladattal még valamennyire foglalkoztak a hallgatók, de a szövegesen megadottat már lerajzolni sem tudták.

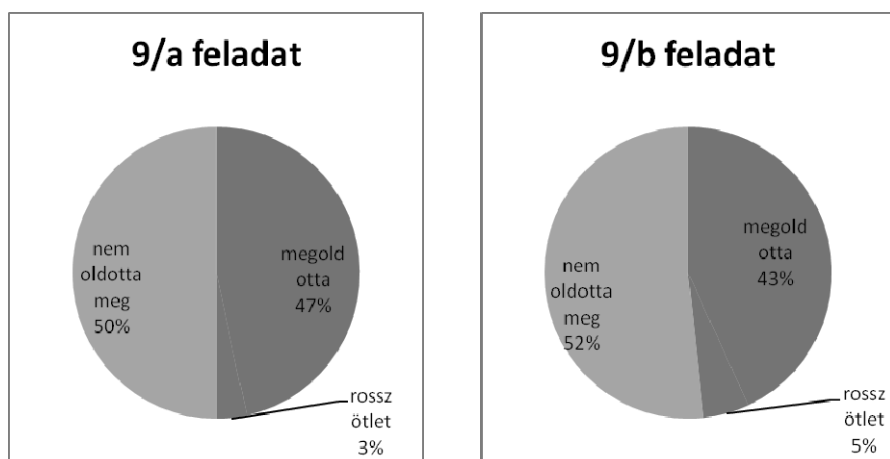


Figure 7. A 9. feladat eredményessége

Ez a feladat a PISA honlapján található, és nem is a matematikai feladatok között, hanem a problémamegoldásnál. Nem kell hozzá sok matematikai ismeret, inkább csak a szöveget kell értelmezni. Meglehető eredmények születtek.

A mérés eredményeit értékelve elmondható, hogy a főiskolákra kerülő diákoknak nincsenek biztos ismereteik matematikából. A szövegesen megadott példák pedig még nagyobb problémákat okoznak nekik. Még az olyan feladat is ahol csak a táblázatból kell adatot kikeresni, és semmi matematikai ismeret nem kell. A hallgatókkal való későbbi beszélgetéskor kiderült, hogy a 9. feladatot sokan azért

hagyták ki, mert túl hosszú volt a szöveg, és még táblázat is volt benne, ezért úgy gondolták, hogy biztosan nehéz. Ez pedig felveti azt a problémát, hogy a matematikai gondok többsége vajon nem abból adódik-e, hogy a tanulók – nem csak az én hallgatóim – nem értik a feladatok szövegét? Hiszen az ilyen típusú feladatoknál a tanulónak kell a szövegből szerzett információk alapján a modellalkotást elvégeznie. Ehhez pedig elengedhetetlen az, hogy megértse amit olvas.

Mi lehet a gond?

Az alsó tagozatosok még jól szerepelnek a diákok a szövegértési feladatokban, de később a PISA esetében épp ebben teljesítettünk a legrosszabbul. Negyedik osztály után már nem fejlesztünk bizonyos képességeket – így az olvasásét sem. Ezzel szemben a hangsúly a speciálisabb, szaktárgyi tudásra helyeződik át. Azaz, hogy az olvasás fejlesztését abbahagyják, a lakosság körében tömeges lesz a funkcionális analfabetizmus.

Másik fontos kérdés, hogy mit, és hogyan tanítsunk a matematika órákon?

Nem jó kizárólag praktikus ismereteket tanítani komolyabb lexikális tudás nélkül. Ha csak a definíció fogalma van meg a tanulóknak, a fogalom képzete nincs, és nem tudják példákhoz kötni, akkor csak bemagolt, jelentés nélküli ismeret lesz. Ezzel szemben, ha törekszünk életből vett példákat bevinni az óránkra, segíthetjük a diákokat a megértésben. Élményt jelenthet számukra a valós problémák vizsgálata, s így ismerkedve a matematikai összefüggésekkel. Könnyebb a hosszútávú memóriából feleleveníteni a definíciókat, ha adott szituációhoz kapcsolódnak. Nálunk az iskolai munka sokszor nem alkalmazkodik a gyerekek fejlődési sajátosságaihoz, előzetes tudásához. A tanulók gyakran kényszerülnek olyasminek a megtanulására, amit nem értenek meg. Ezeket a problémákat ha sikerül legyőzni, akkor talán elérhetjük, hogy a diákok ismét pozitívan viszonyuljanak a matematikához.

Irodalom

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Kompetentnost studenata učiteljskih studija za izvođenje dodatne nastave iz matematike

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Sažetak. Učitelji razredne nastave slobodni su odabrati koje će izvannastavne aktivnosti voditi. Svoj interes usmjerili smo na odabir dodatne nastave matematike kao izvannastavne aktivnosti, a u svojim istraživanjima smo se usmjerili na studente završnih godina učiteljskoga studija kao buduće nositelje dodatne nastave matematike u nižim razredima osnovnih škola.

Ispitali smo sposobnost studenata završnih godina učiteljskog studija za pripremu učenika osnovnih škola za natjecanja iz matematike testiranjem njihove uspješnosti u rješavanju zadataka s regionalnih natjecanja učenika četvrtih razreda osnovnih škola te ju usporedili s uspješnosti učenika u rješavanju istih.

Istraživanje, koje smo proveli na četiri hrvatska sveučilišta (Osijek, Zadar, Split, Zagreb) sa studentima završnih godina integriranog pred-diplomskog i diplomskog sveučilišnog učiteljskog studija, pokazalo je da su učenici uspješniji u rješavanju danih matematičkih problema, ali i da na uspješnost studenata utječe broj odslušanih matematičkih kolegija na studiju.

Ključne riječi: studenti učiteljskih studija, natjecanja iz matematike, dodatna nastava iz matematike, matematički kolegiji na učiteljskom studiju

Uvod

Svaka odgojno-obrazovna ustanova bi trebala organizirati odgovarajuće izvannastavne aktivnosti s ciljem ispunjenja potreba i interesa svojih učenika. U Republici Hrvatskoj je prema Zakonu o odgoju i obrazovanju u osnovnoj i srednjoj školi propisano da se one predvide školskim kurikulumom i godišnjim planom i programom rada škole. Voditelji izvannastavnih aktivnosti učenika, koji pohađaju prva četiri razreda osnovne škole, su učitelji razredne nastave koji sami biraju područje kojim će se dodatno baviti sa svojim učenicima. Prirodno se nameće pitanje jesu li oni dovoljno kompetentni voditi samoinicijativno izabranu izvannastavnu aktivnost. U svojim istraživanjima smo se usmjerili na studente završnih godina

učiteljskoga studija kao potencijalne buduće voditelje dodatne nastave iz matematike kao izvannastavne aktivnosti. Za potrebe ovoga rada dodatnom nastavom iz matematike smatramo pripreme učenika za natjecanja.

Pretpostavili smo da su studenti završnih godina učiteljskog studija kompetentni pripremati učenike nižih razreda osnovnih škola za natjecanja iz matematike te da na njihovu kompetentnost utječu odslušani matematički kolegiji na studiju. S obzirom na postavljene pretpostavke proveli smo istraživanje testiranjem njihove uspješnosti u rješavanju zadataka s regionalnog natjecanja učenika četvrtih razreda osnovnih škola te ju usporedili s uspješnosti učenika u rješavanju istih. Regionalno natjecanje je najviši stupanj natjecanja za učenike osnovnih škola u Republici Hrvatskoj, a razmatramo ga za potrebe ovoga rada. Istraživanje smo proveli na četiri hrvatska sveučilišta (Osijek, Zadar, Split, Zagreb) na uzorku ($n = 195$) studenata završnih godina integriranog preddiplomskog i diplomskog sveučilišnog učiteljskog studija. U nastavku rada, pod pojmom student smatramo studenta završnih godina učiteljskog studija, dok pod pojmom učenik smatramo učenika četvrtog razreda osnovne škole.

Okvir istraživanja

Izvannastavne aktivnosti se mogu odvijati u školi i izvan nje, a možemo ih podijeliti prema namjeni (obrazovne, umjetničke, informativno-poučne) i prema nastavnim područjima (tjelesna i zdravstvena kultura, prirodoslovlje i ekologije, jezično-umjetničko područje, informatičko-matematičko područje) (Jurčić, 2008).

Vođenju izvannastavnih aktivnosti treba pristupiti ozbiljno, jer bez potrebnih ljudskih kompetencija kao i dobre koncepcije i organizacije same aktivnosti, ne može se postići potpunost realizacije (Cardy, Selvarajan, 2006). Kako izvannastavne aktivnosti učenici samostalno odabiru, stupanj njihove motivacije je izrazito visok pa oni ovim oblikom aktivnosti puno lakše i u puno većem opsegu usvajaju nova znanja. Vođeni učiteljem motivatorom učenici istražuju, otkrivaju i usvajaju vještine timskog rada, razvijaju u sebi osjećaj odgovornosti i samopouzdanja te radne navike (Šiljković et al, 2007).

Učiteljeva uloga u vođenju izvannastavnih aktivnosti od iznimne je važnosti. Učitelj mora imati potporu od strane školske uprave kao i slobodu izbora izvannastavne aktivnosti koju će voditi. Osim toga, učitelj mora biti kompetentan odabrati učenika, koji će sudjelovati u izvannastavnoj aktivnosti prema vlastitom interesu i sposobnostima, te biti stručno osposobljen za vođenje aktivnosti. Ovo posljednje mora biti usmjereno u dva pravca: (i) razumijevanje svrhe i cilja izvannastavne aktivnosti, (ii) sposobnost za osmišljavanje pedagoško-metodičkog programa izvannastavne aktivnosti s drugačijim pristupom u odnosu na redovitu nastavu (Jurčić, 2008).

Kao što smo već naveli, jedna od učiteljevih kompetencija i odgovornosti je odabir učenika koji će sudjelovati u izvannastavnoj aktivnosti. S obzirom na trenutnu situaciju u učionicama, može se uočiti da se daroviti učenici češće

zanemaruju nego što se učitelji njima bave. Učitelji više razmišljaju o redovitoj nastavi nego o stručnom usavršavanju i razvijanju svojih kompetencija za rad s nadarenim učenicima (Reis, Renzulli, 2010). Razmišljamo li o matematički darovitim učenicima, oni se razlikuju u trima osobinama vezanima za matematiku: (i) brzina učenja, (ii) dubina razumijevanja, (iii) zanimanje za predmet. Na dodatnoj nastavi iz matematike učitelji trebaju pripremiti učenike za matematička natjecanja koja su mjesta na kojima učenik testira svoje sposobnosti u odnosu na svoje vršnjake.

Aktualni sustav natjecanja za učenike četvrtih razreda osnovnih škola, koje ćemo razmatrati u ovom radu, u Republici Hrvatskoj uključuje sljedeće razine natjecanja: školska natjecanja, za koja učitelji škole sastavljaju zadatke, te općinska ili gradska, županijska i regionalno natjecanje za koja zadatke sastavlja državno povjerenstvo pa su zadaci i kriteriji jedinstveni za svaku kategoriju natjecatelja (Elezović, 2005). Na međunarodnoj razini učenici četvrtih razreda osnovnih škola iz Republike Hrvatske sudjeluju na natjecanju Klokana bez granica, na kojem sudjeluju učenici iz 40-ak zemalja u različitim kategorijama.

Uzorak — podaci i varijable

Nakon što smo identificirali problem upitne kompetentnosti studenata učiteljskih studija kao budućih učitelja razredne nastave za izvođenje dodatne nastave iz matematike, postavili smo dva smjera istraživanja na uzorku studenata populacije od interesa: (i) uspješnost u rješavanju zadataka s regionalnog natjecanja učenika četvrtih razreda osnovnih škola, (ii) utjecaj odslušanih matematičkih kolegija na prethodno opisanu uspješnost. Prateći postavljene smjernice problema, osmislili smo istraživanje i proveli ga na slučajnom uzorku studenata završnih godina integriranog preddiplomskog i diplomskog sveučilišnog učiteljskog studija na četiri hrvatska sveučilišta (Osijek, Zadar, Split, Zagreb) i to u razdoblju od listopada do prosinca 2010. godine.

Testiranje uspješnosti studenata u rješavanju zadataka s regionalnog natjecanja učenika četvrtih razreda osnovnih škola u Republici Hrvatskoj proveli smo na zadacima koje smo preuzeli sa regionalnog natjecanja održanog u svibnju 2010. godine. Studenti su za rješavanje zadataka imali točno 90 minuta, a rješavali su ih pod nadzorom nastavnika nekog matematičkog kolegija na tom sveučilištu. Ovim istraživanjem je obuhvaćeno 195 ispitanika. Nadalje, podatke o uspješnosti učenika četvrtih razreda osnovnih škola u rješavanju zadataka sa spomenutog regionalnog natjecanja preuzeli smo s web-stranice Hrvatskog matematičkog društva, gdje su objavljeni rezultati ovog natjecanja za 2010. godinu, na kojemu su učenici 180 minuta rješavali pet zadataka i to pod nadzorom ispitnog povjerenstva sastavljenog od izabranih nastavnika matematike. Regionalnom natjecanju je te godine pristupilo 153 učenika.

Strukture prethodno opisanih uzoraka studenata i učenika s obzirom na ostvareni broj bodova u rješenjima pojedinog zadataka prikazali smo tablično po zadacima (Tablica 1 — Tablica 5).

ZAD1	STUDENTI ($n = 195$)		UČENICI ($n = 153$)	
Ostvareni broj bodova	Frekvencija	Relativna frekvencija (%)	Frekvencija	Relativna frekvencija (%)
0	56	28,71795	21	13,72549
1	0	0,00000	12	7,84314
2	19	9,74359	4	2,61438
3	0	0,00000	1	0,65359
4	7	3,58974	7	4,57516
5	2	1,02564	3	1,96078
6	6	3,07692	3	1,96078
7	2	1,02564	1	0,65359
8	4	2,05128	2	1,30719
9	4	2,05128	8	5,22876
10	95	48,71795	91	59,47712

Tablica 1. Struktura uzoraka s obzirom na ostvareni broj bodova u prvom zadatku (ZAD1).

ZAD2	STUDENTI ($n = 195$)		UČENICI ($n = 153$)	
Ostvareni broj bodova	Frekvencija	Relativna frekvencija (%)	Frekvencija	Relativna frekvencija (%)
0	104	53,33333	27	17,64706
1	0	0,00000	11	7,18954
2	2	1,02564	28	18,30065
3	1	0,51282	1	0,65359
4	0	0,00000	2	1,30719
5	8	4,10256	4	2,61438
6	2	1,02564	1	0,65359
7	14	7,17949	2	1,30719
8	13	6,66667	2	1,30719
9	0	0,00000	1	0,65359
10	51	26,15385	74	48,36601

Tablica 2. Struktura uzoraka s obzirom na ostvareni broj bodova u drugom zadatku (ZAD2).

ZAD3	STUDENTI ($n = 195$)		UČENICI ($n = 153$)	
Ostvareni broj bodova	Frekvencija	Relativna frekvencija (%)	Frekvencija	Relativna frekvencija (%)
0	156	80,00000	28	18,30065
1	0	0,00000	16	10,45752
2	5	5,56410	25	16,33987
3	0	0,00000	2	1,30719

4	3	1,53846	11	7,18954
5	2	1,02564	2	1,30719
6	2	1,02564	1	0,65359
7	1	0,51282	1	0,65359
8	5	2,56410	4	2,61438
9	0	0,00000	2	1,30719
10	21	10,76923	61	39,86928

Tablica 3. Struktura uzorka s obzirom na ostvoreni broj bodova u trećem zadatku (ZAD3).

ZAD4	STUDENTI ($n = 195$)		UČENICI ($n = 153$)	
Ostvoreni broj bodova	Frekvencija	Relativna frekvencija (%)	Frekvencija	Relativna frekvencija (%)
0	75	38,46154	24	15,68627
1	7	3,58974	20	13,07190
2	14	7,17949	8	5,22876
3	12	6,15385	4	2,61438
4	19	9,74359	2	1,30719
5	12	6,15385	3	1,96078
6	20	10,25641	7	4,57516
7	6	3,07692	3	1,96078
8	12	6,15385	8	5,22876
9	6	3,07692	3	1,96078
10	12	6,15385	71	46,40523

Tablica 4. Struktura uzoraka s obzirom na ostvoreni broj bodova u četvrtom zadatku (ZAD4).

ZAD5	STUDENTI ($n = 195$)		UČENICI ($n = 153$)	
Ostvoreni broj bodova	Frekvencija	Relativna frekvencija (%)	Frekvencija	Relativna frekvencija (%)
0	53	27,17949	16	10,45752
1	1	0,51282	11	7,18954
2	10	5,12821	8	5,22876
3	1	0,51282	6	3,92157
4	5	2,56410	2	1,30719
5	6	3,07692	5	3,26797
6	6	3,07692	6	3,92157
7	6	3,07692	0	0
8	7	3,58974	15	9,80392
9	6	3,07692	3	1,96078
10	94	48,20513	81	52,94118

Tablica 5. Tablica 5 Struktura uzoraka s obzirom na ostvoreni broj bodova u petom zadatku (ZAD5).

Struktura uzorka studenata na kojima je provedeno ovo istraživanje opisana je u donjoj tablici (Tablica 6).

Obilježje	Kategorija	STUDENTI ($n = 195$)	
		Frekvencija	Relativna frekvencija (%)
SPOL	ženski	188	96,41026
	muški	7	3,58974
POHAĐANJE GIMNAZIJE	da	141	72,30769
	ne	54	27,69231
ECTS BODOVI IZ DOSADA ODSLUSANIH MATEMATIČKIH KOLEGIJA	19	86	44,10256
	16	58	29,74359
	15	38	19,48718
	11	13	6,66667

Tablica 6. Struktura uzorka studenata.

Metodologija

Nakon prethodno opisanog prikupljanja podataka provedena je statistička analiza koristeći Soft STATISTICA 7.0 s ciljem procjene populacijskog parametra primjenom intervala pouzdanosti i testiranja statističkih hipoteza izvedenih iz hipoteze istraživanja te su doneseni određeni zaključci o razmatranim populacijama. Populacijski parametar očekivanja smo procijenili intervalom zadane pouzdanosti za velike uzorke, tj. koristeći podatke prikupljene na uzorcima odredili smo intervale koji ih procjenjuju. Za potrebe ovoga rada korištena je pouzdanost od 95% koja osigurava da slučajno izabrani interval pouzdanosti sadrži populacijski parametar s vjerojatnošću od 0,95 (McClave et al., 2001).

Statistički test za testiranje statističke hipoteze provodi se primjenom sljedećih koraka (McClave et al., 2001): (1) postavljanje nulte hipoteze H_0 koja predstavlja status quo, (2) postavljanje alternativne hipoteze H_1 , (3) odabir odgovarajuće test statistike koja će biti primijenjena kod odluke o odbacivanju nulte hipoteze, (4) određivanje područja odbacivanja nulte hipoteze s obzirom na određeni nivo signifikantnosti α , (5) prikupljanje podataka, (6) izračunavanje test statistike, (7) donošenje odluke o odbacivanju nulte hipoteze, (8) zaključivanje o populaciji. Razina signifikantnosti α , koja je primijenjena u statističkim testovima provedenima za potrebe ovoga rada, je 0,05. S ciljem usporedbe očekivanja proveli smo test za testiranje hipoteze o postojanju razlike u očekivanjima velikih uzoraka primjenom z-statistike.

Rezultati

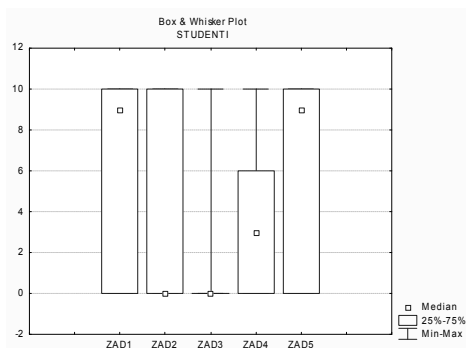
Koristeći informacije iz uzoraka za ostvareni broj bodova u rješenju pojedinog zadatka, odredili smo neke mjere deskriptivne statistike i procijenili očekivanje intervalom pouzdanosti 95%. Dobiveni rezultati dani tablično (Tablica 7) daju

naznaku postojanja razlike u očekivanju broja ostvarenih bodova u rješenjima pojedinih zadataka između studenata i učenika i to u korist učenika. Važno je naglasiti da su pri rješavanju zadataka s regionalnog natjecanja i učenici i studenti koristili metode rješavanja zadataka primjerene učenicima mlađe školske dobi.

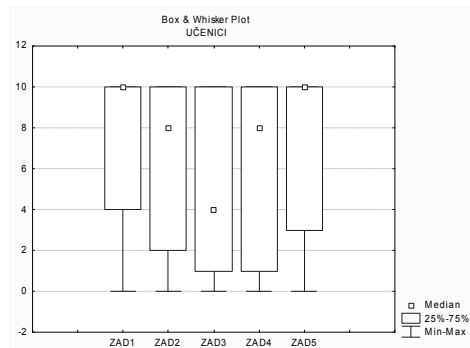
	STUDENTI ($n = 195$)				UČENICI ($n = 153$)			
	Deskriptivna statistika uzorka		Interval pouzdanosti 95% za procjenu očekivanja		Deskriptivna statistika uzorka		Interval pouzdanosti 95% za procjenu očekivanja	
	Očekivanje	Standardna devijacija	Donja granica	Gornja granica	Očekivanje	Standardna devijacija	Donja granica	Gornja granica
ZAD1	5,86667	4,49543	5,23174	6,50159	7,11765	4,06030	6,46911	7,76618
ZAD2	3,95385	4,45978	3,32396	4,58373	5,77124	4,37622	5,07225	6,47023
ZAD3	1,54359	3,36322	1,06858	2,01860	5,22222	4,28465	4,53786	5,90659
ZAD4	3,27692	3,32830	2,80684	3,74700	6,11111	4,24195	5,43356	6,78866
ZAD5	6,16410	4,37586	5,54607	6,78214	7,00000	3,86958	6,38193	7,61807
UKUPNO	20,80513	12,89502	18,98387	22,62638	31,22222	12,03461	29,29999	33,14446

Tablica 7. Deskriptivna statistika uzoraka i procjena očekivanja intervalom pouzdanosti.

Prikupljene numeričke podatke za opisane uzorke studenata i učenika prikazali smo i grafički i to kutijastim dijagramom na bazi medijana za ostvoreni broj bodova u rješenju pojedinog zadatka s obzirom na rješavače zadataka (Slika 1, Slika 2). Također, i iz ovog razmatranja lako je uočiti razliku u ostvarenim bodovima i to pozitivnu upravo na strani učenika.



Slika 1. Kutijasti dijagram za ostvoreni broj bodova u rješenju pojedinog zadatka za studente.



Slika 2. Kutijasti dijagram za ostvoreni broj bodova u rješenju pojedinog zadatka za učenike.

Nadalje, proveli smo jednostrani test za usporedbu očekivanja broja ostvarenih bodova u rješenju pojedinog zadatka u velikim nezavisnim uzorcima studenata i učenika, pri čemu je alternativna hipoteza predstavljala postojanje razlike u očekivanom broju ostvarenih bodova i to u korist učenika ($H_1 : \mu_1 > \mu_2$), a formirali

smo ju na osnovi prethodnih razmatranja. Rezultati testa dani tablično (Tablica 8) na razini značajnosti $\alpha = 0,05$ pokazuju da je očekivanje broja ostvarenih bodova u rješenju pojedinog zadatka statistički značajno veće za rješenja učenika u odnosu na rješenja studenata, što konačno učenike čini statistički značajno uspješnijima u rješavanju svih danih zadataka na razini značajnosti 5%.

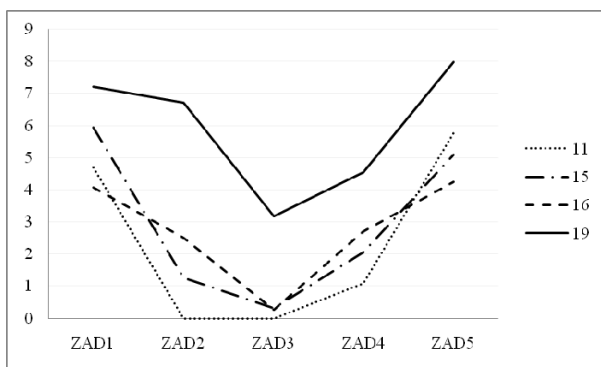
Razmatrano obilježje	p -vrijednost	$\alpha = 0,05$	z	$z_{\alpha} = 1,644854$	H_0
ZAD1	0,0032553383143	$p < \alpha$	2,720888253	$z > z_{\alpha}$	odbaciti
ZAD2	0,0000686269	$p < \alpha$	3,81306549	$z > z_{\alpha}$	odbaciti
ZAD3	0	$p < \alpha$	8,71933034	$z > z_{\alpha}$	odbaciti
ZAD4	$5,75 \dots 10^{-12}$	$p < \alpha$	6,786322167	$z > z_{\alpha}$	odbaciti
ZAD5	0,02952666	$p < \alpha$	1,887796191	$z > z_{\alpha}$	odbaciti
UKUPNO	$4,10782519 \dots 10^{-15}$	$p < \alpha$	7,765870014	$z > z_{\alpha}$	odbaciti

Tablica 8. Rezultati jednostranog testa za usporedbu očekivanja broja ostvarenih bodova u rješenju pojedinog zadatka s obzirom na rješavače.

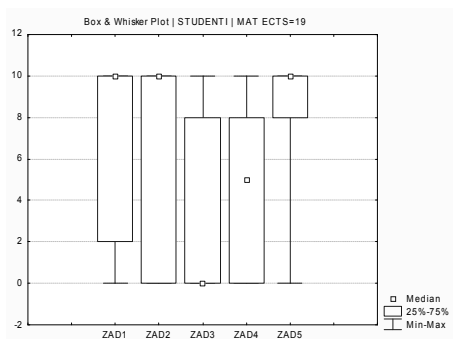
U nastavku smo istražili utjecaj dosada odslušanih matematičkih kolegija na uspješnost studenata u rješavanju danih zadataka, pri čemu su dosada odslušani kolegiji predstavljeni sumom ECTS bodova koji nose (Tablica 6). Deskriptivna statistika uzorka studenata s obzirom na ukupni broj ECTS bodova iz dosada odslušanih matematičkih kolegija kao i procjena očekivanja broja bodova ostvarenih u rješenju pojedinog zadatka intervalom pouzdanosti 95% (Tablica 9, Slika 3 – Slika 5) daju naznaku da studenti s najvećim brojem ECTS-a iz dosada odslušanih matematičkih kolegija ostvaruju najbolje rezultate.

ECTS	n			ZAD1	ZAD2	ZAD3	ZAD4	ZAD5	UKUPNO
19	86	Deskriptivna statistika uzorka	Očekivanje	7,220930	6,709302	3,186047	4,534884	7,976744	29,62791
			Standardna devijacija	4,02463	4,30285	4,27994	3,59967	3,49782	11,21768
		Interval pouzdanosti 95% za procjenu očekivanja	Donja granica	6,35805	5,78677	2,26842	3,76311	7,22681	27,22283
			Gornja granica	8,08381	7,63183	4,10367	5,30665	8,72668	32,03298
16	109	Deskriptivna statistika uzorka	Očekivanje	4,79817	1,77982	0,24771	2,28440	4,73394	13,84404
			Standardna devijacija	4,576065	3,215581	1,434728	2,728762	4,481737	9,421385
		Interval pouzdanosti 95% za procjenu očekivanja	Donja granica	3,92936	1,16931	-0,02469	1,76633	3,88305	12,05531
			Gornja granica	5,66697	2,39032	0,52010	2,80248	5,58484	15,63276

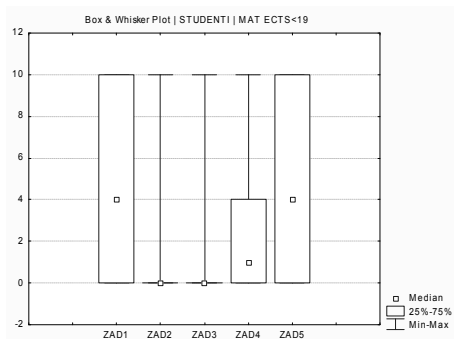
Tablica 9. Deskriptivna statistika uzorka studenata i procjena očekivanja intervalom pouzdanosti.



Slika 3. Linijski dijagram za očekivanje broja bodova u rješenju pojedinog zadatka s obzirom na broj ECTS bodova iz matematičkih kolegija.



Slika 4. Kutijasti dijagram za ostvareni broj bodova u rješenju pojedinog zadatka za studente sa 19 ECTS-a iz matematičkih kolegija.



Slika 5. Kutijasti dijagram za ostvareni broj bodova u rješenju pojedinog zadatka za studente sa manje od 19 ECTS-a iz matematičkih kolegija.

U nastavku smo proveli jednostrani test za usporedbu očekivanja ostvarenog broja bodova u rješenju pojedinog zadatka studenata koji imaju 19 i onih koji imaju manje od 19 ECTS- a iz dosada odslušanih matematičkih kolegija, pri čemu je alternativna hipoteza predstavljala postojanje razlike u očekivanom broju ostvarenih bodova i to u korist studenata s 19 ECTS-a ($H_1 : \mu_1 > \mu_2$), a formirali smo ju na osnovi prethodnog razmatranja. Rezultati testa dani tablično (Tablica 10) na razini značajnosti 5% pokazuju da su studenti sa 19 ECTS-a iz matematičkih kolegija statistički značajno uspješniji u rješavanju svih danih zadataka.

Razmatrano obilježje	p -vrijednost	$\alpha = 0,05$	z	$z_{\alpha} = 1,644854$	H_0
ZAD1	0,000042849868	$p < \alpha$	3,927874986	$z > z_{\alpha}$	odbaciti
ZAD2	0	$p < \alpha$	8,851520848	$z > z_{\alpha}$	odbaciti
ZAD3	$5,24 \dots 10^{-10}$	$p < \alpha$	6,101923784	$z > z_{\alpha}$	odbaciti

ZAD4	$7,58 \dots 10^{-7}$	$p < \alpha$	4,809169076	$z > z\alpha$	odbaciti
ZAD5	0,0000000069416	$p < \alpha$	5,674822095	$z > z\alpha$	odbaciti

Tablica 10. Rezultati jednostranog testa za usporedbu očekivanja broja ostvarenih bodova u rješenju pojedinog zadatka za studente sa 19 ECTS-a i manje.

Zaključak

Provedenim istraživanjem utvrdili smo trenutno stanje kompetentnosti studenata završnih godina učiteljskih studija u Republici Hrvatskoj za pripremu učenika za natjecanja iz matematike u sklopu dodatne nastave iz matematike. Budući da smo uočili kako su učenici, koji su sudjelovali na regionalnom natjecanju iz matematike, uspješniji u rješavanju zadataka sa regionalnog natjecanja u odnosu na studente završnih godina učiteljskog studija, utvrdili smo da trenutne generacije studenata završnih godina učiteljskih studija u Republici Hrvatskoj nisu dovoljno kompetentne za pripremu učenika nižih razreda osnovnih škola za natjecanja iz matematike u svojoj budućoj učiteljskoj praksi. Osim toga, istraživanje je pokazalo da matematički kolegiji koji se izvode na učiteljskom studiju pozitivno utječe na uspješnost studenata u rješavanju postavljenih zadataka što bi dakako trebala biti smjernica učiteljskim studijima u Republici Hrvatskoj za razmatranje svojih programa osposobljavanja budućih učitelja za kvalitetno izvođenje nastave na svim težinskim nivoim primjerenih učenicima nižih razreda osnovnih škola. Slično istraživanje planiramo provesti u sljedećim generacijama završnih godina učiteljskih studija u Republici Hrvatskoj te na taj način evidentirati eventualnu promjenu.

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Prostorno mišljenje studenata nastavničkih studija matematike na Matematičkom odsjeku Prirodoslovno-matematičkog fakulteta Sveučilišta u Zagrebu – sposobnost mentalnog presijecanja

Aleksandra Čizmešija i Željka Milin Šipuš

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Sažetak. U srpnju 2010. godine Ministarstvo znanosti, obrazovanja i športa RH donijelo je *Nacionalni okvirni kurikulum za predškolski odgoj i obrazovanje te opće obvezno i srednjoškolsko obrazovanje* (NOK), s matematičkim područjem kao jednim od osam odgojno-obrazovnih područja u svim odgojno-obrazovnim ciklusima i obaveznim za sve učenike. Osim svrhe i općih ciljeva učenja i poučavanja matematike, usklađenih s definicijom matematičke kompetencije iz *Europskog referentnog okvira o ključnim kompetencijama za cjeloživotno učenje*¹, dokument navodi i očekivana učenička matematička postignuća na kraju svakog ciklusa. Organizirana su u dvije dimenzije – konceptualnu (matematički koncepti) i proceduralnu (matematički procesi).

Budući da je geometrijsko, a napose prostorno mišljenje važna sastavnica matematičke kompetencije, u NOK-u je učeničkim postignućima vezanima uz koncepte Oblik i prostor i Mjerenje ponovo promišljen i reafirmiran njegov razvoj. Kvalitetna provedba ovog dijela matematičkog kurikuluma svakako podrazumijeva i visoku razinu prostornog mišljenja učitelja i nastavnika. Iz tog je razloga nužno ispitati i razinu prostornog zora studenata nastavničkih studija matematike.

Istraživanje čije rezultate prikazujemo nastavak je našeg prethodnog istraživanja prostornog zora studenata nastavničkih smjerova matematike na PMF – Matematičkom odsjeku Sveučilišta u Zagrebu. Cilj je bio utvrditi razinu studentske sposobnosti vizualizacije prostornih odnosa, konkretno, presjeka prostornih oblika ravninom. Studente smo

¹ Dokument *Key Competences for Lifelong Learning – A European Framework* dodatak je dokumentu *Recommendation of the European Parliament and of the Council of 18 December for lifelong learning* (2006/962/EC).

podvrgle standardiziranom testu mentalnog presijecanja (eng. *Mental Cutting Test*), a njihove smo rezultate analizirale i usporedile s rezultatima sličnih studentskih populacija iz susjednih država (Njemačka, Mađarska, Austrija). Uz to, na uzorku studenata prve godine nastavnčkih studija matematike provele smo učeničku aktivnost prepoznavanja i matematičkog opisivanja oblika koje ulivena voda tvori sa stranama posude oblika kvadra pri rotaciji posude oko najkraćeg brida njene osnovke. U priopćenju prikazujemo rezultate navedenog testiranja standardiziranim testom te opisujemo cilj i tijek navedene učeničke aktivnosti, kao i kvalitativnu analizu rezultata njenog provođenja.

Ključne riječi: Nacionalni okvirni kurikulum, matematička kompetencija, prostorno mišljenje, vizualizacija, mentalni presjeci, standardizirani test mentalnog presijecanja, studenti nastavnčkih studija matematike

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Kada tehnologija utječe na učenje?

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Sažetak. U današnje je vrijeme uobičajeno govoriti o tehnologijom podržanom učenju ili e-učenju. No, unatoč tome, postoji ograničen broj istraživanja koja nam odgovaraju na pitanje unaprjeđuje li zaista tehnologija učenje, kvalitetu poučavanja i postizanje ishoda učenja. Naglasimo, prije svega, da mi zagovaramo stav da pedagogija prethodi tehnologiji i da postavljanje obrazovnih ciljeva dolazi prije nego odabir prikladnih IKT alata ili drugih medija za pomoć pri poučavanju. Dilema o adekvatnom korištenju i utjecaju medija na učenje nije nova. Devedesetih godina prošlog stoljeća bila je poznata “debata o medijima” koju su započeli Richard Clark i Robert Kozma, nakon što je Clark (Clark, 1983.) prezentirao svoje viđenje uloge tehnologije u poučavanju i napisao da njen utjecaj nije veći od utjecaja kamiona za prijevoz namirnica na kvalitetu namirnica koje prevozi. Suprotno tom stavu, Kozma (Kozma, 1994.) je naglašavao da je upotreba računalne tehnologije najučinkovitija kada podupire aktivan angažman u okvirima nastavnog plana i programa. U ovom ćemo radu analizirati literaturu koja se bavi pitanjem iz naslova. Nadalje, prezentirat ćemo vlastita iskustva vezana uz učenje i poučavanje kao i rezultate kvalitativnog i kvantitativnog istraživanja koje smo proveli.

Ključne riječi: tehnologija, učenje

Uvod

Od samih početaka sustavno organiziranog procesa učenja, ljudi su koristili razne pomoćne instrumente i alate kako bi podigli produktivnost obrazovnog procesa. Dostupnost i, naravno, egzistenciju tih pomoćnih instrumenata diktiralo je vrijeme. Što to zapravo znači? Uzmimo na primjer fotografiju: fotografija ima vrlo važnu ulogu u procesu učenja te iako se najprije pojavila kao crno-bijela, tijekom godina postala je “obojana”. Nakon toga je čovjek uspio dobiti slike u pokretu, tj. film. A danas, umjesto da te slike u pokretu samo gledamo, možemo na njih utjecati i stvarati vlastite uvjete i okruženja kako bi stekli nova saznanja. Tehnologija napreduje strjelovito i vrijeme zbog toga ima središnju ulogu u raspravi o tehnologijom podržanom učenju. Ne tako davno, kreda i ploča su, zajedno s knjigama, bile osnovni instrument poučavanja, a danas ne možemo

zamisliti predavanje bez upotrebe digitalnih prezentacija na platnu ili ekranu. Govoriti o učenju podržanom tehnologijom ili o e-učenju je danas uobičajeno te se u društvu te sveukupne tehnologije jednim imenom nazivaju informacijske i komunikacijske tehnologije (IKT). Među njima, računalne tehnologije su one koje su se u posljednjih nekoliko desetljeća iznimno razvile. Zbog njihovog naglog razvoja ljudi su se počeli pitati, utječu li uopće na poučavanje i poboljšavaju li učenje i poučavanje ili ne? Ta su pitanja još uvijek otvorena. Računalne su tehnologije u našim učionicama već približno tridesetak godina pa ipak još uvijek postavljamo pitanja o njihovom utjecaju na proces učenja. U ovom radu ćemo se prisjetiti poznate rasprave između Richarda E. Clarka (Clark, 1983.) i Roberta Kozme (Kozma, 1994.) o utjecaju tehnologije, da bi se nakon toga upoznali s najnovijim cjelokupnim pregledom postignuća u visokom obrazovanju (CSLP, 2009.) glede pitanja iz naslova. Na kraju ćemo pokušati usporediti izloženu situaciju s onom na našem fakultetu, Fakultetu organizacije i informatike, Sveučilišta u Zagrebu.

Čuvena “rasprava o medijima”

Čuvena “rasprava o medijima” bila je potaknuta Clarkovim člankom (Clark, 1983.). U članku izjavljuje da uloga tehnologije jest, i da će uvijek biti minimalna. Štoviše, tvrdi da su instrukcijski dizajn i nastavna praksa (nastavne metode) jezgra formalnog učenja, bilo da se radi o nastavi u učionici, udaljenoj ili kombiniranoj (hibridnoj) nastavi. Nadalje tvrdi “da su mediji samo vozila koja isporučuju nastavu i ne utječu na učenikova postignuća ništa više nego što kamion za prijevoz namirnica utječe na našu prehranu”. S druge strane, Kozma (Kozma, 1994.) je bio svjestan razvoja računalne tehnologije i izjavio da se ne smijemo pitati “Utječu li mediji na učenje?”, već “Na koji način možemo upotrijebiti mogućnosti medija da utječemo na učenje?”. Valja naglasiti da je to bilo 1994. godine, prije gotovo 20 godina te 10 godina nakon Clarkovog članka. Kozma nije odvajao medije od nastavnih metoda, već je smatrao da imaju vezani odnos. Zaključio je da je upotreba računalne tehnologije najučinkovitija kada su učenici aktivno angažirani u okvirima nastavnog plana i programa. U njegovom članku, učenici su sami na računalima vježbali gradivo iz fizike te provjeravali svoja problemska rješenja u različitim uvjetima. Takvi učenici su postigli bolje rezultate na testovima od onih koji su pohađali tradicionalnu nastavu u učionici. Clark bi rekao da su u ovom primjeru iz fizike računala bila dio nastavnih metoda te je razlika samo u tome koju metodu odaberete. Moramo priznati da obojica imaju smislene argumente, čak i u suvremenom kontekstu. Posebice, uzmemo li u obzir one članke koji su bili pisani ranih osamdesetih, kad su osobna računala još uvijek bila u povojima, razumljivo je zašto Clark smatra tehnologiju i medije dijelom nastavnih metoda. Zbog preplavljenosti današnjeg procesa učenja računalnom tehnologijom i mi bismo se mogli složiti da većina modernih metoda poučavanja uključuje računalnu tehnologiju te se zapravo sve svodi na odabir odgovarajuće. Moramo se zapitati je li tehnologija zaista unaprijedila i poboljšala proces učenja i pripadne mu ishode ili je samo povećala raspoloživost, dostupnost i količinu nastavnog materijala? A ako su obje ove izjave djelomice točne, kojoj dajemo prednost? To je bila ključna stvar našeg skromnog

istraživanja provedenog među studentima Fakulteta organizacije i informatike o kojem ćemo kasnije govoriti.

Računalne tehnologije

Centar za proučavanje učenja i učinkovitosti (CSLP) na Sveučilištu Concordia, Kanada, proveo je opširno pretraživanje među člancima objavljenim od 1990. godine, a vezanim uz računalnu tehnologiju u visokom obrazovanju (CSLP, 2009.). Cilj istraživanja bio je istražiti postignuti učinak upotrebe računalne tehnologije u učionicama visokoobrazovnog sustava (nastava u učionici). Istraživanje je naišlo na više od 6000 potencijalno odgovarajućih empirijskih studija koje su bile reducirane na uzorak od 231-e reprezentativne studije. Ovdje nećemo ići previše u detalje ovog istraživanja, već ćemo se samo osvrnuti na rezultate. Godina 1990. kao donja vremenska granica istraživanja bila je odabrana zbog toga jer su se računalne tehnologije kao suvremene tehnologije pojavile relativno kasno. Sve u svemu, rezultati su sugerirali da je bolje upotrebljavati tehnologiju, nego ne upotrebljavati je. No, rezultati nisu ništa rekli o tome *koliko* tehnologija unaprijeđuje postignuća u učenju ili *kako* tehnologije najbolje podupiru postizanje ishoda učenja. Uspjeli su pronaći tri dodatna dokaza koji sugeriraju kako je tehnologija povezana s učenjem u visokom obrazovanju. Prvo, u analizi sveukupnih učinaka primjene tehnologije u visokom obrazovanju, pronašli su, čini se, da upotreba tehnologije ima svojih limita kad je u pitanju utjecaj na napredak u učenju. Dakle, možemo povećavati broj tehnoloških aplikacija, ali će u nekom trenutku napredak u učenju dosegnuti svoju gornju granicu. Drugo, pronašli su da tehnologije koje *pomažu u razumijevanju*, definirano u općem smislu, donose znatno bolje rezultate od tehnologije koja se koristi samo za *predočavanje i dostavljanje nastavnog sadržaja*. Nedvojbeno je da tehnološke aplikacije koje zahtijevaju veći angažman nadmašuju one koje zahtijevaju manji, a koji je povezan samo s prihvatanjem i pohranjivanjem sadržaja. Čini se da ovaj rezultat podupire Kozminu tezu (1994.) o prednostima tehnologije. Doduše, to ne isključuje ni Clarkove argumente da su metodika i pedagogija korijen uspjeha u učenju te su postignuća mogla biti unaprijeđena drugačijim načinom koji nema veze s tehnologijom. I treće, pronašli su razlike u razinama zastupljenosti tehnologijom. Uvjeti slabe i srednje zastupljenosti tehnologijom u učionicama vodili su do znatnijeg prosječnog učinka od zastupljenosti tehnologije u učionicama u velikoj mjeri. Vođeni ovim trima spomenutim rezultatima CSLP studije, formirali smo upitnik za studente Fakulteta organizacije i informatike koji će biti tema našeg idućeg odlomka.

Istraživanje na Fakultetu organizacije i informatike (FOI), Sveučilišta u Zagrebu

Pokušajmo najprije objasniti o kakvom se kontekstu ovdje radi. Glavni cilj Fakulteta organizacije i informatike je pružiti studentima prvorazredno obrazovanje na području informacijskih znanosti kao i informacijskih tehnologija. Prisutnost riječi 'tehnologija' u nastavnom programu na neki način podrazumijeva imperativ

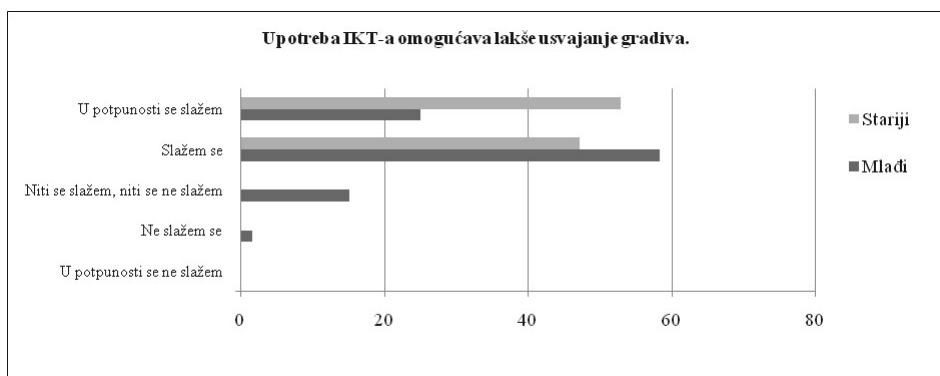
korištenja učenja podržanog tehnologijom tokom preddiplomskog i diplomskog obrazovanja (“koristi tehnologiju da bi poučavao o tehnologiji”). Osim upotrebe IKT-a u predavaonici, e-učenje je, prema odluci profesora, također implementirano na Fakultetskoj razini od 2006. godine temeljem strategije i sustavnog pristupa (Strategija FOI). Sustav za upravljanje učenjem (engl. *Learning Management System* – *LMS*) koji se otad koristi je Moodle koji je zamijenio nekoliko sustava upotrebljivanih prije, ali ne sustavnim načinom. Više o poučavanju i okolini za učenje na FOI-ju može se pročitati u (Divjak i Erjavec, 2007, Divjak i Ostroški, 2009, Divjak et al 2010.).

Što možemo reći o studentima koji upisuju Fakultet? Možemo pretpostaviti da su skloni računalnoj tehnologiji, zbog čega imamo specifičnu situaciju koju vrijedi usporediti s rezultatima studije o računalnim tehnologijama iz prijašnjeg odlomka. Željeli smo vidjeti koja su opažanja studenata što se tiče ove teme. U tu smo svrhu proveli upitnik među 192 studenta prve godine (72% svih studenata) te 17 studenata na predmetu Diskretne strukture s teorijom grafova (85% upisanih na predmet – peti semestar preddiplomskog i prvi semestar diplomskog studija). Do trenutka kad upisuju Diskretne strukture s teorijom grafova, studenti bivaju upoznati s mnogo više e-kolegija i raznolikošću pristupa učenju i poučavanju nego studenti prve godine. Osim toga, spomenuti predmet je dobro opremljen kvalitetnim e-materijalima i IKT alatima pa je vrlo korisno usporediti rezultate tih dviju promatranih studentskih grupa. Svjesni smo činjenice da postoji značajna razlika u veličini i vrsti uzorka pa zbog toga dajemo samo kvalitativnu interpretaciju tih podataka kao i razlike među njima. Nadalje ćemo studente prve godine zvati “mlađim studentima”, a drugu grupu “starijim studentima”. Za početak smo htjeli saznati njihova iskustva s upotrebom IKT-a u učionici iz srednje škole (Tabela 1).

%	Ppt prezentacije	Grafoskop	Sustav za upravljanje učenjem (LMS)	Računala	Bez tehnologije
Mlađi N = 192	77,1	76,1	7,8	29,2	10
Stariji N = 17	47,1	76,5	0	23,5	23,5

Tabela 1. Srednjoškolsko iskustvo (pitanje s višestrukim odgovorima).

Upotreba grafoskopa je još uvijek nešto standardno u srednjoj školi, ali polako biva istisnuta PowerPoint prezentacijama kao što smo i spomenuli prije. U periodu od dvije do tri godine (prosječna razlika u starosti među ispitanim grupama) postotak upotrebe Ppt prezentacija se značajno povećao. Čini se da i LMS-ovi polako pronalaze svoj put u srednje škole, što nije bio slučaj sa starijom grupom. Nakon pitanja o iskustvu s tehnologijama, željeli smo saznati njihove stavove vezane uz tehnologiju u nastavi. Gotovo 85% mlađih studenata i svi stariji studenti su se složili ili u potpunosti složili da upotreba IKT-a omogućava lakše usvajanje gradiva. To potvrđuje našu tvrdnju o studentima sklonim tehnologiji (Slika 1).



Slika 1. Stavovi prema tehnologiji (rezultati u postocima).

U nastavku smo im dali otvoreno pitanje gdje su trebali navesti jedan razlog zbog čega se slažu s prijašnjom izjavom. Približno 75% mlađih studenata i više nego 80% starijih studenata je napisalo neku od varijanti izraza *dostupnost nastavnog materijala*. To pretežno ide u korist Clarkovoj tezi da danas imamo samo novije i kvalitetnije kamione za prijevoz namirnica. U duhu ovih prethodnih dvaju pitanja, u upitniku je studentima uslijedila jedna rečenica koju su trebali završiti odabirom samo jednog od tri ponuđena završetka (Tabela 2).

Smatram da IKT u nastavnom procesu najviše doprinosi zbog... (%)	... veće i lakše dostupnosti nastavnim materijalima.	... materijala koji omogućuju da lakše razumijem gradivo.	... jednostavne komunikacije s nastavnicima i kolegama.
Mlađi	64	32,3	3,7
Stariji	82,3	11,8	5,9

Tabela 2. Doprinos IKT-a.

Dobili smo slične rezultate kao i u prijašnjem (otvorenom) pitanju. Upravo je ovo bio središnji dio upitnika gdje smo mogli identificirati njihova uvjerenja glede doprinosa tehnologije, a koji je bio usko vezan uz drugi rezultat studije iz prošlog odlomka. Kao što i možemo vidjeti iz Tabele 2, većina studenata obiju grupa smatra da je glavni doprinos IKT-a u isporuci informacija, nastavnog materijala, alata te kao baze podataka. To je stvarnost. Ali zbog čega imamo takvu situaciju? Ovdje valja napomenuti da se svi predmeti na FOI-u izvode kao kombinirani e-kolegiji. To znači da studenti imaju i obilje iskustva nastave u učionici i direktne komunikacije s nastavnicima, kao i međusobno, i puno više koriste LMS za materijale i pomoćne alate nego za komunikaciju. Zbog toga su prilikom odgovaranja na ovo pitanje rangirali svoje odgovore i stavili na prvo mjesto ono što im je očiglednije. Po njihovom iskustvu, IKT se najviše koristi za isporuku nastavnog sadržaja, tj. nastavnici najčešće koriste IKT za isporuku nastavnog sadržaja. Jedan od razloga za ovakvu situaciju leži u činjenici da je još uvijek previše studenata u predavačkim i seminarskim grupama te su nastavnici prilično zainteresirani za isporuku materijala putem LMS-a kako bi uštedjeli na vremenu i pripremili kvalitetne zadatke o

kakvima govori Kozma (koji pomažu u razumijevanju). A opet, s druge strane, i za stvaranje e-sadržaja nastavnici trebaju uložiti puno vremena. Stoga, da bi poduprli tvrdnju o kvalitetnim materijalima, pitali smo studente jesu li se ikad susreli, tokom svog perioda studiranja, s materijalima koji su im pomogli u lakšem svladavanju gradiva. Kao rezultat, 82% mlađih i 94% starijih studenata je odgovorilo da se na nekim predmetima susrelo s takvom vrstom materijala. Većina se, dakle, složila da postoje izvrsni pozitivni primjeri. Upitali smo ih također i o negativnim stranama upotrebe IKT-a: jesu li se susreli s predmetom za koji bi mogli reći da sadrži tako veliku količinu nesustavno prezentiranog materijala (dokumente, aplete, animacije, filmove...) ponuđenog u sustavu za e-učenje da zbog svoje opširnosti ne doprinosi usvajanju gradiva. Otprilike polovica obiju grupa je odgovorila da se susrela s barem jednim takvim predmetom. Ovdje imamo primjer gdje studentima pretjerana upotreba IKT-a u nastavnom planu i programu ne pridonosi nužno učenju, niti postizanju ciljeva učenja. Koji je krajnji cilj koji većina studenata ima? Najvažniji cilj im je položiti ispit, a za većinu njih to je i jedini cilj. Ukratko: imate studente orijentirane prema tehnologiji od kojih većina želi uložiti minimalni napor potreban za položiti ispite (pogotovo matematičke ispite, jer matematika nije njihova osnovna djelatnost).

Nadalje, stavili smo im i otvoreno pitanje o tome kakve bi podrške vezane uz IKT voljeli imati više (postojeće ili nepostojeće). Neki odgovori koji pokazuju "tamnu stranu" upotrebe IKT-a se mogu naći u Tabeli 3. Kao što možemo vidjeti, mlađi studenti žele imati sve na pladnju i stavljaju se više u pasivnu, nego u aktivnu ulogu. Na primjer, puno studenata bi željelo imati više "skraćenijeg" materijala za pripremu ispita i kolokvija. Jedan je student čak predložio nastavnicima da pripreme takozvanu kratku verziju zbirke zadataka koja bi sadržavala samo one primjere zadataka koji su sasvim dovoljni za položiti pismeni dio ispita (zato jer je zbirka zadataka preopširna). Malo je pozornosti posvećeno oslanjanju na kreativnost, stoga nastavnici trebaju biti pažljivi da ne ispune sve želje studenata, sputavajući na taj način njihovu kreativnost. Naprotiv, trebamo kod studenata podržavati kreativnost i inovativnost tijekom studiranja jer se danas radna mjesta i karijere značajno oslanjaju na njih. Ohrabrujuće je da su stariji studenti svjesniji kompetencija potrebnih kod zapošljavanja te imaju profinjenije potrebe pri učenju.

	Navedite neke od najkorisnijih materijala dostupnih putem sustava za e-učenje (postojećih ili nepostojećih).
Mlađi studenti (prva godina)	• Kraće elektroničke prezentacije samo sa sadržajem koji dolazi na ispit
	• Više primjera riješenih kolokvija
	• Skripta s riješenim ispitnim pitanjima
	• Više zadataka s rješenjima
	• Popis prezentacija i zadataka
	• Grafičke prezentacije i animacije
Stariji studenti	• Testovi za samoevaluaciju da studenti mogu provjeriti svoje znanje
	• Poveznice na zanimljive stranice

Tabela 2. Najkorisniji IKT materijal.

Zaključak

Upotreba tehnologije u procesu poučavanja je neizbježna jer je ušla u sve aspekte današnjeg života pa su studenti motivirani učiti upotrebom IKT-a. Rezultati istraživanja pokazuju da tehnološke aplikacije koje zahtijevaju više angažmana daju bolje rezultate prilikom učenja. Ako nismo u mogućnosti stvarati takva poboljšanja, već samim povećanjem dostupnosti nastavnog materijala napravili smo pozitivni pomak. No, ne smijemo zaboraviti da pedagogija prethodi tehnologiji i da postavljanje obrazovnih ciljeva dolazi prije nego odabir prikladnih IKT alata ili drugih medija. Stavljanje tehnologije na prvo mjesto može imati suprotan učinak i dovesti do zasićenja i gubitka studentskog angažmana. U tu svrhu trebamo biti svjesni negativnih učinaka upotrebe tehnologije. Na primjer, koristi li se tehnologija isključivo za isporuku materijala i neposrednih uputa kako položiti ispit, imat će negativni učinak na kreativnost i dublje razumijevanje kao i na samoangažman studenata. Upravo zbog toga nastavnici trebaju upoznati studente s različitim pristupima poučavanju, materijalima i alatima koji potiču kreativnost i rješavanje problema. Napokon, u postupku planiranja, nastavnik mora započeti popisom željenih ishoda učenja i mogućih prepreka koje se mogu pojaviti tijekom učenja i realizacije procesa poučavanja. Na temelju toga mora odlučiti o metodama poučavanja i vrednovanja kao i gdje implementirati tehnologiju da bi se unaprijedilo učenje, postigli ishodi učenja i utjelovio proces učenja. Poduprijeti studentsku kreativnost i inovativnost mora biti bezuvjetni imperativ.

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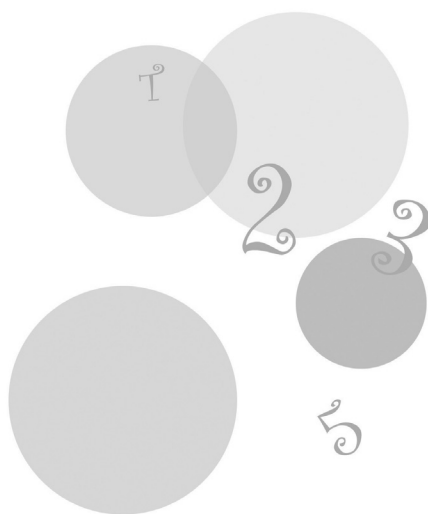
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3.

Kako prema poželjnim ishodima učenja na učiteljskim studijima



U trećem poglavlju monografije autori svojim člancima nude odgovore na pitanja *Kako prema poželjnim ishodima učenja matematike* i što je “školska” matematika. Neki autori poželjnije ishode učenja studenata učiteljskih studija vide kroz češću i osmišljeniju primjenu informacijskih tehnologija. Istražuju kakav je stav studenata učiteljskih studija prema korištenju ICT u nastavi, promišljaju o problemima, kriterijima i metodama odabira on-line materijala za nastavu matematike. Drugi izvještavaju o učinkovitosti originalnih pristupa i postupaka u nastavi matematike, kojima potiču studente prema višim razinama ishoda učenja. Suvremeni uvjeti učenja traže osposobljenog učitelja i učenika/studenta za učenje na daljinu i primjenu inteligentnih sustava te se predlaže u iznovljenim programima učiteljskih studija pronaći prilike za postizanjem tih kompetencija.

Med aksiomi in stvarnostjo

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Povzetek. Sodobno poučevanje in učenje matematike naj bi bilo usmerjeno v dvigovanje ravni matematične pismenosti. To ne pomeni, da se odrekamo matematiki ali njenim teoretičnim osnovam, pomeni veliko več: kar se pri matematiki učimo, naj bi sproti razumeli in logično umeščali med že usvojena znanja, hkrati pa gradili miselne povezave, potrebne za uporabo matematike v vsakdanjem življenju. Le tako se lahko izognemo nezadostnemu funkcionalnemu znanju matematičnih vsebin. Obenem moramo vedeti, da je matematika v funkciji razvoja matematične pismenosti pomembna za vsakogar, ne le za tiste, ki jim je matematika "všeč" in uživajo v njenih lepotah in eleganci.

Matematika je res abstraktna znanost, vendar jo lahko približamo mnogim dijakom z ustreznimi primeri oziroma modeli. Vsak dijak jo lahko doživlja v stvarnosti, bolj ali manj nazorni modeli pa ga usmerijo na pot k "aksiomom". Zavedati se moramo, da se zelo redki posamezniki znajdejo v abstraktnem svetu, velika večina podoživlja ta svet le v stvarni ponazoritvi.

Ključne besede: poučevanje in učenje matematike, matematična pismenost, abstraktni svet, stvarna ponazoritev

Uvod

Vprašati se moramo, s katerimi smotri in cilji upravičimo pouk matematike oziroma kateri so razlogi za to. Ali so učni načrti pravi? Ali imamo tehtne razloge za reformo pouka matematike in za drugačne pristope k pouku? Gotovo so nekateri temeljni razlogi za poučevanje matematike povezani z razvijanjem ustreznih funkcionalnih zmožnosti učeče populacije in v ozadju je premislek, kaj učeča populacija sploh pridobi v okviru pouka matematike.

Vprašanja o upravičenosti poučevanja matematike oziroma o vsebinah, ki naj bi se poučevale določenim skupinam ali na določeni stopnji šolanja, se le redkokdaj postavljajo. Opazna sta vsaj dva vidika poučevanja: eden je povezan s problemom preverjanja znanja ob zaključku določene stopnje šolanja (Jensen, Niss, Wedege, 1998), drugi pa z ravno matematičnih veščin oziroma s potrebo po višanju ravni veščin pri določeni populaciji (OECD, 1995; OECD, 2000).

Potrebe stvarnosti

Pri pouku matematike bi morali usvojiti "uporabno" matematiko oziroma matematiko, ki jo potrebujemo v vsakdanjem življenju. Najprej gre za zmožnost splošnega delovanja v družbi, torej za pridobitev nekaj temeljnih minimalnih znanj in spretnosti, ki bi jih morali usvojiti vsi učenci, razen morda nekaj otrok s specifičnimi motnjami. Vsak posameznik bi moral nadalje razviti kompetence, povezane z delom oziroma poklicem, ki ga opravlja. Vrste kompetenc, znanj in spretnosti so različne za različne poklice, večinoma jih učeča populacija pridobi v okviru specifičnega izobraževanja. Pravzaprav bi morali v šoli pridobiti osnovna razumevanja in sposobnosti, na katerih bi lahko gradili nadaljnja specifična znanja in spretnosti.

Nekaj ljudi potrebuje in želi usvojiti specifična znanja na višji stopnji. To ni cilj, ki bi si ga postavljalo veliko ljudi, vendar bi bilo potrebno motivirati čim več zlasti mlajših, da bi si postavljali dovolj visoke cilje. Nikakor pa ne sme ta cilj prevladovati in izkrivljati kurikulumu za matematiko v osnovni šoli.

V okviru pouka matematike ima velik pomen sam matematični problem, in sicer tako reševanje problema kot postavljanje problema. Res je bilo doslej veliko bolj poudarjeno reševanje problemov, pa še to v veliki meri rutinskih problemov, medtem ko je bilo postavljanje problemov zanemarjeno. Za razumevanje problemov kot zapisov problemskih situacij in za izgradnjo ustreznih strategij reševanja sta reševanje in postavljanje problemov neločljivo povezani. Z njima poiščemo "matematične" povezave med na videz različnimi problemi in pokažemo ustvarjalnost pri reševanju le-teh. S tem je neločljivo povezana tudi samozavest posameznika, ki z uspešnim reševanjem matematičnih problemov dokazuje znanje matematike, izkazuje sposobnost njene uporabe in zaupa v zmožnost usvajanja novih znanj in spretnosti. Z učinkovitim znanjem matematike lahko posameznik rešuje težje matematične probleme in se loteva novih izzivov.

Znanje matematike, ki ga pridobi posameznik v šoli, bi moralo zadoščati, da le-ta uspešno deluje v družbeni stvarnosti in da pripomore k izboljševanju svojega položaja in družbe kot celote. Posameznik bi moral zaznati, tolmačiti, oceniti in kritično presojati matematične vidike stvarnosti, pa naj bo to v ekonomskem, političnem ali socialnem smislu.

Ne smemo pozabiti na matematiko kot del kulture in zgodovine ter družbe nasplo. To pomeni razumevanje matematike v vsej njeni vlogi in ne le kot skupen znanj in spretnosti. Žal se tako v šoli kot v samostojnem učenju matematike največkrat le-to spoznava v smislu kopičenja takih in drugačnih formul in receptov, kar ne odseva veličine matematične kulture, ampak zgolj drobce, izkrušene iz matematičnega mozaika. Poznavanje zgodovinskega razvoja matematike, družbenega konteksta začetkov matematičnih pojmov, njene simbolike, teorije in problemov je pomembno že zaradi zavedanja razvoja človeške kulture, pomembno pa je tudi za motivacijo pri učenju oziroma poglobljanju znanja matematike.

Posameznik lahko šele po več letih učenja matematike spoznava matematiko kot disciplino. Zave se kvalitativnega in intuitivnega razumevanja neka-

terih pojmov, kot so simetrija, vzorec, neskončnost, struktura, paradoks, rekurzija, dokaz ipd. V matematiki najdemo veliko najglobljih, najmogočnejših in najvzemirljivejših idej, ki jih je razvilo človeštvo.

Matematično izobraževanje

Zavedati se moramo, da "šolska" matematika ni enolično definirana, odvisna je od družbenih vrednot in kulturnega okolja. Gotovo ni identična "čisti" matematiki, pač pa je neki izbor prirejenih matematičnih vsebin. Izbrano je tudi zaporedje, po katerem se usvajajo te vsebine, način poučevanja in preverjanja znanja.

Koristnost "čiste" in "šolske" matematike je v današnjem svetu daleč precejšena, zato argument koristnosti nima posebne teže pri upravičevanju njenega poučevanja skozi ves čas obveznega šolanja, čeprav je splošno privzeto, da je matematika močno prisotna s svojo družbeno uporabnostjo v skoraj vseh segmentih sodobnega življenja, od izobraževanja, gospodarstva, industrije itd. Praktična matematika se je stalno pomembno razvijala ob "čisti" matematiki. Tudi dandanes se matematične študije s področja računovodstva, zavarovalništva, managementa in informacijske tehnologije pridobivajo znotraj strokovnih institucij z zelo majhnim vložkom "čiste" matematike (Ernest, 2000).

Sodobna družba in sodobno življenje sta globoko matematizirana, na vsakem koraku srečujemo matematiko, marsikaj je urejeno z umeščenimi kompleksnimi numeričnimi in algebrničnimi sistemi, pa najsi bo to v veleblagovnicah, trgovanjem z delnicam, davčnem sistemu, sistemu socialnega varstva, izobraževalnem sistemu, industriji itd. S temi avtomatiziranimi sistemi opravljamo zelo kompleksne naloge zbiranja podatkov in izvrševanja določenih dejavnosti, s pomočjo matematike urejamo veliko vidikov našega življenja z zelo malo posegi v sisteme in pregledi le-teh potem, ko so vzpostavljeni. Kljub temu pa ni potrebe po nekem večjem oziroma poglobljenem znanju matematike, da bi lahko uspešno delovali v družbi, pri čemer seveda ne mislimo, da bi obvladovali te sisteme. Uspeh neke populacije na mednarodnem preverjanju znanja matematike gotovo ne zagotavlja gospodarskega in družbenega razcveta, je pa potrebna neka ozka elita, ki nadzira delovanje teh sistemov, in skupina strokovnjakov, ki jih servisira. Cilje splošnega matematičnega izobraževanja zato ne smemo postaviti glede na specifične potrebe te zelo majhne skupine ljudi.

Kot kaže, večina ljudi ne potrebuje poglobljenega znanja matematike, manjšina, ki uporablja znanja matematike, pa si le-ta večinoma pridobi izven šolskega izobraževanja. Gre za neke vrste "paradoks pomembnosti", saj se v družbi sočasno izkazuje objektivna pomembnost in subjektivna nepomembnost matematike (Niss, 1994). Čeprav družba postaja vedno bolj matematizirana, se s tem matematiziranjem ukvarja le peščica ljudi. Zmotno je torej poudarjati, da bi morali vsi pridobiti kar največ znanja matematike zaradi vedno večje matematiziranosti družbe nasploš.

V drugi polovici 20. stoletja se je težilo k vzpostavljanju boljšega izobraževanja, svetovna javnost je postala pozorna na problem nepismenosti in iskala poti za izkoreninjenje le-tega. Ob tem je bilo izpostavljeno tudi učenje in

poučevanje matematike, s katerim bi višali raven matematične pismenosti. Znano je gibanje "matematika za vse". Velika večina učencev pa ne dosega pričakovane ravni znanja in razumevanja matematike ter ne razvije ustreznih spretnosti glede na načrtovan učni načrt. Zaradi tega so vedno glasnejša razmišljanja o diferenciranem puku matematike, t. i. "resno poučevanje matematike" naj bi omejili na majhno skupino učencev, ki bi poglobljala znanje matematike oziroma razvijala matematične kompetence že z razumnim vložkom svojega truda in časa (Niss, 2002). To seveda ne pomeni, da za veliko večino učencev ne potrebujemo pouka matematike, pač pa da so zanje potrebni drugačni pristopi pri usvajanju in utrjevanju znanja matematike ter višanju ustreznih kompetenc, ki pripomorejo k višanju ravni matematične pismenosti.

V proces pouka matematike bi morali vključiti tudi elemente razumevanja in zavedanja matematike, ne le razvijanja sposobnosti. Jezik pouka matematike je poln ukazov, učenec mora opraviti zdaj to, nato ono – praktično ne srečamo pogovora o postopku, vzrokih za izbrano pot reševanja problema, razumevanju nekega pojma ipd. Postavlja se sicer vprašanje, kaj pomeni razumevanje in zavedanje matematike. Ernest (2000) navaja elemente, ki naj bi jih vsebovala razumevanje in zavedanje matematike, med njimi:

- kvalitativno razumevanje nekaterih temeljnih matematičnih pojmov, kot so neskončnost, simetrija, struktura, rekurzija, dokaz, kaos, naključnost itd.,
- razumevanje glavnih vej in konceptov matematike ter občutenje njihove povezanosti in soodvisnosti, pa tudi enotnosti matematike,
- zavedanje, kako matematično mišljenje pronica v vsakdanje življenje in kje vse je prisotno, čeprav ne govorimo o matematiki,
- kritično razumevanje uporabe matematike v družbi: prepoznavanje, tolmačenje, vrednotenje in kritično ocenjevanje umeščenosti matematike v družbene in politične sisteme,
- zavedanje zgodovinskega razvoja matematike, družbenega konteksta njenih začetkov, simbolike, teorij in problemov,
- občutenje matematike kot osrednjega elementa kulture, umetnosti in življenja, ki je danes tako kot v preteklosti tesno povezana z znanostjo, tehnologijo in vsemi vidiki človeške biti.

Nikakor pa ne bi smeli zanemariti vrzeli med postavljenimi cilji pouka matematike in učinkom, ki se izkazuje kot rezultat le-tega. Najsi bodo cilji pouka še tako plemeniti, visoko leteči ter postavljeni s takimi in drugačnimi nameni, na koncu mora biti preverjeno, ali so bili doseženi. Vsak premislek o učnem načrtu za matematiko mora vključevati tri ravni:

- načrtovan oziroma planiran učni načrt,
- sprejet oziroma izveden učni načrt in
- "usvojen" učni načrt, to je učenčeve pridobitve in učne izide.

Prav ugotovitev, kateri cilji pouka matematike so doseženi v šolski praksi, bi morala prvenstveno vplivati na sam pouk matematike. Ker je poučevanje načrtovana dejavnost, bi morali biti izraženi cilji tesno povezani z realizacijo ciljev poučevanja.

Če te povezave ni, se pojavlja neko neravnovesje in neskladnost, ki se izkazuje v nelagodju učiteljev in učencev.

Koliko abstrakcije?

Še vedno prevladuje prepričanje da so matematične resnice univerzalne, neodvisne od človeka, njegove kulture in vrednot – bile naj bi odkrite, ne izumljene. Dejansko pa tega ne moremo jemati kot temelja matematike – matematika se je vendarle gradila v družbenem okolju in je izdelek kulture ter zmotljiva kot vsaka druga veja znanosti. Družbeni konstruktivizem poudarja družbeni in kulturni izvor matematike ter postavlja upravičenost matematičnega znanja na njegove kvazi-empirične temelje. Potrebno je navesti dve predpostavki družbenega konstruktivizma:

- predpostavko realnosti: obstoj trdnega fizičnega sveta, kot nam narekuje zdrava pamet, in
- predpostavko družbene stvarnosti: vsaka razprava predpostavlja obstoj človeške vrste in jezika.

S predpostavkama o obstoju družbene in fizične stvarnosti lahko postavimo načela družbenega konstruktivizma tako, da prvima načeloma, ki veljata za osnovni načeli konstruktivizma, dodamo še štiri:

- znanja ne pridobivamo na pasiven način, pač pa ga učenec aktivno gradi,
- funkcija spoznavanja (kognicije) je prilagodljiva in služi pri organizaciji izkustvenega sveta, ne pri odkrivanju ontološke stvarnosti,
- osebnostne teorije, ki izhajajo iz organizacije izkustvenega sveta, se morajo ujemati z omejitvami, ki izhajajo iz fizične in družbene realnosti,
- teorije so podvržene ciklusu teorija – napoved – test – neuspeh – prilagoditev – nova teorija,
- nastajanje družbeno dogovorjene teorije o svetu, družbeni vzorci in pravila o uporabi jezika,
- matematika je teorija oblik in struktur, ki nastane znotraj jezika (Ernest, 1998).

Matematični koncepti torej nastajajo z abstrakcijo neposredne izkušnje v fizičnem svetu, s posploševanjem in reflektivno abstrakcijo predhodno izgrajenih konceptov, v iskanju smisla ob pogovoru z drugimi ter z različnimi kombinacijami teh načinov. Matematika je veja znanosti oziroma človekovega znanja, tesno povezana z drugimi. Jezik omogoča oblikovanje teorij o družbeni in fizični stvarnosti, le-te pa so z dialogom med ljudmi in interakcijo s fizičnim svetom podvržene izboljševanju – boljšem ujemanju z omejitvami, ki izhajajo iz fizične in družbene stvarnosti. Matematika je soudeležena v tem izpopolnjevanju, tudi ujemanje matematičnih struktur na področjih izven matematike se stalno preverja. Matematika se tako razvija ob reševanju praktičnih problemov družbe in t. i. nerazumna učinkovitost matematike ni neki slučajni čudež, pač pa posledica empiričnih in jezikovnih zametkov ter funkcije matematike (Ernest, 1998).

Potrebno je izpostaviti, da je poučevanje matematike zelo odvisno od učitelja, njegovih prepričanj in njegovega pojmovanja matematike. Pojmovanje, kaj matematika sploh je, pa vpliva na pojmovanje, kako bi se morala predstaviti. Način, po katerem jo nekdo predstavlja, kaže na njegovo mnenje o tem, kaj je v matematiki najbolj bistveno. Odgovor na vprašanje, kateri je najboljši način poučevanja, je torej v resnici podan v odgovoru na vprašanje, za kaj gre pri matematiki (Hersh, 1986).

V razni literaturi o poučevanju in učenju so poudarjeni situacijski faktorji, ki pogojujejo tako učiteljeve kot učenčeve izkušnje v procesu izobraževanja oziroma pri pouku matematike (Cole, 1990). V posamezni državi oziroma skupnosti so prisotne vrednote, prepričanja in tradicija določenega izobraževalnega sistema, ki se izražajo v sprejetem učnem načrtu, izobraževalni praksi, organizaciji šolskega sistema, pa tudi v pričakovanjih učencev, učiteljev, staršev, vodstev šol in drugih zainteresiranih subjektov.

Kdaj znamo matematiko

Kot smo že poudarili, je odgovor na vprašanje, zakaj naj bi se učili matematiko, odvisen od vloge, ki jo ima matematika v družbi, in od same filozofije matematičnega izobraževanja. Če upoštevamo izobraževalni, družbeni in politični vidik, lahko (uradne) razloge za matematično izobraževanje razdelimo v tri skupine; matematično izobraževanje namreč:

- prispeva k tehnološkemu in socialno-ekonomskemu razvoju širše družbene skupnosti,
- prispeva k političnemu, ideološkemu in kulturnemu vzdrževanju in napredku družbe,
- zagotavlja pogoje, da posamezniki uspešno delujejo v različnih družbenih sferah: izobraževanju, poklicu, zasebnem in družabnem življenju ter kot državljani nasploh (Niss, 1996).

Ko analiziramo cilje matematičnega izobraževanja kot odsev potreb družbe, se pojavijo težave pri odkrivanju in identificiranju dejanskega učinka matematičnega izobraževanja na te potrebe. Večina raziskav s področja matematičnega izobraževanja sloni na predpostavki, da matematično izobraževanje vsebuje neke "notranje" vrednote, pri čemer je mišljen pomen, ki naj bi se izkazoval s pozitivno vrednostjo matematičnega izobraževanja, zajamčeno že z dejstvom, da gre za izobraževanje na področju matematike (Skovsmose, 2006). Skovsmose (2006) izpostavlja to predpostavko kot zagotavljanje, da "edukatorji matematike" lahko delujejo kot "ambasadorji" matematike in z gotovostjo je delovanje usmerjeno v splošno dobro.

Rezultat matematičnega izobraževanja naj bi se vendarle izkazoval z nekim znanjem matematike. Kaj pa pravzaprav pomeni, da nekdo zna matematiko? Namesto pojma matematično znanje se bolj pojavlja izraz matematične kompetence, ko želimo celovito predstaviti "izhodno stanje" posameznika. Ernest (2004) razlikuje med "koristnostjo" in "relevantnostjo":

- Koristnost pomeni ozko pojmovano uporabnost, ki se pokaže takoj ali v kratkem času, ne da bi upoštevali širši kontekst ali dolgoročne cilje. Utilitaristično izobraževanje prikaže matematiko kot ozko tehnično vedo, izolirano od vrednot in družbenega konteksta.
- Relevantnost je vidna kot relacija med tremi pojmi (S, P, C), pri čemer je S situacija, dejavnost ali objekt, kateremu je pripisana ta relevantnost, P je posameznik ali skupina ljudi, ki pripisuje relevantnost situaciji S , C pa je cilj, ki pooseblja vrednote posameznika oziroma skupine v tem primeru. Objekt S je torej relevanten, če ga tako ocenjuje posameznik P pri doseganju cilja C . Primer: “šolska” matematika (objekt S) je po mnenju mnogih politikov in odgovornih za izobraževanje (skupina P) relevantna zaradi razvijanja matematičnih kompetenc in spretnosti, potrebnih za tehnološke poklice (cilj C).

V iskanju odgovora na vprašanje, kaj pomeni znati matematiko, je Ernest (2004) razlikoval več različnih ciljev, ki naj bi jih dosegli z učenjem in poučevanjem matematike, in sicer: utilitaristično znanje, praktično znanje, povezano s službo, strokovno znanje na višji stopnji, razumevanje matematike, matematična samozavest in razumevanje same matematike kot del kulture. Ti cilji niso med seboj izključujoči in niso nujno pomembni oziroma potrebni za vse učence, kot celota pa s pripadajočimi sposobnostmi oziroma zmožnostmi predstavljajo “matematično kompetentnost”.

Kdor bi usvojil vse te cilje, bi bil matematično kompetenten, morda bi tedaj rekli, da zna matematiko (Wedge, 2009). Wedge (2009) predstavlja utilitaristično znanje in praktično znanje, povezano s službo, kot zmožnosti “narediti nekaj” (znati, kako), torej koristni za učence in študente kot bodoče zaposlene. Strokovno znanje na višji stopnji je za nekatere koristno, za druge relevantno, medtem ko je razumevanje matematike relevantno za vse (vedeti, da). Matematična samozavest in razumevanje same matematike kot del kulture predstavljata afektivno in družbeno razsežnost matematične kompetentnosti.

Matematika v realnem svetu

Sodobno poučevanje in učenje matematike naj bi bilo bolj povezano z vsakdanjim življenjem. Ena ključnih komponent naj bi bila samostojna izgradnja pojmov in konceptov preko reševanja problemov “realnega” sveta, s čimer postavljamo “naravno” vez z matematiko. Z oblikovanjem modelov, ki so uzaveščeni v smislu matematiziranih modelov in hkrati prilagojeni realni situaciji, podpiramo matematično mišljenje. Ta močna vez z realnim svetom zagotavlja uporabnost pridobljenega znanja, saj odkrivamo realnost kot vir in kot področje uporabe. Otrok ob tem uporablja vse svoje specifične zmožnosti v praksi.

Model, ki je nastal iz konteksta, ima jasen didaktični pomen in učencu veliko pomeni, saj je osmišljena reprezentacija matematičnega pojma ali koncepta. Primer modela številskega poltraka, ki se nadgradi v model realne osi, ima smisel, če ga učenec usvoji kot reprezentacijo naravnih in celih števil, kasneje kot reprezentacijo

racionalnih in končno realnih števil. Zaradi precejšnje abstraktnosti, čeprav gre za dokaj nazoren grafični prikaz, ga najbrž učenci ne razumejo v začetnih letih šolanja, ko šele usvajajo pojem števila kot količine. Učitelj mora biti previden pri uporabi modelov, saj je vsiljevanje ponazoril, ki jih učenci ne povežejo s pojmom, ki naj bi ga ponazarjala, nesmiselno in celo škodljivo.

Primeren model je običajno most med neformalnim in formalnim znanjem ter podpira proces vertikalne matematizacije. Freudenthal (1991) omenja horizontalno matematizacijo kot povezavo med vsakdanjim svetom in svetom simbolov, medtem ko naj bi bila vertikalna matematizacija "gibanje" znotraj sveta simbolov. Poudarja, da ni ostre ločnice med obema in da sta tako horizontalna kot vertikalna matematizacija pomembni za razvoj osnovnih matematičnih pojmov in konceptov ter formalnega matematičnega jezika.

Proces učenja se začne z realističnim problemom. Z dejavnostmi znotraj horizontalne matematizacije učenec zgradi neformalni ali formalni matematični model. Z reševanjem problema, primerjanjem postopkov reševanja in tolmačenjem le-teh je učenec v fazi vertikalne matematizacije, ki se zaključi z rešitvijo problema. Tolmačenje rešitve in strategije reševanja pokaže uporabo pridobljenega matematičnega znanja.

Model je pomemben del učenčevega matematičnega razvoja, saj lahko isti model uporabi v različnih kontekstih. Učitelj bi se moral vedno zavedati pomena, ki ga ima model, obenem pa tudi dejstva, da vsi učenci ne zgradijo identičnih modelov. Pouk bi moral biti naravnano tako, da bi ustrezal vsem učencem, ne glede na to, kakšen model so zgradili za posamezen matematični pojem oziroma koncept, če je le ta model res v skladu z matematičnim pojmom oziroma konceptom.

Ker so le nekateri učenci zmožni abstraktnega mišljenja, so le njihovi modeli lahko "dovolj" abstraktni. Ti učenci so na primerni stopnji razvoja na primer sposobni razumeti pojem neznanke, črkovne oznake za števila ipd. Učenci, ki ne zmorejo take abstrakcije, v bistvu nikoli ne znajo "računati" s črkovnimi oznakami, pač pa zgolj reproducirajo posamezne naučene postopke. Ker ne najdejo povezave med matematiko, ki jo (po)znajo, in kontekstom, ki ga ne razumejo, lahko le memorirajo pravila in procedure, ki zanje nimajo nobenega smisla.

Za učence, ki so sposobni abstrakcije, lahko izhajamo tudi iz realističnih problemov, ki niso neposredno vezani na stvarno situacijo, saj jim abstraktni svet ni tuj in imajo predstav o njem. Veliko drugih učencev lahko razvije znanja in sposobnosti, ki jim omogočajo uspešno profesionalno in osebno rast, seveda pa je zanje matematični svet vedno povezan s stvarno ponazoritvijo.

Sklep

Na koncu je potrebno poudariti, da je kvaliteten pouk matematike eden izmed najpomembnejših faktorjev, ki vplivajo na učenčeve in dijakove dosežke oziroma znanje pri matematiki. Če želimo imeti dober pouk, moramo kvalitetno izobraziti in doizobraževati učitelja. Tudi za profesorja matematike, ki bo uspešno razvijal pri

učencu matematično znanje in pismenost, je nujno, da najprej sam razume matematiko, pri čemer ni mišljena le sposobnost rokovanja z vsebinami matematike. Globlje mora razumeti koncepte in procese ter le-te logično razložiti z uporabo ustreznega matematičnega jezika in primerov.

Zmožnost (bodočega) učitelja, da poišče več reprezentacij posameznega pojma ali vsebine ter da uvidi in nazorno pokaže povezave med različnimi matematičnimi predstavami ali zamislimi, je tesno povezana z učiteljevo samozavestjo oziroma zaupanjem v lastno znanje matematike. Pozitivne izkušnje z matematiko skupaj z ustreznim zaupanjem v lastno znanje ugodno vplivajo tako na uspešne predstavitve matematičnih vsebin na nastopih, ki jih imajo bodoči učitelji matematike, kot tudi na kakovostno poučevanje matematike. Tudi bodoči profesor matematike bi torej moral imeti v okviru študija najprej možnost razumeti matematiko in se šele nato usmeriti v didaktiko matematike. Mnogi študenti imajo namreč zakoreninjena taka prepričanja o matematiki in o učenju, s katerimi bi zelo težko uspešno poučevali matematiko.

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Szép kövezések, szép matematika!?!

Emil Molnár

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Összefoglaló. Iskolás füzeink “kockás papírja”, az utcák (fekete-fehér) négyzetes kövezései felvethetik a következő játékos problémát: *Hogyan alakítsuk át a négyzetháló oldalszakaszait, úgyhogy az így kapott oldalak továbbra is egyformák (egybevágók) maradjanak?* Pontosabban fogalmazva megkívánjuk, hogy az új minta minden egybevágósága, mely egy tetszőleges oldalt egy másik ilyenbe visz, képezze a (végtelenbe kiterjesztett) mintát önmagára (hagyja változatlanul).

Kiderül, hogy pontosan 14 ilyen négyszögminta-típus van, néha két különböző alakú (pl. fekete és fehér) négyszöggel. Az ilyen kövezések gyűjteménye nagyon szép lehet, esetleg hasznos is utcáink, lakásaink díszes kövezésére.

Vázolommajd a probléma megoldását, mely igényes és fontos matematikai fogalmakhoz vezet (sokszög-kövezés súlyponti háromszögelése, szomszédsági műveletek, szomszédsági mátrix, a kövezés szimmetriacsoportja, D-szimbólum, stb.). Mindezek szórakoztató módon is tárgyalhatók, például egy középiskolai matematika-szakkörben, vagy egyszerűsítve, látványos ábrákkal az általános iskolában is.

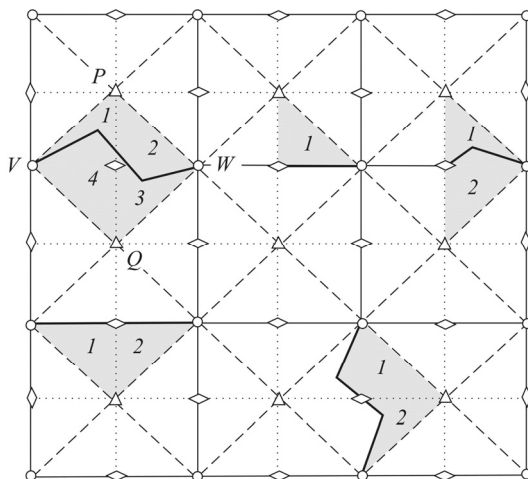
Prok István kollégám [P] nagyon szép programot készített az euklideszi sík kristálycsoportjairól interaktív felhasználásra (szabad letöltésre), 3–5. ábráink ezzel készültek

Az ilyenfajta euklideszi síkbeli kövezések teljes osztályozása (nemcsak négyszögekkel) az egyetemistáknak is érdekes lehet, talán a tisztelt Hallgatóságnak is az. Ezért néhány hivatkozást is megadok, ahol még továbbiak is vannak.

Bevezetés

Az összefoglalóban megfogalmazott problémát hosszabb kísérletezéssel vezethetjük be. Négyzethálós “kockás” papírral próbálkozva (az 1. ábra méretében), a tanulók jó és hibás mintákat készíthetnek, akár egyénileg, akár csoportos munkával. Fokozatosan közelítsenek a helyes ábrákhoz. A tanár minél kevesebbet segítsen, de a hibákra mutasson rá. Valószínűleg a második vagy harmadik foglalkozáson juthatnak el a tanulók ahhoz a gondolathoz, melyet most már tömören megfogalmazunk (lásd bővebben és részletesebben az [M94] dolgozatban tudományos szinten).

Az 1. ábrán a hagyományos négyzethálós papíron Δ jelek mutatják a (2-dimenziós) négyzetlapok középpontjait, $\diamond$ jelek mutatják az (1-dimenziós) élek felezőpontjait, a (0-dimenziós) csúcsokat $\circ$ jelöli. Ezekkel szemben rendre bevezetjük a ——— folytonos, - - - - - szaggatott, pontozott háromszögszoldalakat. Ezzel megkapjuk a négyzetháló úgynevezett baricentrikus (súlyponti) háromszögfelbontását, melyet C -vel jelölünk. Ez a fogalom fontossá válik a négyzetháló leírásánál (de később a sokszögekkel történő kövezéseknél, sőt akár görbevonalú idomokkal való "topológikus" kövezéseknél is; megfelelő módosításokkal persze [LMS94]).



1. ábra. A négyzetháló baricentrikus (súlypontok szerinti) felosztása. 4 baricentrikus háromszög $VQWP$ éltartománya. Az éltartomány szimmetriáinak jellemzése kisebb alaptartományokkal.

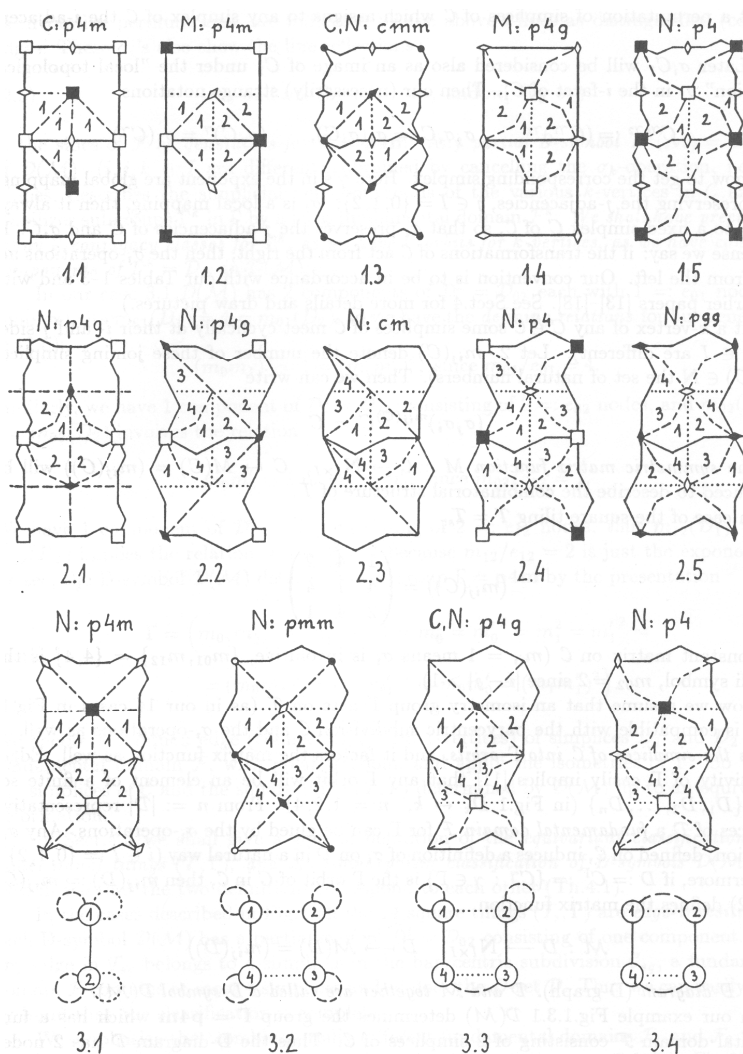
4 baricentrikus háromszög (1, 2, 3, 4 az 1. ábra bal felső részén) támaszkodik bármely ——— élhez mindkét oldalról. Ezek képezik a $VPWQ$ éltartományt, melynek fontos szerep jut a lehetséges élmódosítások jellemzésében. Ugyanis egy élet szinte szabadon rajzolhatunk a $VPWQ$ belsejében a V és W csúcsok között. Ezután jön egy síkbeli *egybevágóság-csoport* (mint ismétlődési szabály), mely az éltartománnyal – benne a módosított éllel –, mint *alaptartománnyal* (ismétlődő egységgel) kikövezi a teljes négyzethálót (lássuk ezt a 2. ábrán: 2.3, 2.4, 2.5, 3.2, 3.3, 3.4).

De ez a módosított él szimmetrikusan is választható, pl a középpontjára nézve (az 1. ábra jobb alsó képén, 2. ábra: 1.3, 1.5.). De lehet önmagára is szimmetrikus (1. ábra bal alsó képe, 2. ábra: 1.2, 1.4). Itt az él nem változhat, de a félél-tartományba rajzolt címkével jelezhetjük a helyzetet. De lehet a módosított él szimmetrikus az eredeti élfelező merőlegesre (1. ábra jobb felső képe, 2. ábra: 2.1, 3.1). Továbbá lehet szimmetrikus mindkét utóbbi tüköregyenesre (1. ábra felső középső képe, 2. ábra: 1.1), de ez éppen az eredeti négyzethálót jelenti.

Hátra van még az a feladat, hogyan helyezzük el a fenti éltartományokat szabályosan (ahogy az összefoglalóban is megfogalmaztuk), tehát hogy "topológikus négyszögekövezést" kapjunk, úgyhogy bármely két új élet kiválasztva létezzen

a síknak olyan egybevágósága, mely az első élet a második élre, a teljes síkbeli kövezést önmagára képezze.

Ehhez az utolsó lépéshez **szükségünk van** a 17 síkbeli kristálycsoport ismeretére, ahogy azt a [P] program feldolgozza, vagy **legalább azoknak a lehetőségeknek az áttekintésére, hogy az éltartomány vagy fenti kisebb részei** (1. ábra) **alaptartomány (ismétlődő egység) legyen.** Ehhez a feladathoz az éltartomány négyszügnei (vagy fenti részeinek) csak nem sok oldalpárosítását kell áttekintennünk.



2. ábra. Az egyforma élű négyszög-kövezések 14 családja: 1. Egyforma kövek élszimmetriával; 2. Csúcsszimmetriájú egyforma kövek; 3. Különböző (fekete-fehér) kövek D-diagrammokkal.

Ez azt jelenti, hogy *a vizsgált problémának “elméleti hozadéka” is van. Megismerünk vele néhány síkbeli kristálycsoportot* (7-et a 17 közül). Ambíciózus tanárkollégáknak ez vonzó kísérlet lehet, akár középiskolai, akár általános iskolai matematika szakkörben.

Összeállítottam az *éltranzitív* (homogén, egybevágó élű) *négyszögkitöltések* teljes listáját az [M94] dolgozathoz, ahol az általánosabb, tudományosan is újdonságot jelentő *térbeli kockakitöltések analóg problémáját is láthatjuk*. Az Olvasó további síkbeli általánosítást talál a [LMS94] dolgozatban.

A megoldás

A 2. ábrán láthatjuk az *éltranzitív* *négyszögkitöltések* 14 típusát, ahol a számozott baricentrikus háromszögekkel jellemzett éltartományok végül is két *négyszögtípushoz* vezethetnek. A listát 3 részre: 1 – 3 osztottuk.

Az 1. osztály tagjai: 1.1 – 1.5, ahol az *él* szimmetriacsoportja (másképpen mondva, az élközéppont stabilizátor csoportja) a két szomszédos *négyszöglapot* egymásba viszi.

A 2. osztály is 5 kövezésből áll: 2.1 – 2.5, ahol ugyancsak egyforma *négyszöglapokat* kapunk, de nem az *élstabilizátorral*.

A 3. osztálynak 4 meglepő kövezése van: 3.1 – 3.4 két (mondjuk fekete-fehér) *négyszögtípussal*. Ezek nem képeződnek egymásra, csak az *élek*.

Mindegyik kövezésnél feltüntettük, hogy *konvex* (C), *nem-konvex* (N) *négyszögekkel* megvalósíthatók-e (néha mindkettő előfordulhat, gondoljuk meg!). Ezenkívül a kövezés *kristálycsoportja* is szerepel (7 csoport a 17 közül).

D-szimbólumokról. Illusztráció számítógéppel rajzolt képekkel

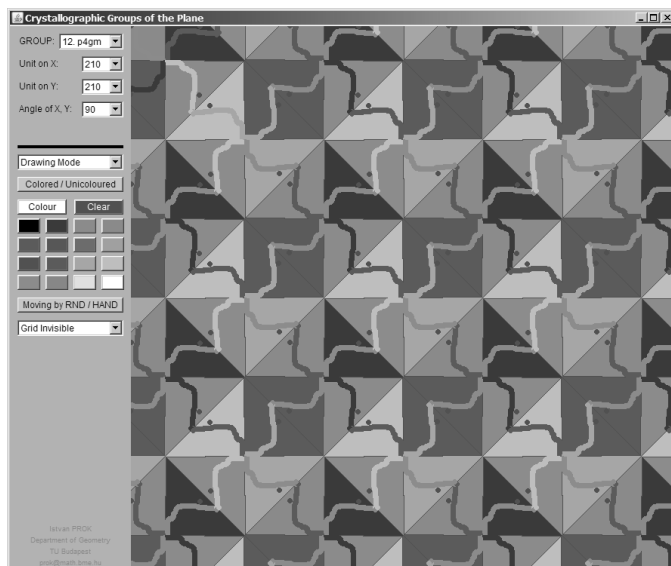
A 3. osztály mindegyik képénél *feltüntettük a később* (D, M) *-mel jelölt u.n. D-szimbólumot*, mely egy D szomszédsági diagrammal (szomszédsági gráf) nagyon fontossá vált (B.N. Delone, M.S. Delaney, A.W.M. Dress kezdeményezésére). (Keresse meg az Olvasó a többi kövezés D -szimbólumát is!.)

Ez az absztrakció következő szintjét jelenti, az egyetemi hallgatóknak, vagy egy tudományos konferencia hallgatóságának figyelmét hívhatjuk fel rá. Természetesen itt csak jelezhetjük ezt a fontos kutatási módszert. A sík másfajta kövezéseit, vagy – ami tudományosan is még fontosabb – szabályos kövezéseket kereshetünk vele *homogén terekben* (például állandó görbületű terekben, így az E^3 euklideszi tér [M94], [DHM93] csak speciális eset), *ahol még sok megoldatlan probléma van.*

Minden D -diagrammhoz, D -hez (2. ábra: 3.1 – 3.4) tekintjük a négyzethálós C -vel jelölt baricentrikus háromszögelését (1. ábra). Az éltartomány minden egyes háromszögét (legfeljebb 1, 2, 3, 4) egy csúcs jelöli a D diagrammban. A pontozott élék, most a C és $D\sigma_0$ operációját is jelölik. A szaggatott - - - - élék a C és $D\sigma_1$ operációját, a folytonos ——— élék a C és $D\sigma_2$ operációját tüntetik fel, mint

a D diagramm szomszédságait (a D gráf címkézett vagy színezett éleit), ugyanúgy ahogy a C megfelelő baricentrikus háromszögei szomszédosak.

Vegyük észre az 1. ábrán, hogy az éltartomány vagy a kisebb alaptartományok képeit is meghatározzák majd ezek a szomszédsági operációk.



3. ábra. Prok István programja színes vagy fekete-fehér minták rajzolását kínálja. A [P] honlapról szabadon letölthető.

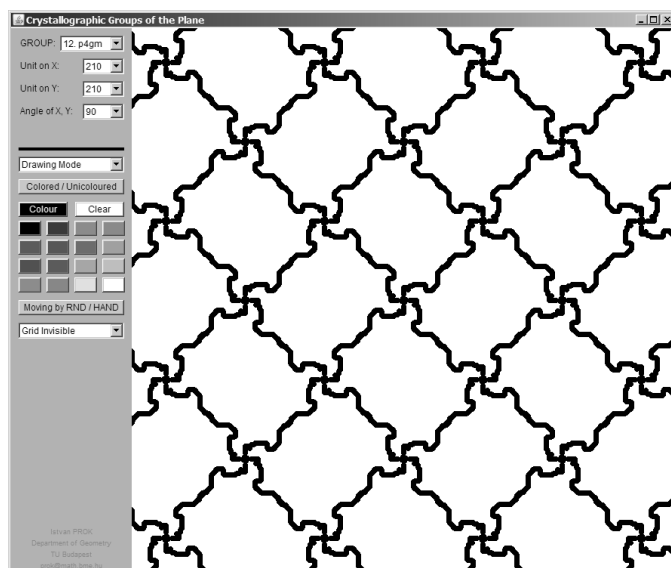
Ez azt is jelenti majd, hogy D a *keletkező kövezés* Γ -val *jelölt teljes szimmetriacsoportját meghatározza*, ha még megadjuk azt a (szimmetrikus) u.n. szomszédsági mátrix függvényt, $M = m_{ij} - t$, mely $D - n$ lesz értelmezve, és megmondja, *hány i és j oldalú baricentrikus háromszög* (majd ennek a fele szerepel a mátrixban) *veszi körül a megfelelő k -jelű háromszögcsúcsot a C -ben és ennek megfelelően D -ben: $\{i, j, k\} = \{0, 1, 2\}$* . Ez azt is jelenti majd, hogy a D diaszgramm bármely eleme (csúcsa) a C baricentrikus felbontás egy háromszög-osztályának, egy a Γ csoport szerinti pályájának (orbit-jának) felel meg. Úgyhogy megkívánjuk: Γ gartsa meg a C felbontás szomszédsági viszonyait.

Most az M szomszédsági mátrixfüggvény állandó $\begin{pmatrix} 1 & 4 & 2 \\ 4 & 1 & 4 \\ 2 & 4 & 1 \end{pmatrix}$

a D és a C minden eleméhez (baricentrikus háromszögehez), ahol a négyszögműkövezésben 4 négyszög találkozik minden csúcsban.

Példaként tekintsük a 2. ábra: 3.3 képét és diagrammját, ahol a szomszédsági operációkat involutív (másodrendű) permutációként (ez általánosan így van) adhatjuk meg:

$\sigma_0 : (12)(34)$; $\sigma_1 : (1)(2)(34)$ as the loops show it at vertex 1 and 2; $\sigma_2 : (14)(23)$.



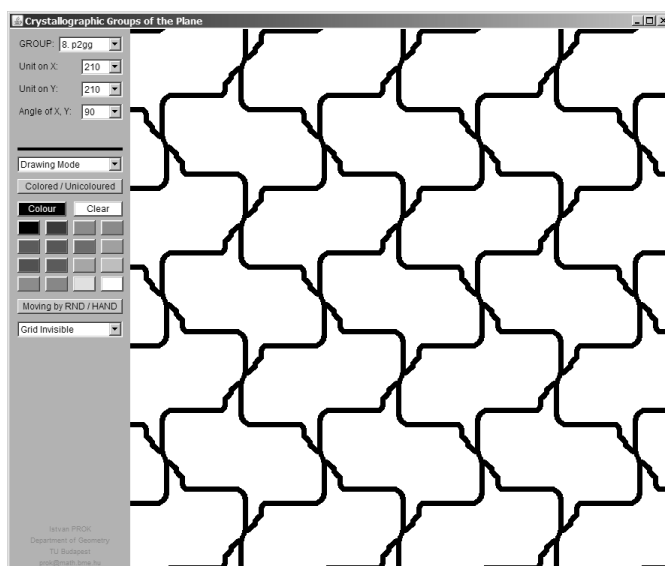
4. ábra. Fekete-fehér minta az előbbi 12. **p4gm** csoport szerint.

Itt $(\sigma_1 \sigma_0)$ az 1, 2 C-beli háromszögekkel, mint lapcentrum (2-centrum) körül egy Γ -ra vonatkozó tükörsarkot jelent, $(\sigma_1 \sigma_0)$ a 3, 4 C-beli háromszögekkel, mint lapcentrum (2-centrum) körül egy Γ -ra vonatkozó 4-forgást jelez az $m_{01} = m_{10} = 4$ mátrixelemek miatt, (gondosan ellenőrizzük!). A $(\sigma_2 \sigma_1)$ kompozíció (mindig ellentétes sorrendű végrehajtást jelent mostani megállapodásunk szerint) egyenestükrözést jelent az 1, 2, 3, 4 C-beli háromszögek közös csúcsa (0-centruma) szerint, megintcsak az $m_{12} = m_{21} = 4$ mátrixelem miatt (ellenőrizzük!). A $(\sigma_2 \sigma_0)$ kompozíció a Γ -ban triviális stabilizátort jelent, ahogy az 1, 2, 3, 4 háromszögek élcentrumai (1-centruma) körül leolvashatjuk, újracsak $m_{02} = m_{20} = 2$ miatt. Tehát az éltartomány éppen a **p4g**, bővebb jelöléssel **p4gm**, csoport alaptartománya lesz. Ezt az eljárást algoritmizálhatjuk, programozhatjuk számítógépre, ahogy ez általánosabban a térre is megtörtént a [DHM93] dolgozatban.

A $\Gamma = \mathbf{p4g}$ csoport szerinti másik éltranzitív kövezést mutat a 3. ábra, mely a [P] programmal készült (lásd még a 2. ábra: 2.1 képét), ahol egy élmotívumot rajzoltunk a Γ csoport alaptartományába. A színezett képbe rajzolás nagyon tetszetős és szórakoztató lehet.

Ez a módszer az általános. Mind a 14 típust megrajzolhatjuk ezzel a stratégiával, továbbá másfajta éltranzitív (homogén egybevágó élű) kövezéseket is [LMS94].

*Befejezésül: két (nem túl szép) fekete-fehér számítógépes rajz, melyet a szerző készített [P] segítségével (4-5. ábrák). Az elsőt éppen az előbbi mintára rajzoltam a $\Gamma = \mathbf{p4g}$ (Fig. 4) csoport szerint. A második kép (5. ábra) a **pgg** csoport szerint készült (keressük meg a 2. ábrán!).*



5. ábra. A 8. **p2gg** csoport szerinti minta.

Folytassák a rajzolást, kérem!!!

Szeretném **Dr. Szirmai Jenő** kollégámnak megköszönni a dolgozat elkészítéséhez adott segítségét.

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Upotreba tehnologije za unaprjeđenje nastave matematike: perspektiva, problemi, kriteriji i metoda odabira

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Sažetak. Već je u osamdesetim godinama prošlog stoljeća prepoznato da “se nalazimo na pragu revolucije u matematici – kako o njoj razmišljamo, kako je koristimo i kako je možemo učiti. Revolucija je usmjerena na računalo kao matematički alat. Kao kod svake revolucije, ljudi odabiru strane – neki bi htjeli istražiti ulogu računala u matematici, dok drugi smatraju da je korištenje računala u matematici prijevara” [12]. Istraživanja pokazuju da je unatoč dostupnosti hardvera i softvera matematičko osoblje u srednjim školama rijetko koristilo računala u poučavanju matematike [15]. Iznosimo neke od razloga takve situacije. Nadalje, svjesni smo da “razvoj matematike leži na razvoju tehničkih znanosti, biomedicine, ekonomije i informacijskih tehnologija. Studenti u tim područjima trebaju matematičko obrazovanje ali njihovo predznanje, sposobnosti i stavovi prema matematici se naveliko razlikuju” [8]. U radu prezentiramo neke resurse koji su dostupni na Internetu i koji bi mogli unaprijediti nastavu matematike. “Kako Internet postaje veći rastućom stopom, nemoguće je napraviti preciznu sliku stanja njegovog razvoja” [7], pa tako ni razvoja takvih materijala. U radu uvodimo metodologiju odabira materijala definiranjem kriterija i metode odabira najboljeg prikladnog materijala. Kriteriji osiguravaju bolje ostvarivanje ishoda učenja i smanjenje rizika napuštanja materijala jer nisu prikladni za nastavnike iz razloga što su presloženi ili vremenski prezahtjevni za korištenje.

Ključne riječi: nastava matematike, korištenje tehnologije u nastavi, odlučivanje

Uvod

Internet i tehnologija nude nove materijale za poučavanje, programske pakete i ostale resurse koji se mogu koristiti u nastavi da bi se pojačalo studentsko učenje i svladavanje gradiva. Često se raspravlja da li tehnologija, globalno gledajući, obogaćuje proces poučavanja u smislu boljeg ostvarivanja ciljeva učenja.

Prenosimo nekoliko argumenata za i protiv upotrebe tehnologije. Neki od argumenata za upotrebu su bolja predanost studenata učenju, stimulirani kognitivni rast i rezultati, bolje poznavanje slabosti u poznavanju nastavnog gradiva te veća fleksibilnost poučavanja kroz velik broj brzo dostupnih informacija u usporedbi s klasično unaprijed pripremljenim predavanjem. Argumenti protiv su potreba promjene stila poučavanja da bi se prilagodilo tehnologiji i otvoreno pitanje o metodološkom pristupu koji će osigurati poboljšane rezultate studenata.

Uloga tehnologije može ukratko biti opisana izjavom *National Research Councila*: “Proces upotrebe tehnologije za poboljšanje učenja nikad nije čisto tehničke prirode, odnosno određen svojstvima tehnologije ili programskog paketa. Kao i udžbenik ili neki drugi kulturalni objekt, tehnološki resursi za obrazovanje djeluju u društvenoj okolini te trebaju biti vođeni komunikacijom između polaznika i nastavnika” [2].

U stručnoj literaturi prepoznato je da tehnologija sama po sebi ne osigurava uspjeh procesa poučavanja [1]. Pripreme za nastavu uz upotrebu tehnologije zahtijevaju više truda i vremena nego klasična priprema. Koristi od uloženog truda nisu vidljive u kratkom roku i rijetko su adekvatno prepoznate u radnoj ili široj zajednici. Motiv za ulaganje takvog truda često je unutarnja motivacija nastavnika vođena entuzijazmom pripreme za “bolju” budućnost ili traganjem za samoostvarenjem.

Znanost i praksa poučavanja prepoznale su i veliku tehnološku promjenu svijeta [9],[11],[12],[15]. Ta promjena povlači i potrebu za promjenom metodološkog pristupa poučavanju matematike. Proces poučavanja trebao bi biti usmjeren na stjecanje trajnog i korisnog znanja, vještina i sposobnosti. Upotreba tehnologije može proces poučavanja učiniti suvremenim. Upotrebom tehnologije može se uštedjeti vrijeme na operativnom dijelu procesa poučavanja. Uštedeno vrijeme može se upotrijebiti za objašnjavanje osnovnih matematičkih ideja (osnovno znanje), diskusije ili kreativno matematičko razmišljanje (refleksivno znanje).

Prepoznato je da upotreba tehnologije ne znači nužno i bolje obrazovanje. Neki od mogućih negativnih efekata na studente mogli bi biti pad upornosti, strpljenja, točnosti i koncentracije – što nastavnici matematike obično smatraju da matematika daje njihovim studentima.

Tehnologija i on-line aplikacije za poučavanje mogu posjedovati velik problem – biti jako zanimljive i zahtijevati puno vremena, a biti od male edukativne koristi. Postojanjem svijesti o njima i uz relativno malen trud vjerojatnost da zapnemo s takvim aplikacijama može se značajno smanjiti. Upotrebom tehnologije nastaje još jedna prilika. Možemo se pitati želimo li da naš matematički predmet odgovara na pitanje “što ako” umjesto na klasično pitanje “zašto”. Odgovor na takvo pitanje relativno nam jasno daje odgovor na pitanje da li bi upotreba tehnologije u nastavi bila poželjna ili čak nužna.

Teorija odlučivanja

Odluka o upotrebi tehnologije može imati značajan utjecaj na proces poučavanja i učenja. Upotreba nove tehnologije, programskih paketa ili on-line materijala pruža nove prilike i prijetnje, ima svoje prednosti i slabosti. U ovom dijelu prikazujemo metodu za iskorištavanje poželjnih, odnosno izbjegavanje nepoželjnih aspekata.

Proces poučavanja je dinamičan proces. Prije poduzimanja akcije u poučavanju nastavnik bi trebao promisliti hoće li akcijom ostvariti željeni ishod. U procesu odlučivanja trebao bi uzeti u obzir nove informacije, ukoliko su dostupne, i biti otvoren za povratne informacije. Nastavnici uglavnom odluke donose na temelju kreativnosti, iskustva i intuicije. Neke akcije utječu na ostvarivanje ciljeva predmeta i mogu biti nazvane strateškim. Odluke o tim akcijama ne bi trebale biti donošene samo na tim principima. Sustavni analitički pristup bi također trebao biti upotrijebljen. Postoje različiti metodološki pristupi i matematički modeli opisivanja problema i argumentiranja odluke. Sustavni analitički pristup smanjuje vjerojatnost donošenja loše odluke, jer je odluka bazirana na logici, uzima više dostupnih informacija (po mogućnosti sve) i alternativa. Ispuštanje neke alternative, koja ne mora biti nužno značajna može dovesti do pogrešnog odabira. Također je poznato da dobra odluka može rezultirati neželjenim ishodom, ali generalno analitički pristup u donošenju odluke bolji je nego onaj baziran samo na intuiciji. Jedna od strateških odluka je odluka o upotrebi programskog paketa u poučavanju i odabir prikladnijeg ukoliko je potreban.

Teorija odlučivanja proučava proces donošenja odluka na sustavni i analitički način. Okolina u kojoj je potrebno donijeti odluku može biti više ili manje deterministička ili slučajna. Stoga se odluke mogu donositi u uvjetima sigurnosti i u uvjetima nesigurnosti i rizika. Iako ne možemo biti u potpunosti sigurni u posljedice naših akcija, problem odlučivanja koji razmatramo može se karakterizirati kao odlučivanje u uvjetima sigurnosti. Donositelj odluke također bi trebao biti svjestan da odluka često nije jednokratna, nego je jedna u nizu međusobno povezanih i sigurno će utjecati na odluke koje će se donositi kasnije.

U radu smo se odlučili za upotrebu metodologije višekriterijskog odlučivanja Analitički hijerarhijski proces (AHP). Postupak upotrebe metode AHP može se ukratko opisati kao provođenje sljedećih koraka:

1. Modeliranje problema kao hijerarhije koja se sastoji od cilja odlučivanja, alternative za njegovo ostvarivanje i kriterija za vrednovanje alternativa.
2. Određivanje prioriteta među elementima hijerarhije donošenjem niza prosudbi baziranih na usporedbi elemenata u parovima.
3. Spajanje prosudbi u konačne prioritete u hijerarhiji.
4. Provjeravanje konzistentnosti prosudbi.
5. Donošenje konačne odluke bazirane na rezultatima procesa odlučivanja [16].

Postoje razni komercijalni i besplatni alati izrađeni za analizu višekriterijskog donošenja odluka na temelju usporedbi u parovima kao što su *Expert Choice*, *Decision Lab*, *D-Sight*, *ERGO*, *MakeItRational*. Naši izračuni provedeni su u alatu Expert Choice.

Obrazovni programski paketi

Obrazovni programski paketi su programski paketi čija je primarna namjena korištenje u poučavanju ili učenju. U konačnici su razmatrana četiri programska paketa za upotrebu na predmetu Odabrana poglavlja matematike. U nastavku se nalazi opis tih paketa.

GeoGebra je alat za poučavanje i učenje geometrije, algebre i matematičke analize. Napisana je u Javi pa se stoga može pokrenuti na mnogim popularnim operacijskim sustavima. Pogodna je za razvoj nastavnih materijala iz matematike u raznim oblicima i za sve razine matematičkog obrazovanja. *GeoGebra* u sebi ima ugrađen Kartezijev koordinatni sustav pa omogućuje unos geometrijskih naredbi (crtanje točaka, pravaca, vektora, okomica, simetrala kutova,...), ali isto tako i algebarskih naredbi (uređeni par koordinata, jednačbe krivulja, funkcije,...). Ovaj dvostruki prikaz matematičkih objekata, geometrijski i algebarski, jedan je od najvećih prednosti *GeoGebre*. Transformacija objekata u geometrijskom prozoru utječe na promjenu odgovarajućih izraza u algebarskom prozoru i obrnuto, promjena izraza u algebarskom prozoru utječe na odgovarajuće objekte u geometrijskom prozoru. Nadalje, *GeoGebra* sadrži i tablični prozor koji je također na dinamički način povezan sa geometrijskim i algebarskim prozorom pa je na taj način spremna i za razne statističke naredbe [10].

Mathematica je trenutno najmoćniji i najviše korišteni sustav za kompjutorsku algebru. *Mathematica* se sastoji od tri glavne vrste računanja: numeričkog, simboličkog i grafičkog. Za razliku od tradicionalnih programskih jezika kao što su C i C++, sastoji se od puno većeg broja tipova podataka. Najopćenitiji tip podataka u *Mathematici* je simbolički izraz koji omogućuje reprezentaciju veoma apstraktnih matematičkih i nematematičkih objekata. *Mathematica* u sebi sadrži veliki broj implementiranih funkcija, ali isto tako ima vrlo moćan vlastiti programski jezik koji omogućuje nekoliko različitih načina programiranja:

- *Proceduralno programiranje* s blokovnom strukturom, uvjetima, petljama i rekurzijama
- *Funkcijsko programiranje* s lambda-računom i operatorskim funkcijama
- *Rule-based* programiranje s ispitivanjem uzoraka

Najveći nadostatak *Mathematice* je njezina cijena jer je ona ipak preskupa za prosječne korisnike [18].

Maxima je moćan sustav za kompjutorsku algebru napisan u programskom jeziku Lisp koji kombinira numeričke, simboličke i grafičke mogućnosti. To je besplatni open source program koji se neprekidno poboljšava. U usporedbi s *Mathematicom*, *Maxima* ima puno slabije i jednostavnije sučelje, ali ima veliku prednost u cijeni. Trenutno, *wxMaxima* i *XMaxima* su dva najpopularnija grafička sučelja za *Maximu*. Za nove korisnike je najbolje da počnu koristiti *wxMaxima* sučelje jer je ono bogato raznim menijima i tipkama koje sadrže najvažnije i najviše korištene *Maxima* naredbe. Kroz menije i tipke novi korisnici u *wxMaximi* postepeno uče *Maxima* sintaksu. Nadalje, *wxMaxima* ima svoju vlastitu implementaciju za lijepi prikaz matematičkih izraza. *XMaxima* je puno siromašniji grafičko sučelje sa vrlo malim brojem naredbi u menijima, ali je zato puno stabilnije sučelje od *wxMaxime*. Stoga iskusniji korisnici radije koriste *XMaxima* sučelje jer oni već znaju *Maxima* sintaksu i puno brže im je napisati ime naredbe, nego tražiti pojedinu *Maxima* naredbu u menijima (ukoliko se uopće ta naredba nalazi u nekom od menija *wxMaxime*). *XMaxima* je također puno brže sučelje za testiranje i isprobavanje koda od *wxMaxime* [13], [14].

SAGE je open source matematički alat napisan u veoma moćnom i popularnom programskom jeziku Python. Većina matematičkih alata sadrži svoj vlastiti programski jezik, dok je *SAGE* izgrađen na postojećem programskom jeziku Python

kao njegovo moćno matematičko proširenje. Stoga su iskusni Python korisnici ujedno i iskusni *SAGE* korisnici. Početnici moraju najprije naučiti Python kako bi bili spremni za rješavanje matematičkih problema u *SAGE*-u. Dakle, oni ne moraju učiti novi specijalizirani matematički programski jezik i to je jedna od velikih prednosti *SAGE*-a. Dok većina matematičkih alata funkcioniraju kao zasebne jedinice, *SAGE* ima drukčiji pristup. Za neke algoritme daje vlastitu implementaciju, dok neke algoritme koristi od ostalih open source matematičkih alata (*Maxima*, *GAP*, *BLAS*, *LAPACK*, matematički *Python* paketi, itd.). U *SAGE* je uključeno oko stotinjak open source programa koji omogućuju proučavanje elementarne, više i primijenjene matematike kao što su elementarna i napredna algebra, matematička analiza, elementarna i napredna teorija brojeva, kriptografija, numerika, teorija grupa, kombinatorika, teorija grafova, linearna algebra, itd. Sa *SAGE*-om se može komunicirati iz komandne linije ili preko web preglednika koji mu služi kao grafičko sučelje. Preko web preglednika *SAGE* se može spojiti na lokalnu instalaciju *SAGE*-a na disku ili pak na *SAGE* server na internetu. *SAGE* server na internetu je prekrasna stvar za windows korisnike jer je prirodna domena za *SAGE* operacijski sustav *Linux*. Windows korisnici moraju instalirati virtualnu mašinu ako žele pokrenuti *Linux* i *SAGE* lokalno na windowsima. *SAGE* trenutno najbolje radi sa Firefox web preglednikom [17].

Primjer: odabir programskog paketa

U ovom dijelu predstavljamo proces donošenja odluke o odabiru obrazovnog programskog paketa.

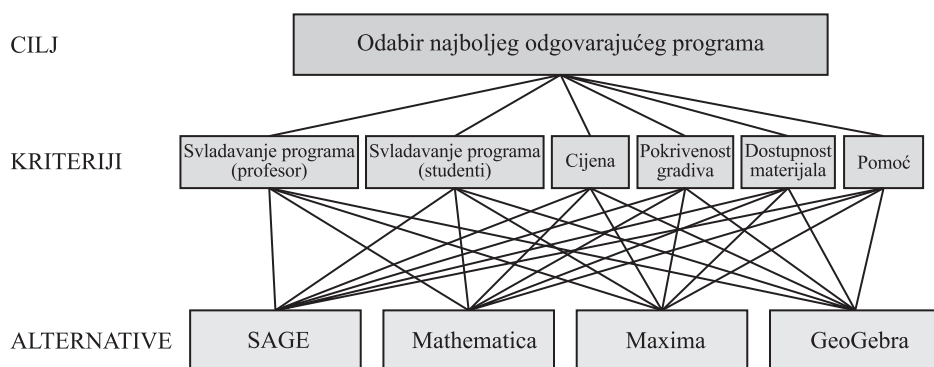
A. Cilj odlučivanja, alternative i kriteriji

Odabrana poglavlja matematike je predmet na drugoj godini preddiplomskog studija informatike. Upotreba tehnologije očekuje se na svim predmetima pa nema potrebe za donošenjem odluke je li potrebna ili nije. Cilj odlučivanja je odabir najprikladnijeg programskog paketa za ostvarivanje ciljeva predmeta. U dijelu obrazovni programski paketi opisane su četiri alternative koje dolaze u razmatranje. Kriteriji za vrednovanje alternativa su svladavanje programa (nastavnik), svladavanje programa (studenti), cijena, pokrivenost gradiva, dostupnost gotovih materijala, dostupnost korisničke pomoći.

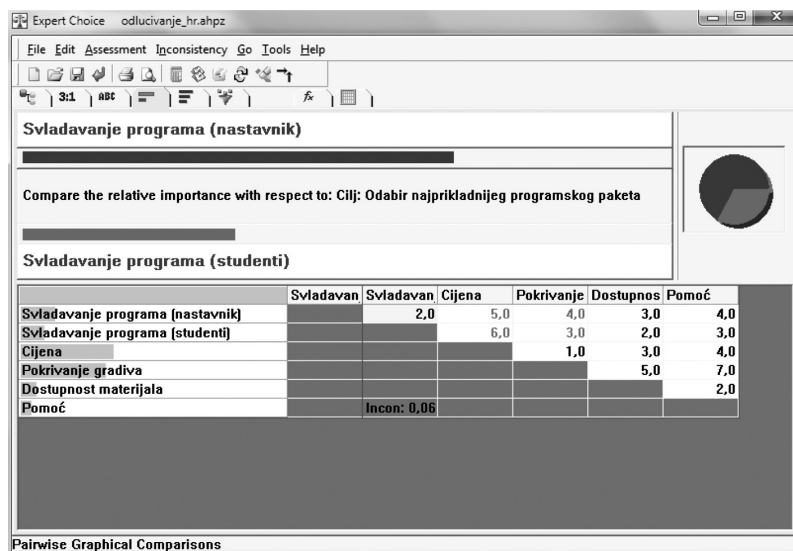
B. Određivanje prioriteta među elementima

Slika 1. prikazuje hijerarhijsku strukturu problema odlučivanja.

Slika 2. prikazuje usporedbe važnosti kriterija odlučivanja u parovima. Usporedbe u parovima su rađene prema 1-9 skali preporučenoj u AHP metodologiji. Konačno rangiranje kriterija daje sljedeće težine kriterija: cijena (34,8%), pokrivanje gradiva (32,5%), svladavanje programa (nastavnik) (12,9%), svladavanje programa (studenti) (9,0%), dostupnost gotovih materijala (6,5%), dostupnost korisničke pomoći (4,2%).



Slika 1. Hijerarhijska struktura problema odlučivanja.



Slika 2. Usporedba važnosti kriterija u parovima.

Slična metoda upotrijebljena je za usporedbe u parovima alternativa u odnosu na zadovoljavanje svakog od kriterija.

C. Konačni prioriteti i konzistentnost

Skupivši i analizirajući sve usporedbe možemo zaključiti da je *SAGE* najprikladniji programski paket za upotrebu na predmetu Odabrana poglavlja matematike. Tablica 1. *Ukupni prioriteti u procesu odlučivanja* prikazuje rangiranje alternativa prema svakom kriteriju te konačno rangiranje alternativa. Provjerena je konzistentnost usporedbi. Omjer inkonzistentcije svake tablice usporedbi je manji od 0,10 što je prihvatljivo prema AHP metodologiji.

	Svladavanje programa (nastavnik)	Svladavanje programa (studenti)	Cijena	Pokrivanje gradiva	Dostupni materijali	Pomoć	Ukupni bodovi
Težine	12,9%	9,0%	34,8%	32,5%	6,5%	4,2%	
<i>SAGE</i>	0,124	0,218	0,320	0,293	0,362	0,236	0,275
<i>Mathematica</i>	0,319	0,078	0,040	0,477	0,375	0,472	0,265
<i>Maxima</i>	0,074	0,149	0,320	0,186	0,165	0,106	0,207
<i>GeoGebra</i>	0,484	0,555	0,320	0,044	0,098	0,186	0,253

Tablica 1. Usporedba važnosti kriterija u parovima.

D. Konačna odluka

Koristeći AHP metodologiju za višekriterijsko odlučivanje možemo zaključiti da je *SAGE* naprikladniji programski paket za upotrebu na predmetu Odabrana poglavlja matematike. Neke od prednosti paketa *SAGE* su cijena, dostupnost gotovih materijala i pokrivanje nastavnog gradiva. Pokazalo se da je najbolji iako uspoređujući s ostalim programima *SAGE* je najbolji samo u cijeni (besplatan kao i neki drugi među njima), a po svakom ostalom kriteriju postoji barem jedan koji je bolji od njega. Na primjer, *Mathematica* bolje pokriva gradivo, ima više gotovih materijala, bolju korisničku pomoć, ali je komercijalni paket studentima relativno kompliciran za svladavanje. Svladavanje paketa *SAGE* je jednostavnije jer su studenti do izvođenja predmeta upoznati s programiranjem u *Pythonu*. Nastavnici su bolje upoznati s *Mathematicom* pošto su diplomirali ili doktorirali matematiku, ali to nije dovoljno dobar razlog za stvaranje učenja rada u *Mathematici* obaveznim za studente informatike. *GeoGebra*, iako besplatna i laka za naučiti ne pokriva gradivo u dovoljnoj mjeri da bi bila najbolji odabir.

Zaključak

Tehnologija je povećala mogućnosti unapređenja poučavanja matematike. Postoje pozitivni i negativni aspekti upotrebe tehnologije. Priprema za poučavanje matematike uz upotrebu tehnologije zahtijeva više vremena. Uloženi trud ne pruža rezultate u kratkom vremenu pa nije prepoznat u zajednici kao što bi trebao biti. Upotreba tehnologije otvara novu perspektivu na metodologiju poučavanja. Zbog promjene svijeta u kojem živimo, u nekim aspektima, upotreba tehnologije postaje nužna za ostvarivanje ciljeva učenja. U našem radu prikazali smo primjenu AHP metode, metodologije višekriterijskog odlučivanja, na problem odabira najprikladnijeg programskog paketa za korištenje u nastavi. U procesu odlučivanja koristili smo šest kriterija i četiri alternative.

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Kakva je budućnost integracije ICT-a u poučavanje matematike

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Sažetak. Posljednjih godina se potreba za uvođenjem ICT-a u nastavni proces nameće kao jedna od neizbježnih promjena u obrazovnom sustavu. Današnje generacije učenika u tolikoj su mjeri ovladale ICT-om u svakodnevnom slobodnom vremenu, da ovu promjenu u obrazovnom sustavu ne možemo promatrati kao ulaganje u bolju budućnost, već kao nužnost kako bi se zadržao korak s tehnologijom i učenicima. Razmatrajući integraciju ICT-a u nastavu matematike, jasno je da zamjena ploče i krede digitalnim prezentacijskim materijalom ne obuhvaća sve ono što tehnologija i matematika mogu zajedno stvoriti.

Jedan od važnih preduvjeta za kvalitetnu integraciju ICT-a u nastavu matematike jest sam učitelj, tj. njegovo znanje, volja i želja da unaprijedi nastavu matematike te ju svojim pristupom poučavanju približi generacijama današnjih učenika.

Cilj ovog rada je utvrditi spremnost budućih predavača matematike na osnovnoškolskim i srednjoškolskim razinama za integraciju ICT-a u nastavu matematike. Faktori utjecaja na opisanu spremnost, koji će biti razmotreni ovim radom, su fakultetsko obrazovanje budućeg predavača i njegove vlastite inicijative s obzirom na osobne digitalne kompetencije i infrastrukturu.

Istraživanjem, koje smo proveli na uzorku studenata završnih godina Učiteljskog fakulteta u Osijeku ($n = 196$) i završnih godina sveučilišnog nastavničkog studija matematike i informatike na Odjelu za matematiku u Osijeku ($n = 36$), smo pokazali da identificirani aspekti utječu na opisanu spremnost te da fakultetsko obrazovanje uzrokuje različitost u stavovima budućih predavača prema njihovim osobnim digitalnim kompetencijama potrebnim za kvalitetnu integraciju ICT-a u nastavu.

Ključne riječi: nastava matematike, integracija ICT-a, fakultetsko obrazovanje, vlastite inicijative, digitalne kompetencije

Uvod

Informacijsko komunikacijska tehnologija (kratica: ICT) promijenila je naše svakodnevne aktivnosti na mnogo načina. Te promjene su najvidljivije na mlađim članovima našeg društva pa tako i na učenicima osnovnih i srednjih škola. S obzirom da ICT igra sve važniju ulogu u društvu, posebno ako uzmemo u obzir socijalnu, ekonomsku i kulturnu ulogu računala i Interneta, jasno je da je došlo vrijeme za stvarni ulazak ICT-a na područje obrazovanja. Kombinacija ICT-a i Interneta nedvojbeno otvara mnogobrojne mogućnosti za kreativnost i inovativnost, ali također i za približavanje gradiva generacijama današnjih učenika.

S obzirom da su neki autori još 1980. godine predviđali da će do 2000. godine glavne metode poučavanja podrazumijevati primjenu računala, na svim razinama i u svim područjima (Bork, 1980), možemo zaključiti da je i prije 30 godina primjena ICT-a u nastavi bila predmet proučavanja i istraživanja.

U ovom radu bazirali smo se na uvođenje ICT-a u nastavu matematike iz jednostavnog razloga što je matematika jedan od školskih predmeta u kojem proces vizualizacije gradiva olakšava njegovo shvaćanje, a upravo bi upotreba ICT-a trebala olakšati proces vizualizacije.

Budućnost integracije ICT-a u nastavu matematike nedvojbeno ovisi o budućim naraštajima predavača matematike, a to su: (i) studenti učiteljskih studija, koji će matematiku predavati u nižim razredima osnovnoškolskog obrazovanja, (ii) studenti sveučilišnog nastavničkog studija matematike, koji će matematiku predavati u višim razredima osnovnoškolskog obrazovanja i u svim razredima srednjoškolskog obrazovanja. Iz toga razloga je istraživanje provedeno upravo na te dvije populacije studenata.

Pretpostavili smo da fakultetsko obrazovanje predavača matematike uzrokuje različitost u stavovima prema samoinicijativama usmjerenima ka vlastitim digitalnim kompetencijama i infrastrukturi te smo ovim radom istražili koji je smjer utjecaja. Osim toga smo ispitali kako fakultetsko obrazovanje i opisane vlastite inicijative utječu na spremnost predavača za primjenu ICT-a u nastavi matematike.

Pojam kompetencije podrazumijeva znanja, vještine i stavove potrebne za obavljanje nekog posla. Posebna vrsta kompetencija su digitalne kompetencije koje uključuju sigurno i kritičko korištenje ICT-a na poslu, u slobodno vrijeme i u komunikaciji (Marcetić, 2010). Pod osnovnim digitalnim kompetencijama podrazumijeva se korištenje multimedijske tehnologije za pronalaženje, pristup, pohranu, proizvodnju, predstavljanje i razmjenu informacija i komuniciranje i sudjelovanje u Internet mreži (Ala-Mutka, 2008). Konačno, digitalne kompetencija uključuje pouzdanu i kritičku uporabu ICT-a za zapošljavanje, učenje, samorazvoj i sudjelovanje u društvu (Marcetić, 2010).

Okvir istraživanja

Na koji način mladi nastavnici mogu doprinijeti svojim znanjima o različitim oblicima korištenja ICT-a istraživao je švedski izvanredni profesor na Linköping sveučilištu, Sven Andersson. Istraživanje se provodilo u švedskim osnovnim

školama i kao konačni zaključak vezano za primjenu ICT-a u njihovom poslu, učitelji su zaključili da su nove tehnologije unaprijedile njihov odnos prema pronalaženju znanja za razvoj vlastitih kompetencija, pronalaženju nastavnih materijala i metodoloških ideja te suodnos s učenicima i kolegama. Unaprijeđenje njihovih digitalnih kompetencija dalo im je ideju i inspiraciju za razvoj metoda učenja, njihovo vlastito znanje o računalima ili početnu točku za predstojeće ICT aktivnosti u školama (Andersson, 2006).

Istraživanje koje su proveli Sang, Valcke, van Braak i Tondeur govori o utjecaju spola, konstruktivističkih uvjerenja učitelja, samoinicijative u nastavi, samoinicijative u korištenju računala te stav o računalima. Istraživanje je provedeno na 727 studenata jednog kineskog sveučilišta. Rezultati su pokazali da je potencijalna integracija ICT-a u korelaciji sa svim navedenim varijablama, osim spola (Sang, 2010).

U članku autora Drent i Mellissen govori se o utjecajima koji stimuliraju ili ograničavaju inovativnu upotrebu ICT-a od strane učitelja u Nizozemskoj. Neki od faktora koji su proučavani su: pedagoški pristup, ICT kompetencije te poduzetnost učitelja. Rezultati su pokazali da svi navedeni faktori imaju utjecaj na inovativnost u upotrebi ICT-a. Autori zaključuju da su ICT kompetencije uvjet za upotrebu ICT-a u nastavi, ali da na inovativnu upotrebu ICT-a utječu i neki drugi faktori (Drent, 2008).

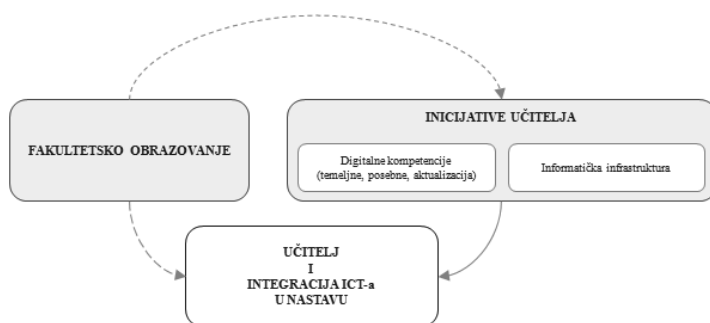
Na području Flandrije, za razliku od Velike Britanije i Kanade, ICT kompetencije nisu uključene u nacionalni kurikulum, postavljene su samo smjernice za škole kako usmjeriti inovativne obrazovne procese u postupku integracije ICT-a u nastavu. Ova studija istražuje kako i u kojoj mjeri, škola općenito i nastavnici osobno, provode nova očekivanja proizašla iz te odluke. Konkretno, ispituje se koje ICT kompetencije učitelji trenutno usvajaju (za stvarnu upotrebu u nastavi), a koje kompetencije namjeravaju usvojiti u budućnosti (preferiraju njihovo korištenje). Istraživanje je pokazalo da je većina učitelja upoznata s pojmom ICT-a, ali ipak 2,6% ispitanika nije koristilo računalo, niti u razredu niti za pripremu nastave. Od ukupnog broja sati koje tjedno provedu za računalom, najviše se odnosi na stručnu pomoć, zatim slobodno vrijeme i na kraju na nastavu. Glavni rezultat istraživanja pokazuje da učitelji u osnovom obrazovanju teže povećanju i proširenju svojih ICT kompetencija (Tondeur, 2007).

Model, uzorak, podaci

Početak istraživanja označen je diskusijom o aspektima pojedinog od ovim radom izdvojenih faktora utjecaja na spremnost na integraciju ICT-a u nastavu matematike što je rezultiralo uvođenjem pripadnog modela situacije (Slika 1) i izradom upitnika za prikupljanje podataka o populacijama od interesa.

Samoinicijative učitelja s ciljem kvalitetne primjene ICT-a u nastavi matematike razmatrali smo iz perspektive osobnih digitalnih kompetencija te informatičke infrastrukture samog učitelja. O digitalnim kompetencijama smo promišljali u smislu stjecanja, posjedovanja, usavršavanja i aktualizacije, a raščlanili smo ih na temeljne i posebne. Posjedovanje temeljnih digitalnih kompetencija analizirali smo kroz posjedovanje znanja o sljedećem: osnovni dijelovi računala, osnovni

pojmovi o Internetu, primjena elektroničke pošte, izrada digitalnog tekstualnog i prezentacijskog dokumenta, izrada proračunske tablice, crtanje i obrada fotografije, izrada i objava web-stranice te primjena Interneta kao dodatnog izvora informacija potrebnih za izradu nastavnih materijala. Nadalje, posjedovanje posebnih digitalnih kompetencija razmotrili smo kroz posjedovanje znanja i vještina za rad s nekim specijaliziranim računalnim alatima prikladnima za primjenu u nastavi matematike kao što su pametna ploča i edukacijski matematički softveri. Posebnu smo pozornost pridodali postojanju svijesti o nužnosti konstantnog usavršavanja i aktualizacije digitalnih kompetencija. Informatičkom infrastrukturom nastavnika modelirana je osobna situacija nastavnika u pogledu informatičke opremljenosti.



Slika 1. Konceptualni model

U ovom radu je primijenjen online upitnik kao metoda prikupljanja podataka koji su prikupljeni iz uzorka studenata završnih godina Učiteljskog fakulteta u Osijeku ($n = 196$) i završnih godina sveučilišnog nastavničkog studija matematike i informatike na Odjelu za matematiku u Osijeku ($n = 36$). Od studenog 2010. g. do siječnja 2011. prikupljena su 232 odgovora ispitanika iz populacija od interesa. Struktura uzorka opisana je u donjoj tablici (Tablica 1).

Obilježje	Kategorija	Frekvencija	Relativna frekvencija (%)
STUDIJ	učiteljski	196	84, 48
	nastavnički – matematika	36	15, 52

Tablica 1. Struktura uzorka

Odgovori prikupljeni iz uzorka su diskretni kvantitativni podaci, a pohranjeni su primjenom smislene cjelobrojne numeričke skale od 1 do 5. Oni označavaju ocjenu dane tvrdnje, pri čemu 1 označava “Uopće se ne slažem”, a 5 označava “U potpunosti se slažem”.

Tablično je dana statistička distribucija varijabli od interesa koje modeliraju ovim radom razmotrene aspekte samoinicijativa učitelja s obzirom na vlastite digitalne kompetencije i infrastrukturu (Tablica 2).

Razmatrano obilježje	Relativne frekvencije vrednovanja					Deskriptivna statistika uzorka	
	1	2	3	4	5	Očekivanje	Standardna devijacija
Temeljne digitalne kompetencije	0,006705	0,021073	0,068966	0,144636	0,758621	4,627395	0,760839
Posebne digitalne kompetencije	0,281609	0,100575	0,21408	0,217672	0,186063	2,926006	1,478185
Konstantna aktualizacija digitalnih kompetencija	0,018103	0,064655	0,244828	0,326724	0,34569	3,917241	1,003039
Informatička infrastruktura	0,006466	0,036638	0,090517	0,19181	0,674569	4,491379	0,854369

Tablica 2. Statistička distribucija varijabli od interesa

Metodologija

Nakon što smo iz opisanog uzorka prikupili podatke o varijablama od interesa proveli smo statističku analizu usmjerenu ka procjeni populacijskih parametara te testiranju statističkih hipoteza izvedenih iz hipoteze istraživanja s ciljem donošenja zaključaka o razmatranim populacijama.

Procjenu očekivanja i proporcije proveli smo primjenom intervala pouzdanosti za velike uzorke iz podataka prikupljenih na uzorku, a pouzdanost primijenjena za potrebe ovoga rada je 95%.

Testiranje statističke hipoteze proveli smo kroz sljedeće korake (McClave et al., 2001): (1) uvođenje nulte hipoteze koja predstavlja status quo, (2) uvođenje alternativne hipoteze, (3) izbor prikladne test statistike, (4) određivanje područja odbacivanja nulte hipoteze s obzirom na postavljeni nivo signifikantnosti α , (5) prikupljanje podataka iz uzorka, (6) izračunavanje test statistike, (7) donošenje odluke o odbacivanju nulte hipoteze, (8) zaključivanje o populaciji. U statističkim testovima provedenima za potrebe ovoga rada primijenjena je razina signifikantnosti $\alpha = 0,05$. S ciljem donošenja zaključka o razlikama u očekivanjima i proporcijama proveli smo odgovarajuće testiranje hipoteza za velike uzorke primjenom z -statistike.

Rezultati

Ispitali smo kako fakultetsko obrazovanje utječe na prosudbu vlastitih inicijativa usmjerenih ka stjecanju, posjedovanju, usavršavanju i aktualizaciji vlastitih digitalnih kompetencija i infrastrukture.

Iz podataka prikupljenih na uzorcima studenata za razmatrane aspekte samoinicijativa odredili smo mjere deskriptivne statistike te procijenili očekivanja intervalom pouzdanosti 95%. Dobiveni rezultati su opisani tablično (Tablica 3, Tablica 4), a daju naznaku višeg vrednovanja svih razmatranih aspekata samoinicijativa od strane studenata nastavničkog studija matematike u odnosu na studente učiteljskog studija.

Razmatrano obilježje	Deskriptivna statistika uzorka		Interval pouzdanosti (95%) za procjenu očekivanja	
	Očekivanje	Standardna devijacija	Donja granica	Gornja granica
Temeljne digitalne kompetencije	4,619615	0,770176	4,583649	4,655580
Posebne digitalne kompetencije	2,876701	1,468281	2,792696	2,960705
Konstantna aktualizacija digitalnih kompetencija	3,892857	1,002423	3,830019	3,955695
Informatička infrastruktura	4,461735	0,878015	4,374547	4,548922

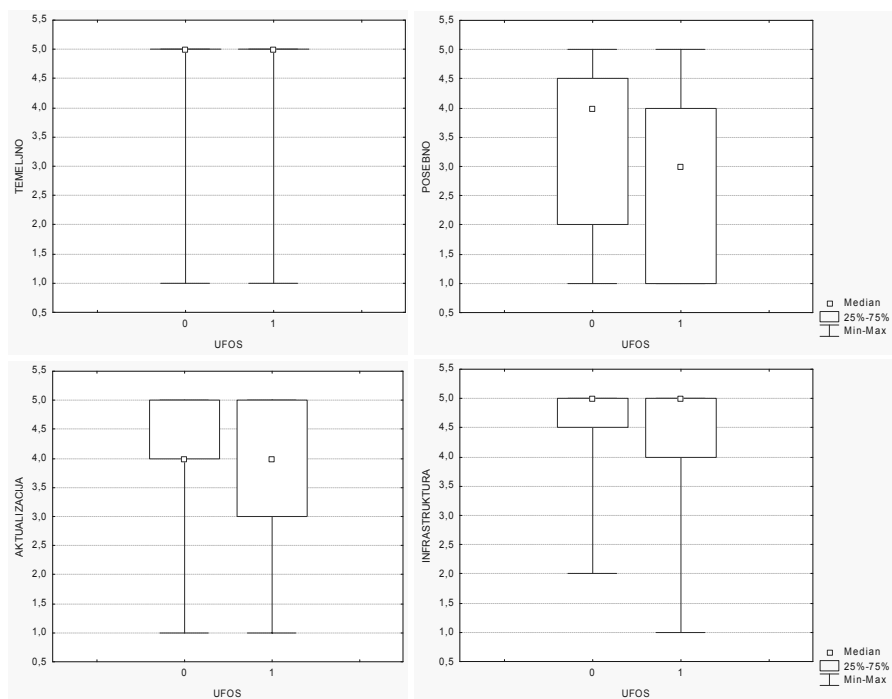
Tablica 3. Deskriptivna statistika i intervali pouzdanosti za procjenu očekivanja za uzorak studenata učiteljskog studija

Razmatrano obilježje	Deskriptivna statistika uzorka		Interval pouzdanosti (95%) za procjenu očekivanja	
	Očekivanje	Standardna devijacija	Donja granica	Gornja granica
Temeljne digitalne kompetencije	4,669753	0,707465	4,592430	4,747076
Posebne digitalne kompetencije	3,194444	1,506317	2,992427	3,396462
Konstantna aktualizacija digitalnih kompetencija	4,050000	0,998742	3,903103	4,196897
Informatička infrastruktura	4,652778	0,695250	4,489402	4,816154

Tablica 4. Deskriptivna statistika i intervali pouzdanosti za procjenu očekivanja za uzorak studenata nastavničkog studija matematike

Kvantitativne podatke prikupljene na uzorcima studenata prikazali smo i grafički na bazi p -postotnih vrijednosti i raspona (Slika 2), a s obzirom na dvije razmotrene kategorije studenata.

Nadalje smo proveli jednostrani test za usporedbu očekivanja za velike uzorke, kojim smo usporedili prosudbu ispitanika o opisanim vlastitim inicijativama usmjerenima na digitalne kompetencije i infrastrukturu, a s obzirom na fakultetsko obrazovanje. Alternativna hipoteza je predstavljala postojanje razlike u očekivanjima prosudbi razmatranih aspekata samoinicijativa i to u korist studenata nastavničkog studija matematike, a dizajnirana je temeljem prethodne rasprave. Iz rezultata prikazanih tablično (Tablica 5) na razini značajnosti $\alpha = 0,05$ zaključujemo: (i) nemamo dovoljno argumenata da bismo odbacili hipotezu o jednakosti očekivanja u razmatranim populacijama od interesa, (ii) postoji statistički značajna razlika u očekivanjima vrednovanja posljednja tri aspekta samoinicijativa i to u korist studenata nastavničkog studija matematike.



Slika 2. Kategorizirani kutijasti dijagrami aspekata samoinicijativa

Razmatrano obilježje	p -vrijednost	$\alpha = 0,05$	z	$z_{\alpha}=1,644854$	H_0
Temeljne digitalne kompetencije	0,12383	$p > \alpha$	1,15604	$z < z_{\alpha}$	ne odbaciti
Posebne digitalne kompetencije	0,00211	$p < \alpha$	2,86061	$z > z_{\alpha}$	odbaciti
Konstantna aktualizacija digitalnih kompetencija	0,026241	$p < \alpha$	1,939156	$z > z_{\alpha}$	odbaciti
Informatička infrastruktura	0,020156	$p < \alpha$	2,050542	$z > z_{\alpha}$	odbaciti

Tablica 5. Rezultati jednostranog testa za usporedbu očekivanja vrednovanja samoinicijativa s obzirom na fakultetsko obrazovanje

U nastavku smo pomnije analizirali svaki od prethodno uvedenih aspekata samoinicijativa nastavnika određivanjem mjera deskriptivne statistike i procjenom očekivanja intervalom pouzdanosti 95% (Tablica 6, Tablica 7).

Razmatrano obilježje	Deskriptivna statistika uzorka		Interval pouzdanosti (95%) za procjenu očekivanja	
	Očekivanje	Standardna devijacija	Donja granica	Gornja granica
Poznavati osnovne dijelove računala	4,571429	0,771279	4,462777	4,680080
Poznavati osnovne pojmove o Internetu	4,760204	0,504999	4,689064	4,831344

Znati koristiti elektroničku poštu	4,877551	0,372526	4,825073	4,930029
Znati izraditi tekstualni dokument	4,903061	0,359188	4,852462	4,953661
Znati izraditi prezentacijski materijal	4,877551	0,411758	4,819546	4,935556
Znati izraditi proračunsku tablicu	4,397959	0,919752	4,268392	4,527526
Poznavati rad u nekom programu za crtanje i obradu fotografije	4,448980	0,854820	4,328560	4,569400
Znati izraditi i objaviti web-stranicu	3,658163	1,185591	3,491147	3,825179
Znati koristiti Internet kao dodatni izvor informacija potrebnih za izradu nastavnih materijala	4,816327	0,482450	4,748363	4,884290
Važnost sposobnosti za rad sa tzv. pametnom pločom	3,790816	1,053501	3,642408	3,939225
Važnost poznavanja nekog od specijaliziranih programskih alata za matematičku edukaciju	4,112245	0,904492	3,984828	4,239662
Osobna sposobnost rada u <i>The Geometer's Sketchpad-u</i>	3,448980	1,087198	3,295824	3,602135
Osobna sposobnost rada u <i>GeoGebri</i>	2,316327	1,293748	2,134074	2,498579
Osobna sposobnost rada u <i>Wolfram Mathematici</i>	1,301020	0,720540	1,199517	1,402524
Osobna sposobnost rada s tzv. pametnom pločom	1,923469	1,163223	1,759604	2,087335
Aktualizacija digitalnih kompetencija nastavnika matematike je neophodna za adekvatnu primjenu ICT alata u nastavi.	3,811224	0,877103	3,687666	3,934783
Kako bi nastavnik matematike aktualizirao svoje ICT vještine te kvalitetno i primjereno integrirao aktualne ICT alate u nastavu, treba redovito čitati odgovarajuće informatičke časopise.	3,678571	1,054295	3,530051	3,827092
Kako bi nastavnik matematike aktualizirao svoje ICT vještine te kvalitetno i primjereno integrirao aktualne ICT alate u nastavu, treba redovito pratiti relevantne web portale.	3,750000	1,019678	3,606356	3,893644
Kako bi nastavnik matematike aktualizirao svoje ICT vještine te kvalitetno i primjereno integrirao aktualne ICT alate u nastavu, treba pohađati informatičke seminare.	4,112245	0,975417	3,974836	4,249654
Kako bi nastavnik matematike aktualizirao svoje ICT vještine te kvalitetno i primjereno integrirao aktualne ICT alate u nastavu, treba samoinicijativno i samostalno proučavati ICT alate.	4,112245	1,001360	3,971182	4,253308
Preduvjet za kvalitetnu pripremu nastave matematike je posjedovanje osobnog računala (kod kuće).	4,469388	0,879441	4,345499	4,593276

Preduvjet za kvalitetnu pripremu nastave matematike je omogućen pristup Internetu (kod kuće).	4,454082	0,878772	4,330288	4,577876
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Tablica 6. Deskriptivna statistika i intervali pouzdanosti za procjenu očekivanje za uzorak studenata učiteljskog studija

Razmatrano obilježje	Deskriptivna statistika uzorka		Interval pouzdanosti (95%) za procjenu očekivanja	
	Očekivanje	Standardna devijacija	Donja granica	Gornja granica
Poznavati osnovne dijelove računala	4,583333	4,349266	4,817401	0,691789
Poznavati osnovne pojmove o Internetu	4,805556	4,647486	4,963625	0,467177
Znati koristiti elektroničku poštu	4,972222	4,915830	5,028614	0,166667
Znati izraditi tekstualni dokument	4,972222	4,915830	5,028614	0,166667
Znati izraditi prezentacijski materijal	4,972222	4,915830	5,028614	0,166667
Znati izraditi proračunsku tablicu	4,888889	4,781047	4,996731	0,318728
Poznavati rad u nekom programu za crtanje i obradu fotografije	4,722222	4,548541	4,895903	0,513315
Znati izraditi i objaviti web-stranicu	4,611111	4,392914	4,829308	0,644882
Znati koristiti Internet kao dodatni izvor informacija potrebnih za izradu nastavnih materijala	4,944444	4,831661	5,057228	0,333333
Važnost sposobnosti za rad sa tzv. pametnom pločom	4,138889	3,845558	4,432220	0,866941
Važnost poznavanja nekog od specijaliziranih programskih alata za matematičku edukaciju	4,666667	4,485810	4,847523	0,534522
Osobna sposobnost rada u <i>The Geometer's Sketchpad-u</i>	4,083333	3,849266	4,317401	0,691789
Osobna sposobnost rada u <i>GeoGebri</i>	3,361111	2,861523	3,860699	1,476536
Osobna sposobnost rada u <i>Wolfram Mathematici</i>	3,166667	2,783012	3,550321	1,133893
Osobna sposobnost rada s tzv. pametnom pločom	1,750000	1,412859	2,087141	0,996422
Aktualizacija digitalnih kompetencija nastavnika matematike je neophodna za adekvatnu primjenu ICT alata u nastavi.	3,972222	3,736607	4,207837	0,696362
Kako bi nastavnik matematike aktualizirao svoje ICT vještine te kvalitetno i primjereno integrirao aktualne ICT alate u nastavu, treba redovito čitati odgovarajuće informatičke časopise.	3,777778	3,406153	4,149402	1,098339
Kako bi nastavnik matematike aktualizirao svoje ICT vještine te kvalitetno i primjereno integrirao aktualne ICT alate u nastavu, treba redovito pratiti relevantne web portale.	3,861111	3,463615	4,258607	1,174802

Kako bi nastavnik matematike aktualizirao svoje ICT vještine te kvalitetno i primjereno integrirao aktualne ICT alate u nastavu, treba pohađati informatičke seminare.	4,083333	3,727317	4,439349	1,052209
Kako bi nastavnik matematike aktualizirao svoje ICT vještine te kvalitetno i primjereno integrirao aktualne ICT alate u nastavu, treba samoinicijativno i samostalno proučavati ICT alate.	4,555556	4,306992	4,804119	0,734631
Preduvjet za kvalitetnu pripremu nastave matematike je posjedovanje osobnog računala (kod kuće).	4,666667	4,424022	4,909311	0,717137
Preduvjet za kvalitetnu pripremu nastave matematike je omogućen pristup Internetu (kod kuće).	4,638889	4,407948	4,869830	0,682549

Tablica 7. Deskriptivna statistika i intervali pouzdanosti za procjenu očekivanje za uzorak studenata nastavničkog studija matematike

U nastavku smo za svaki od prethodno uvedenih aspekata samoinicijativa primijenili test za testiranje statističke hipoteze o jednakosti očekivanja za velike uzorke, pri čemu je alternativna hipoteza predstavljala postojanje razlike u očekivanjima i to u korist studenata nastavničkog studija matematike. Na prihvaćenoj razini značajnosti $\alpha = 0,05$ zaključili smo:

- Među ranije navedenim temeljnim digitalnim kompetencijama jedino ne postoji statistički značajna razlika u vrednovanju potrebe poznavanja osnovnih dijelova računala i osnovnih pojmova o Internetu između dvije skupine od interesa ($p = 0,46289$; $p = 0,29858$), dok su studenti nastavničkog studija matematike potrebu posjedovanja i/ili stjecanja svih ostalih znanja ove kategorije ocijenili statistički značajno višom ocjenom u odnosu na studente učiteljskih studija ($p < \alpha$).
- Potrebu posjedovanja i/ili stjecanja svih ovim istraživanjem obuhvaćenih posebnih digitalnih kompetencija studenti nastavničkog studija matematike su ocijenili statistički značajno višom ocjenom ($p < \alpha$).
- Obje skupine studenata su u jednakoj mjeri prepoznale potrebu za konstantnom aktualizacijom digitalnih kompetencija praćenjem odgovarajućih informatičkih publikacija i sudjelovanjem u edukacijama ($p > \alpha$).
- Studenti nastavničkog studija matematike su statistički značajnije višom ocjenom vrednovali potrebu samoinicijativnog i samostalnog proučavanja ICT alata s ciljem aktualizacije ICT vještina te kvalitetne integracije aktualnih ICT alata u nastavu matematike ($p = 0,000885$).
- Ne postoji statistički značajna razlika u ocjeni potrebe posjedovanja određene razine informatičke infrastrukture nastavnika koju smo promotрили kroz posjedovanje osobnog računala i pristupa Internetu ($p = 0,072$; $p = 0,07746$) u populacijama od interesa.

Potom su ispitanici ocijenili jesu li ih njihove vlastite inicijative, koje poduzimaju s ciljem aktualizacije svojih znanja, vještina i informatičke infrastrukture, učinile spremnima za integraciju ICT-a u nastavu matematike. Intervalom pouzdanosti 95% smo procijenili očekivanja ocjena opisane spremnosti te za studente nastavnčkog studija matematike dobili interval $[3,776023; 4,335088]$, a za studente učiteljskog studija dobili interval $[3,501265; 3,784449]$. Osim toga, proveli smo test za testiranje hipoteze o jednakosti očekivanja navedenih ocjena, gdje je alternativna hipoteza predstavljala postojanje razlike u očekivanju navedene ocjene i to u korist studenata nastavnčkog studija matematike, a postavljena je temeljem ranijih analiza. Rezultati provedenog testa ($p = 0,003934$) pokazuju da su na razini značajnosti 5% studenti nastavnčkog studija matematike statistički značajno više ocijenili razmatranu spremnost uvjetovanu poduzetim samoinicijativama.

Naposljetku smo još intervalom pouzdanosti 95% procijenili proporcije studenata, koje su samoinicijative učinile spremnima za primjenu ICT-a u nastavi matematike, tj. one koji su svoju spremnost vrednovali ocjenom barem 4. Tako smo za studente nastavnčkog studija matematike dobili interval $[0,54397; 0,84492]$, a za studente učiteljskog studija interval $[0,48656; 0,62568]$. Primjenom iste metode procijenili smo i proporcije studenata, koje samoinicijative nisu učinile spremnima za primjenu ICT-a u nastavu matematike, tj. one koji su svoju spremnost vrednovali ocjenom najviše 2. Na ovaj način smo za studente učiteljskog studija dobili interval $[0,07229; 0,162404]$ dok u uzorku studenata nastavnčkog studija matematike takvih studenata nema. Osim toga, proveli smo i test za testiranje hipoteze o proporcijama spremnih (ocjena barem 4), odnosno nespremnih (ocjena najviše 2), studenata za primjenu ICT-a u nastavi matematike u razmatranim populacijama. Alternativna hipoteza je predstavljala postojanje razlike u proporcijama i to u korist studenata nastavnčkog studija matematike, odnosno u korist studenata učiteljskog studija, a postavljena je temeljem prethodne analize. Rezultati provedenih testova ($p = 0,054888486$; $p = 0$) na razini značajnosti 5% pokazuju: (i) nemamo dovoljno argumenata da bismo odbacili hipotezu o jednakosti proporcija studenata koji su spremni za primjenu ICT-a u nastavu matematike, (ii) statistički je značajno veća proporcija studenata učiteljskog studija, koji nisu spremni na primjenu ICT-a u nastavi matematike, u odnosu na proporciju takvih studenata nastavnčkog studija matematike.

Zaključak

Rezultati dobiveni ovim istraživanjem ukazuju da fakultetsko obrazovanje uzrokuje različitost u stavovima prema samoinicijativama usmjerenima ka vlastitim digitalnim kompetencijama i infrastrukturi. Pokazali smo da viša razina informatičke obrazovanosti uzrokuje pozitivnije stavove o primjeni razmotrenih vlastitih inicijativa što smo i očekivali. Kako program učiteljskog studija pokriva manje informatičkih područja i pokriva ih na nižoj razini nego program nastavnčkog studija matematike, očekivano je da će studenti nastavnčkog studija matematike

bolje vrednovati samoinicijative usmjerene ka digitalnim kompetencijama i infrastrukturi što je i pokazalo ovo istraživanje.

Međutim, svakako treba istaknuti da su i studenti učiteljskog studija i studenti nastavničkog studija matematike prepoznali potrebu za konstantnom aktualizacijom digitalnih kompetencija praćenjem odgovarajućih informatičkih publikacija i sudjelovanjem u edukacijama. Ovaj nalaz nam daje pravo da tvrdimo da se sa studentima učiteljskog studija radi u pravom smjeru u smislu informatičke edukacije, jer je svijest o nužnosti aktualizacije znanja i vještina izuzetno važna u radu s ICT alatima.

Nastojat ćemo pratiti razliku u ovim radom razmatranim fenomenima između ove dvije populacije od interesa. S obzirom na činjenicu da je ICT sve većim dijelom uključen u nastavni proces, sami studenti bi trebali shvatiti da su njihove digitalne kompetencije spona koja ih veže uz kreativnost i inovativnost na području primjene ICT-a u nastavi. Ovo istraživanje je polazna točka većeg istraživanja kojim je planirano obuhvatiti različite populacije budućih i sadašnjih predavača matematike na osnovnoškolskim i srednjoškolskim razinama.

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Utjecaj stila učenja studenata na odabir izbornog modula

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Sažetak. Ovaj rad istražuje postoji li veza između stila učenja studenata i upisanog izbornog modula u okviru studijskoga programa, te može li se metodama strojnog učenja predvidjeti uspješnost studenta na temelju stila učenja i izbornog modula. U istraživanju su sudjelovali studenti pete godine diplomskog studija čiji je stil učenja bio ispitan upotrebom indeksa stila učenja Feldera i Solomanove. Istražuju se i korelacije između dimenzija stila učenja detektiranih upotrebom indeksa stila učenja Feldera i Solomanove te korelacije između uspješnosti studenata, odabranog izbornog modula i dimenzija stila učenja. U obradi podataka koristila se metoda ANOVA te je testirana klasifikacijska metoda strojnog učenja za predviđanje uspješnosti studenata ovisno o izabranom modulu i njihovom stilu učenja. Rezultati su pokazali da studenti s dominantnim aktivnim, vizualnim, senzornim i intuitivnim dimenzijama odabiru podjednako sve izborne module, studenti s naglašenom sekvencijalnom dimenzijom češće biraju modul razvojnog smjera, a studenti s izraženom globalnom dimenzijom preferiraju modul s pojačanim predmetima iz informatike. Radom se ukazuje na potrebu prilagođavanja stila poučavanja u pojedinim modulima stilovima učenja studenata koji su upisali taj modul. Rezultati dobiveni ovim radom mogu poslužiti kao smjernice kod dizajniranja nastavnih materijala i u organizacijske svrhe kod planiranja nastave, a s ciljem poboljšanja studentskih ishoda učenja.

Ključne riječi: stil učenja, ANOVA, metode strojnog učenja, izborni modul, uspješnost studiranja

Uvod

U preporukama Bolonjske deklaracije¹ posebna pozornost posvećena je izbornosti, tj. prilikama studenata da sami kreiraju sadržaje svojega studija. Stoga 2005. godine

¹ Tuning, 2005., EUA tuning educational structures in Europe, dostupno na internetskoj adresi: <http://www.let.rug.nl/TuningProject/index.htm> (uvid 4. Ožujka 2005.).

visokoobrazovne ustanove izrađuju nove programe u okviru kojih sadržaje za izvjestan postotak vremena studiranja prepuštaju izboru studenata u skladu s njihovim interesima. Paralelno se u Hrvatskoj radi na preustroju Visokih učiteljskih škola u Učiteljske fakultete. Ukida se dotadašnja praksa studija pojačanoga programa iz pojedinoga predmeta kojim su se studenti učiteljskih studija osposobljavali za dvije djelatnosti: a) učitelja u primarnom obrazovanju i b) predmetnoga učitelja djece starosti do 14 godina. No, kreatori programa učiteljskih studija i dalje ustraju na potrebi da se u okviru učiteljskih studija pruži prilika studentima za postizanje dodatnih kompetencija u skladu s njihovim interesima, a sa ciljem kvalitetnije izobrazbe djece u primarnom obrazovanju. To je bilo i u skladu s bolonjskim preporukama. Stoga se kreiranju učiteljskih studijskih programa prišlo na tri razine: obvezni kolegiji, kolegiji izbornoga modula i slobodni izborni kolegiji. Izborni modul na učiteljskim studijima tada je definiran kao skup kolegija koji svojim sadržajima uže usmjeravaju studenta prema dodatnim kompetencijama. U programu učiteljskih studija kreiranom na Visokoj učiteljskoj školi u Osijeku, a koji je 2005. godine pozitivno recenziran² od strane Nacionalnoga vijeća za visoko obrazovanje, kolegiji izbornoga modula i slobodni izborni kolegiji pokrivaju skoro 30% sveukupnoga ECTS opterećenja studenta.

Nakon prvog semestra studenti integriranog preddiplomskog i diplomskog sveučilišnoga studija Učiteljskog fakulteta u Osijeku opredjeljuju se za jedan od izbornih modula koji ih osposobljavaju za specifična područja odgoja i obrazovanja djece mlađe školske dobi. Ponuđena su im tri modula pri čemu ih modul razvojnog usmjerenja (modul A) s izabranim predmetima iz pedagogije i psihologije šire osposobljava za razumijevanje specifičnih pitanja odgoja i obrazovanja djece, informatički modul (modul B) ih temeljitije osposobljava za korištenje informacijskih tehnologija u odgojno-obrazovnom procesu te za informatički odgoj i obrazovanje djece u primarnom obrazovanju, dok ih modul stranih jezika (modul C) dodatno osposobljava za rano poučavanje djece mlađe školske dobi stranom jeziku.

S obzirom na rezultate prethodnih istraživanja drugih autora (Pask, 1988 u Hativa, Birenbaum, 2000; Naimie i dr. 2010) koji pokazuju da usklađenost stila učenja i stila poučavanja utječe na uspješnost studenata, cilj ovog rada bio je ispitati postoji li veza između stila učenja studenata i upisanog izbornog modula, kako bi se na temelju tih rezultata pristupilo usklađivanju stila poučavanja sa stilom učenja, a u svrhu povećanja uspješnosti studiranja. Za potrebe rada provedeno je vlastito istraživanje 2010./11. akademske godine među studentima pete godine Učiteljskog fakulteta u Osijeku. Za testiranje stila učenja studenata upotrijebljen je indeks stila učenja Feldera i Solomanove te su prikupljeni podatci i o uspješnosti studenata. Istraživala se korelacija između dimenzija stila učenja detektiranih indeksom stila učenja Feldera i Solomanove te odabranog izbornog modula, proučavala se razlika u stilovima učenja ovisno o odabranom izbornom modulu koristeći metodu jednosmjerne analize varijance, dok se za razvrstavanje studenata prema uspješnosti koristila klasifikacijska metoda k-najbližeg susjeda (eng. k-nearest neighbor).

² Ministar znanosti, obrazovanja i športa RH, Dragan Primorac, potpisao je 16. lipnja 2005. godine dopusnicu Visokoj učiteljskoj školi u Osijeku za izvođenje integriranoga preddiplomskog i diplomskog petogodišnjeg sveučilišnoga učiteljskog studija.

O indeksu stila učenja

Felder i Silverman (1988) predložili su model stila učenja koji je klasificirao ispitanike po sklonostima prema jednoj od četiri dimenzije: senzorno-intuitivno dimenziji, vizualno-verbalno dimenziji, aktivno-refleksivno dimenziji i sekvencijalno-globalnoj dimenziji. Svaka od navedenih dimenzija razlikuje dva osnovna tipa. U svom radu (Felder, Silverman, 1988) oni navode osnovne karakteristike svake dimenzije te ističu razlike između svakog tipa navedene dimenzije. Tako navode da ispitanici koji preferiraju senzorno učenje vole podatke, činjenice i eksperimentiranje, dok oni koji su intuitivni tip više vole principe i teorije. Oni koji imaju vizualni tip najlakše pamte informacije ukoliko su im one predstavljene slikovnim prikazima, grafovima, filmovima i demonstracijama, dok verbalni tip lakše pamti ono što je izgovoreno ili napisano. Ispitanici koji imaju aktivni tip najlakše uče isprobavajući stvari te vole rad u grupi, dok oni s istaknutim refleksivnim tipom preferiraju samostalni rad. Sekvencijalni ispitanici uče novo gradivo u rastućim koracima, dok globalni uče novo gradivo u "većim skokovima".

Kao nastavak tog istraživanja, Felder i Soloman (1991) razvili su indeks stila učenja kao instrument (upitnik) koji ispituje sklonosti ispitanika prema dimenzijama modela stila učenja Feldera i Silvermanove. Sastoji se od 44 pitanja pri čemu je za svaku dimenziju postavljeno 11 pitanja na koja se odgovara s odgovorima *a* ili *b*. Broj odgovora *b* oduzima se od broja odgovora *a* za svaku grupu pitanja tako da je za svaku dimenziju rezultat broj između -11 i 11. Ukoliko se apsolutna vrijednost dobivenog rezultata nalazi u intervalu [1,3] tada govorimo o uravnoteženom odnosu unutar dimenzija, ako je rezultat u intervalu [5,7] tada govorimo o umjerenoj sklonosti prema jednoj dimenziji, a ukoliko je rezultat u intervalu [9,11] tada govorimo o izrazitom preferiranju jedne dimenzije.

Pregled prethodnih istraživanja

Istraživanja su pokazala da u slučajevima kada je stil poučavanja sličan stilu učenja studenata tada studenti lakše usvajaju gradivo i efikasniji su u učenju od onih studenata kod kojih se stil učenja ne podudara sa stilom poučavanja (Pask, 1988, u Hativa, Birenbaum, 2000). Naimie, Siraj i dr. (2010) istraživali su utjecaj stila poučavanja i stila učenja studenata te utjecaj njihovog podudaranja ili nepodudaranja na uspješnost studenata. Na uzorku od 310 studenata koristeći indeks stila učenja Feldera i Solomanove, promatranja i intervjue za prikupljanje podataka, pokazali su da podudaranje stila učenja i poučavanja može povećati uspješnost studenata. Nepodudaranje se pojavljuje kada studentova preferirana metoda procesiranja informacije nije u skladu s preferiranim stilom poučavanja učitelja/profesora što može uzrokovati slabu učinkovitost studenta zbog njegove demotivanosti, odnosno studentu može biti dosadno te može biti nezainteresiran, lošije napisati ispite, postati obeshrabren te u nekim slučajevima promijeniti kolegij koji sluša (Felder, Spurlin, 2005). Amir i Jelas (2010) također istražuju o podudaranju stila poučavanja i stila učenja studenata. Na uzorku od 545 studenata i

120 kolegija upotrebljujući prevedenu verziju Grasha-Riechmanovog³ inventara stila poučavanja i stila učenja proučavali su koji su stilovi učenja dominantni na Sveučilištu Kebangsaan, Malaysia.

Metodologija istraživanja

Istraživanje se provelo ak. god. 2010./2011. na studentima pete godine Učiteljskog fakulteta u Osijeku. Uzorak je obuhvatio 109 studenata. Instrument korišten za ispitivanje stila učenja studenata je prevedeni na hrvatski jezik indeks stila učenja Feldera i Solomanove koji je dostupan u obliku besplatne web aplikacije. Kod obrade i analize podataka koristio se program Statistica 8. U cilju otkrivanja postoji li razlika između stilova učenja studenata i njihove sklonosti odabiru modula upotrijebljena je statistička metoda jednosmjerne analize varijance (ANOVA). S obzirom da je ANOVA pokazala određene zavisnosti, pretpostavilo se da je moguće kreirati model strojnog učenja koji će na temelju ulaznih varijabli o stilu učenja studenata procijeniti u koji izborni modul studenta treba razvrstati. U tu svrhu testirana je klasifikacijska metoda k -najbližeg susjeda.

Metodologija analize varijance (ANOVA)

Analiza varijance (ANOVA) je statistička metoda koja se koristi za ispitivanje statistički značajnih razlika između srednjih vrijednosti dviju ili više grupa, pod pretpostavkom da je uzorak populacije normalno distribuiran (NIST/SEMATECH, 2010). Osnovna formula za izračun varijance s^2 od n mjerenja je (McClave, Benson, 1998):

$$s^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n - 1} \quad (1)$$

gdje je $\bar{y}$ srednja vrijednost n mjerenja. Kod ANOVE, varijacija zavisne varijable je podijeljena u komponente koje odgovaraju različitim vrijednostima nezavisnih varijabli tj. varijabli grupe. Cilj ANOVE je podijeliti ukupnu varijaciju kod podataka u dio s obzirom na slučajnu grešku i dijelove s obzirom na promjene u vrijednostima nezavisne varijable. U našim eksperimentima, svaki od tipova stilova učenja zasebno se koristio kao zavisna varijabla, dok je izabrani izborni modul bio korišten kao nezavisna varijabla (valoriziran kao modul A, B i C). Deskriptivna statistika pokazuje da su podatci o stilu učenja normalno distribuirani, prema tome pretpostavka za ANOVU je bila zadovoljena. Test statistika, korištena za testiranje jednakosti tretmana srednjih vrijednosti je F test. Kritična vrijednost je tablična vrijednost za F distribuciju, temeljena na odabranoj p -razini i stupnjevima slobode.

³ eng. izraz: Grasha-Riechman Teaching and Learning Style Inventories.

Metodologija k -najbližeg susjeda

Metodologija k -najbližeg susjeda je metoda koja uči na primjerima, a koristi se za klasifikaciju ulaznih vektora na temelju najbližih primjera u ulaznom prostoru, premda se može koristiti i kod regresijskog tipa problema. To je jedna od najčešće korištenih metoda strojnog učenja, uspješno je primijenjena u mnogim područjima, kao što je rudarenje podacima, statistički raspoznavanje uzoraka, obrada slike i mnoga druga (Teknomo, 2011). Govindarajan i RM. Chandrasekaran (2010) su pokazali da je učinkovita u rudarenju teksta za izravni marketing. Prednost ove metode je u njezinoj jednostavnosti uporabe i tumačenja, a slabost je u vremenski zahtjevnoj obradi kod procesa pretraživanja koji je proporcionalan veličini uzorka za treniranje (Kudo et al, 2003).

Metoda djeluje na način da je ulazni vektor klasificiran većinom glasova svojih susjeda. Susjedi označavaju ulazne vektore za koje je poznata ispravna klasifikacija. Novi ulazni vektor dodjeljuje se onoj kategoriji izlazne varijable koja je najčešća među svojim k najbližim susjedima, gdje je k pozitivan broj, obično postavljen na male vrijednosti. Ako je $k = 1$, tada je ulazni vektor jednostavno dodijeljen klasi najbližeg susjeda. U našim eksperimentima, veličina k je određena u postupku n -struke unakrsne validacije (zbog malog uzorka, korištena je vrijednost $n = 3$), koja je pokazala da je $k = 2$ je najbolji izbor za promatrane podatke). Kako bi se mjerila udaljenost između ulaznog vektora i njegovih susjeda, koristila se euklidska udaljenost gdje je udaljenost između točaka p i q duljina dužine koja ih spaja.

U n -dimenzionalnom prostoru, ako su $\mathbf{p} = (p_1, p_2, \dots, p_n)$ i $\mathbf{q} = (q_1, q_2, \dots, q_n)$ dvije točke, onda Euklidova udaljenost od $\mathbf{p}$ do $\mathbf{q}$, ili od $\mathbf{q}$ do $\mathbf{p}$ je dana (Teknomo, 2011):

$$\begin{aligned} d(p, q) = d(q, p) &= \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2 + \dots + (q_n - p_n)^2} \\ &= \sqrt{\sum_{i=1}^n (q_i - p_i)^2} \end{aligned} \quad (2)$$

Ova metoda se koristi u našim eksperimentima za potrebe klasifikacije studenata prema njihovoj uspješnosti studiranja, koja je mjerena prosječnom ocjenom studiranja. U fazi predprocesiranja, izlazna varijabla (tj. prosječna ocjena) je pretvorena u binarni format, tako da vrijednost 0 ukazuje na uspjeh ispod 4, dok je vrijednost 1 značila uspjeh jednak ili veći od 4 (ocjene su izražene u rasponu 1 do 5). Ulazne varijable su bile osam tipova stila učenja i odabrani modul. Kako bi se izvršila metoda k -najbližeg susjeda, uzorak podataka je nasumično podijeljen na uzorak za treniranje (75% podataka) i uzorak za testiranje (25% podataka). Model je procijenjen na uzorku za treniranje, dok je uzorak za testiranje korišten za procjenu njegovu točnost. Matrica konfuzije koja prikazuje točnu i netočnu klasifikaciju prikazana je za dobiveni model, i stopa točnosti klasifikacije (tj. stopa pogodaka, eng. hit rate) izračunava se zasebno za kategoriju 0 i 1 kategorije. Konačna ocjena uspješnosti modela provodi se na temelju prosječne stope klasifikacije obje kategorije izlazne varijable.

Rezultati

U istraživanju se najprije analizirala raspodjela sklonosti ispitanika prema svakoj dimenziji indeksa stila učenja Feldera i Solomanove. U Tablici 1 prikazana je klasifikacija sklonosti ispitanika na izrazitu, umjerenu sklonost prema dimenzijama indeksa stila učenja Felder i Solomanove te na uravnotežen odnos između dimenzija.

Učiteljski fakultet ($N = 109$)

Sklonost	Stil učenja							
	A	R	SE	I	VI	VE	SEK	GL
Izrazita	11	0	3	0	3	0	18	0
Umjerena	49	2	37	2	14	3	50	1
Uravnotežen odnos	39	8	52	15	57	32	30	10

A – aktivna, R – refleksivna, SE – senzorna, I – intuitivna, VI – vizualna, VE – verbalna, SEK – sekvencijalna, GL – globalna

Tablica 1. Distribucija stila učenja.

Preporuka je (Felder, Spurlin, 2005.) promatrati samo ispitanike koji imaju umjerenu ili izrazitu sklonost prema nekoj dimenziji, pa su za potrebe ovog istraživanja ti ispitanici svrstani u jednu kategoriju kao što je prikazano u tablici 2. Za kategoriju izrazite/umjerene sklonosti uzimaju se vrijednosti od 5 do 11 kod svake dimenzije indeksa stila učenja Feldera i Solomanove, a za uravnoteženi odnos vrijednosti od 3 do –3.

Rezultati su pokazali da 55.05% ispitanika ima sklonost prema aktivnoj dimenziji, 15.60% prema vizualnoj, a 0.92% prema globalnoj dimenziji.

Aktivna – refleksivna			Senzorna – intuitivna		
UM _A /I _A	U _{A-R}	UM _R /I _R	UM _{SE} /I _{SE}	U _{SE-I}	UM _{SE} /I _I
55,05%	43,12%	1,83%	36,70%	61,47%	1,83%
Vizualna – verbalna			Sekvencijalna - globalna		
UM _{VI} /I _{VI}	U _{VI-VE}	UM _{VE} /I _{VE}	UM _{SEK} /I _{SEK}	U _{SEK-GL}	UM _{GL} /I _{GL}
15,60%	81,65%	2,75%	62,38%	36,70%	0,92%

*UM – umjerena prednost, I – izrazita sklonost, U – uravnotežen odnos

Tablica 2. Klasifikacija prema intenzitetu sklonosti.

Prije same analize varijance napravljena je korelacijska analiza kako bi se istražilo postojanje linearnih veza između izabranog modula kao zavisne varijable i dimenzija stila učenja kao nezavisnih varijabli. Rezultati su pokazali da je sekvencijalno-globalna dimenzija u korelaciji sa odabranim modulom, no tipovi sekvencijalno-globalne dimenzije nisu u korelaciji s aktivnim, refleksivnim, vizualnim i verbalnim tipom stila učenja (na razini značajnosti $p < 0.05$).

Rezultati ANOVE su pokazali da postoji statistički značajna veza ($p < 0.05$) između kategorija modula i sekvencijalnog stila učenja ($F = 4.124$, $p = 0.019$), te između kategorija modula i globalnog stila učenja ($F = 4.124$, $p = 0.019$), tj. da se studenti koji su izabrali različite module razlikuju međusobno i po stilu učenja. Također, primijetilo se da se aktivni, vizualni, refleksivni, senzorni i intuitivni tipovi uklapaju u sve module, sekvencijalni tipovi pretežno biraju razvojni modul (modul A), a globalni tipovi češće biraju informatički modul (modul B). Detaljni rezultati metode ANOVA prikazani su u Tablici 3.

Stil učenja	Stupnjevi slobode	F	p
aktivni	106	0,478765	0,620881
refleksivni	106	0,478765	0,620881
senzorni	106	0,197495	0,821086
intuitivni	106	0,197495	0,821086
vizualni	106	0,161496	0,851079
verbalni	106	0,161496	0,851079
sekvencijalni	106	4,124447	0,018839
globalni	106	4,124447	0,018839

Table 3. Rezultati metode ANOVA.

Kako bi se istražilo postoji li mogućnost predviđanja da li će student biti iznad prosječno uspješan ukoliko je poznat modul koji želi izabrati, te informacije o njegovom stilu učenja, kreiran je model za klasifikaciju studenata s obzirom na uspješnost, pri čemu je korištena metoda k najbližeg susjeda. Rezultati u obliku stopa točnosti za pojedinu kategoriju studenata prikazani su u Tablici 4.

Algoritam za računanje udaljenosti	Stopa pogodaka za kategoriju 0 (studenti s prosjekom ispod 4)	Stopa pogodaka za kategoriju 1 (studenti s prosjekom iznad 4)	Prosječna stopa pogodaka
Euklidski, $k = 2$	75.00%	62.00%	68.5%

Table 4. Rezultati metode k-najbližeg susjeda u klasificiranju studenata prema uspjehu dobiveni na uzorku za testiranje.

Tablica 4 pokazuje da metoda k najbližeg susjeda uspješno razvrstava 75% studenata koji pripadaju u kategoriju 0 (onih koji imaju prosječnu uspješnost studiranja ispod 4), dok model točno prepoznaje 62% studenata koji imaju prosjek iznad ili jednak 4 (kategorija 1). Dakle, manje uspješni studenti su lakši za prepoznavanje od onih vrlo uspješnih prema primijenjenoj metodi. Prosječna stopa točnosti modela za obje kategorije studenata je 68.5%, što je bolje od slučajnog odabira, pa se model prema tome može smatrati uspješnim u klasificiranju studenata prema uspješnosti studiranja.

Zaključak

Cilj rada bio je utvrditi dominantni stil učenja studenata primjenom indeksa stila učenja Feldera i Solomanove, istražiti postoji li veza između dominantnog stila

učenja studenata i odabranog izbornog modula koristeći jednosmjernu analizu varijance, a zatim s pomoću metode strojnog učenja k -najbližeg susjeda kreirati model za predviđanje uspješnosti studenata s obzirom na izborni modul i stil učenja.

Istraživanje je pokazalo da većina naših ispitanika ima najmanju sklonost prema refleksivnom, intuitivnom i globalnom tipu stila učenja, a najviše ih izrazitu ili umjerenu sklonost imaju prema sekvencijalnom tipu. Rezultati ANOVE su pokazali da postoji statistički značajna veza ($p < 0.05$) između sekvencijalno-globalne dimenzije stila učenja i odabranog izbornog modula. Također, pokazalo se da su sekvencijalni i globalni tip stila učenja u korelaciji s odabranim modulom kojeg pohađaju studenti. Metoda k -najbližeg susjeda je na temelju ulaznih varijabli o stilu učenja studenata i izbornom modulu producirala prosječnu stopu točnosti od 68.5% u klasifikaciji studenata i time pokazala da je moguće kreirati model strojnog učenja koji će moći predvidjeti da li će student biti iznadprosječno uspješan ili ne. S obzirom da je ovo preliminarno istraživanje, dobiveni rezultati moraju se ograničiti na promatrani uzorak, te u budućem istraživanju poraditi na povećanju točnosti modela testiranjem i drugih metoda.

Kako je rad pokazao da studenti koji su izabrali pojedini modul imaju različite stilove učenja, preporuka za daljnja istraživanja je raditi na usklađivanju stila poučavanja odgovarajućim stilovima učenja, kako bi se olakšalo svladavanje gradiva što većem broju studenata i time povećala uspješnost studiranja. Na taj način, ovisno o dominantnosti stila učenja studenata, poučavatelji bi prilagodbom nastavnih materijala i metoda pridonijeli poboljšanju ishoda učenja.

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Samoeфикаsnost IKT studenata prve godine

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Sažetak. Svrha ove studije je istraživanje i vrednovanje postavki Bandurine socijalne kognitivne teorije samoeфикаsnosti na uzorku studenata prve godine Fakulteta organizacije i informatike na Sveučilištu u Zagrebu. Naše istraživanje je provedeno u ljetnom semestru akademske godine 2009./2010. u nastavi matematike na uzorku od 202 studenata. Nakon konfirmatorne faktorske analize, izdvojili smo 5 faktora koji mogu utjecati na studentsku samoeфикаsnost: iskustvena vještina uz društveni utjecaj, psihološka stanja, posredno iskustvo, ocjene i obiteljska potpora. Otkrili smo razlike u samoeфикаsnosti onih studenata koji su položili ispit iz matematike i onih koji ga nisu položili. Tri faktora utječu na samoeфикаsnost i uspješnih i manje uspješnih studenata: iskustvena vještina uz društveni utjecaj, psihološka stanja i posredno iskustvo. Od preostala dva, faktor ocjena utječe na samoeфикаsnost uspješnih studenata, a obiteljska potpora na samoeфикаsnost manje uspješnih. Više je razloga zašto je samoeфикаsnost važna za učenike i studente, a među najbitnijim jest to da je akademska samoeфикаsnost dobar pokazatelj akademskog postignuća.

Кljučne riječi: matematika, samoeфикаsnost, iskustvena vještina, posredno iskustvo, društveni utjecaj, psihološka stanja

Uvod

Samoeфикаsnost se definira kao stupanj uvjerenja koji ima pojedinac u svoje akademske sposobnosti da može uspješno ostvariti ponašanje koje je potrebno da bi se postigao određeni ishod (Bandura 1977).

Samoeфикаsnost je temeljito istraživana u posljednjih nekoliko desetljeća. Pokazalo se da postoje različite veze između nje i *straha od matematike* (osjećaji napetosti i anksioznosti koji utječu na baratanje brojevima i rješavanje matematičkih zadataka), *straha od ispitivanja*, *samopoštovanja* (opće vrednovanje sebe), *samoregulacije* (misli se na osviještenost i usklađivanje procesa mišljenja i strategija, te uključuje odabir i razvoj odgovarajućih strategija da bi se postigli eksplicitni i implicitni ciljevi učenja (Duncan & McKeachie, 2005)), *uspješnosti u rješavanju zadataka*, *matematičkog postignuća*, . . .

Na primjer, Jain i Dowson (2009) su pokazali da su faktori samoregulacije i samoeфикаsnosti pozitivno i statistički značajno povezani jedan s drugim, ali da su negativno povezani sa strahom od matematike. Hoffman (2010) je pokazao da je samoeфикаsnost važan pokazatelj *učinkovitosti rješavanja zadataka* za manje složene probleme o čemu svjedoči veća točnost i smanjeno *vrijeme rješavanja zadataka*. Ti rezultati daju nove informacije o ulozi straha od matematike i samoeфикаsnosti u odnosu na vrijeme rješavanja zadataka. Studenti s većim samopouzdanjem u rješavanje zadataka će pokazati veću točnost u tom dijelu zato jer ne moraju trošiti vrijeme na metode smanjivanja stresa (Hoffman, 2010).

Bandurina teorija samoeфикаsnosti

Bandura (1997) smatra da su uvjerenja o eфикаsnosti ključan čimbenik u sustavu ljudskih sposobnosti. Pokazuje se da različiti ljudi sa sličnim vještinama, ili pak ista osoba u različitim okolnostima može izvesti neku radnju loše, dobro ili izvrsno, ovisno o promjenama u vjerovanjima o vlastitoj eфикаsnosti.

Bandurina teorija pretpostavlja da na samoeфикаsnost utječu četiri faktora: *iskustvena vještina* (ME), *posredno iskustvo* (VE), *društveni utjecaj* (SP) i *psihološka stanja* (PH). Po toj teoriji, na samoeфикаsnost snažno utječu prethodna postignuća. Na primjer, pokazuje se da je iskustvena vještina posebice snažna kada pojedinci svladaju prepreke ili uspiju u zahtjevnoj zadaći pa tako uspješan rezultat u kolegiju može imati trajne učinke na samoeфикаsnost. Primjeri posrednog iskustva su: praćenje drugih i promatranje uspješnih kolega, učitelja i odraslih. Pod društvenim utjecajem misli se na roditeljsku, učiteljsku ili kolegijalnu potporu koja može poboljšati studentovo samopouzdanje u vlastite akademske sposobnosti. Primjeri psiholoških stanja su snažna anksioznost, stres, ćudljivost, umor – sve to može oslabiti samoeфикаsnost. Također, Bandurina teorija samoeфикаsnosti pretpostavlja da se ponašanje može mijenjati u skladu s očekivanjima o samoeфикаsnosti, koja mogu jačati ili slabiti, ovisno o različitoj povratnoj informaciji koju pojedinac dobiva (Bandura 1977).

Istraživanje

Naše istraživanje je provedeno u ljetnom semestru akademske godine 2009./2010. u nastavi matematike na uzorku od 202 studenta. Istraživanje je provedeno on-line unutar sustava za e-učenje Moodle i studenti su mu pristupili krajem drugog semestra. Ispunjavanje upitnika bilo je dobrovoljno, ali su se studenti motivirali s 1 bodom (od ukupno 100 koliko je moguće dobiti različitim aktivnostima na predmetu).

Postavili smo sljedeća istraživačka pitanja:

1. Koji faktori utječu na samoeфикаsnost u učenju matematike cijele grupe IKT studenata?
2. Da li postoje razlike u samoeфикаsnosti između uspješnih i manje uspješnih studenata, tj. onih koji su položili matematiku putem kontinuiranog praćenja

tijekom semestra ili temeljem ispita na kraju semestra, te onih koji nisu položili matematiku?

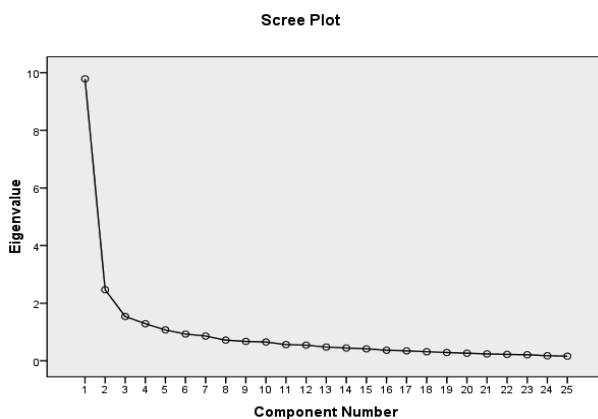
Anketa koju smo koristili preuzeta je iz "*Sources of self-efficacy in mathematics: A validation study*" (Usher, Pajares, 2009). Pitanja su bila prevedena na hrvatski i prilagođena za danu grupu studenata (IKT studenti prve godine na matematičkim predmetima). Dodano je jedno pitanje. Sve promjene su naznačene u popisu pitanja.

Pitanja iz ankete:

1. (ME1) Dobivam dosta bodova i dobre ocjene *na kolokvijima* i testovima iz matematike. (*Modificirano.*)
2. (ME5) Oduvijek sam dobar/-a u matematici.
3. (ME10) Čak i kad učim puno, postižem loše rezultate iz matematike.
4. (ME15) Dobio/-la sam dobre ocjene na zadnjem *kolokviju* iz matematike. (*Modificirano.*)
5. (ME20) Dobro mi ide rješavanje zadataka.
6. (ME25) Dobro rješavam čak i najteže zadatke.
7. (VA2) Kad vidim starije studente da im matematika ide dobro i ja se jače trudim.
8. (VA6) Kad vidim svojeg nastavnika kako rješava zadatke i ja mogu zamisliti da rješavam zadatak na isti način.
9. (VP11) Kad vidim da mojim kolegama/-icama matematika ide bolje nego meni, onda se i ja više trudim.
10. (VP16) Kad vidim svojeg kolegu/-icu kako rješava zadatke i ja mogu zamisliti da rješavam zadatak na isti način.
11. (VS21) Zamišljam si da mogu uspješno riješiti i izazovnije matematičke zadatke.
12. (VS9) Natječem se sam sa sobom u postizanju sve boljih rezultata.
13. (P3) Moj nastavnik/asistent mi je rekao da sam dobar/-a iz matematike.
14. (P7) Drugi su mi rekli da imam talent za matematiku.
15. (P12) Odrasli iz moje obitelji su me pohvalili kad sam bio/bila dobar/-a u školskoj matematici. (*Modificirano.*)
16. (P17) *Odrasli iz moje obitelji me i dalje pohvale kad postignem dobar rezultat iz kolokvija ili testa iz matematike.* (*Dodano.*)
17. (P22) Nagrađen/-a sam za svoje matematičke sposobnosti.
18. (P14) Kolege studenti/-ce su mi rekli da sam dobar/-a u matematici.
19. (P4) Moje kolege/-ice vole raditi matematiku sa mnom jer misle da mi dobro ide.
20. (PH8) Već time što sjedim na satu matematike osjećam se nervozno.
21. (PH13) Rješavanje zadataka iz matematike troši svu moju energiju.
22. (PH18) Osjećam se opušteno čim počnem raditi matematiku.
23. (PH23) Mozak mi zablokira i ne mogu jasno razmišljati kad radim matematičke zadatke.
24. (PH19) Loše se osjećam kad mislim na učenje matematike.
25. (PH24) Cijelo tijelo mi je napeto kad moram raditi matematiku.

Modificirali smo pitanja ME1, ME15 i P12 te dodali pitanje P17. Modifikacija je bila potrebna jer je originalna anketa bila rađena za učenike srednjih škola, a ne za studente te smo u pitanja ME1 i ME15 dodali “na kolokvijima”. Pitanje iz ankete “*Odrasli iz moje obitelji me hvale da sam dobar učenik iz matematike.*” podijelili smo na pitanja P12 i P17 čime smo htjeli razlikovati pohvalu odraslih iz obitelji za uspjeh učenika u školskoj matematici i pohvalu odraslih za uspjeh studenta na fakultetskoj matematici.

Kaiser-Meyer-Olkinova (KMO) mjera za cijelu grupu studenata je 0,916 što potvrđuje da su prikupljeni podaci prikladni za provođenje faktorske analize.



Slika 1. Dijagram za 25 varijabli. Na x-osi su varijable, a na y-osi svojstvene vrijednosti.

Faktori	Svojstvene vrijednosti		
	ukupno	% varijance	kumulativni %
1	9,789	39,156	39,156
2	2,467	9,867	49,022
3	1,538	6,151	55,174
4	1,287	5,148	60,322
5	1,074	4,297	64,619
6	,928	3,714	68,333
7	,858	3,432	71,764
8	,717	2,867	74,632
9	,673	2,692	77,323
10	,653	2,611	79,934
11	,562	2,247	82,181
12	,544	2,177	84,358
13	,476	1,903	86,262
14	,443	1,774	88,035
15	,412	1,649	89,685
16	,365	1,460	91,145
17	,343	1,373	92,518

18	, 314	1, 257	93, 775
19	, 287	1, 147	94, 922
20	, 263	1, 052	95, 974
21	, 238	, 952	96, 926
22	, 222	, 888	97, 815
23	, 210	, 840	98, 655
24	, 176	, 704	99, 359
25	, 160	, 641	100, 000

Tablica 1. Svojsvene vrijednosti, pojedinačni i kumulativni postoci varijance varijabli za cijelu grupu studenata.

Rezultati

Nakon konfirmatorne faktorske analize na *cijeloj grupi studenata* izdvojili smo 5 faktora (Slika 1. i Tablica 2.) koji mogu utjecati na samoefikasnost studenata, a to su: iskustvena vještina uz društveni utjecaj, psihološko stanje, posredno iskustvo, ocjene i obiteljska potpora. Postotak varijance objašnjene s tih 5 faktora je 64,619% (Tablica 1.)

	Faktori				
	Iskustvena vještina i društveni utjecaj	Psihološko stanje	Posredno iskustvo	Ocjene	Obiteljska potpora
ME1	, 320	–, 265	, 149	,801	, 116
VA2	, 039	, 027	,785	–, 047	, 147
P3	, 433	–, 088	, 368	, 405	–, 105
P4	,710	–, 135	, 301	, 277	, 121
ME5	,755	–, 403	–, 033	, 048	, 124
VP6	, 374	–, 380	,529	–, 012	–, 181
P7	,827	–, 233	, 119	–, 012	, 035
PH8	–, 123	,762	–, 197	–, 268	–, 007
ME10	–, 345	, 491	, 092	–, 260	, 273
VP11	–, 013	, 031	,798	, 112	, 171
P12	, 445	–, 174	, 159	–, 068	,630
PH13	–, 106	,715	, 107	–, 031	–, 082
ME15	, 240	–, 178	, 065	,852	, 002
VP16	, 173	–, 182	,671	, 092	–, 010
P17	, 002	–, 043	, 248	, 106	,824
PH18	, 400	–,591	, 196	, 185	, 094
ME20	,677	–, 329	, 107	, 344	, 088
VS21	,698	–, 223	, 079	, 170	–, 029
P22	, 403	–, 238	, 301	, 240	, 144

PH23	—, 402	,631	—, 054	—, 094	, 030
ME25	,686	—, 368	, 050	, 282	, 146
VS9	, 170	—, 091	,537	, 158	, 251
P14	,737	—, 185	, 208	, 332	, 058
PH19	—, 341	,731	—, 222	—, 156	—, 147
PH24	—, 261	,795	—, 147	—, 057	—, 084

Tablica 2. Rotirana matrica komponenta za cijelu grupu studenata.

Zatim smo cijelu grupu studenata podijelili na dvije podgrupe: *uspješni* (kolokvirali ili položili matematiku na ispitu) i *manje uspješni* (koji nisu položili matematiku).

KMO za studente iz grupe uspješni je 0,868, a za grupu manje uspješni 0,821 što također potvrđuje da su podaci prikladni za provođenje još dviju konfirmatornih faktorskih analiza.

Otkrili smo razlike u samoeфикаsnosti onih studenata koji su položili matematiku i onih koji nisu položili. Tri su zajednička faktora koja utječu na samoeфикаsnost i uspješnih i manje uspješnih studenata: iskustvena vještina uz društveni utjecaj, psihološka stanja i posredno iskustvo. Od preostala dva faktora, faktor ocjena utječe na samoeфикаsnost uspješnih studenata, a faktor obiteljske potpore na samoeфикаsnost manje uspješnih.

Postotak varijance objašnjene s četiri faktora koja utječu na samoeфикаsnost uspješnih studenata je 59,458%, dok je postotak varijance objašnjene s četiri faktora koja utječu na samoeфикаsnost manje uspješnih studenata 56,423%.

Ako faktorsko opterećenje varijable faktorom ispod 0,5 smatramo zanemarivim, te ako izbacimo iz faktorske analize varijable P3 i VS9 u grupi uspješni, te varijable ME1, ME10 i ME15 u grupi manje uspješni dobit će se da je postotak varijance objašnjene s 4 faktora za grupu uspješni 62,479% (i KMO 0,870), a za grupu manje uspješni 59,310% (i KMO 0,826).

S obzirom na značenje varijable P3 (*“Moj nastavnik/asistent mi je rekao da sam dobar/-a iz matematike.”*) možemo zaključiti da nastavnikova pohvala ne utječe na samoeфикаsnost uspješnih studenata. To ne čudi jer nastavnik, koji radi s velikom grupom studenata, nema prilike upoznati svakog pojedinog studenta i uspostaviti komunikaciju kakva je u manjim grupama.

Varijabla VS9 (*“Natječem se sam sa sobom u postizanju sve boljih rezultata”*) ne utječe na samoeфикаsnost uspješnih studenata jer većina dobrih studenata uglavnom želi samo sakupiti onoliko bodova koliko im je potrebno da bi položili predmet. To također potvrđuje i činjenica da je mali postotak dobrih, vrlo dobrih i odličnih ocjena na kraju semestra.

Dalje, može se zaključiti da varijable povezane s dobrim ocjenama i bodovima (ME1 i ME15) ne utječu na samoeфикаsnost manje uspješnih studenata jer do trenutka ispunjavanja ankete nisu dobili bodove koje oni percipiraju dobrim bodovima. To potvrđuje i varijabala ME10 (*“Čak i kad puno učim, postižem loše rezultate iz matematike.”*) koja se našla zajedno s ostale dvije varijable u grupi izbačenih varijabli.

Diskusija i zaključci

Na cijeloj grupi studenata, kao i na dvije podgrupe, spojila su se dva Bandurina faktora – iskustvena vještina i društveno uvjerenje – što ne iznenađuje jer su studenti koji imaju spoznaju o svojim dotadašnjim uspješnim postignućima češće pohvaljeni zbog istih. Obratno, oni studenti koji doživljavaju svoj trud u matematici kao uzaludan, skloniji su tumačiti povratnu informaciju na način da nisu dovoljno sposobni (Usher, Pajares, 2009; Bandura, 1997).

Kod uspješne grupe studenata posebno se izdvojio faktor ocjene – pitanje ME1 (*“Dobivam dosta bodova i dobre ocjene na kolokvijima i testovima iz matematike.”*) i ME 15 (*“Dobio/-la sam dobre ocjene na zadnjem kolokviju iz matematike.”*) – što i dalje potvrđuje da postoji snažna pozitivna veza između samoeфикаsnosti i matematičkog postignuća (Ayotola, Adeđeji, 2009).

Kod manje uspješne grupe studenata se uz faktor obiteljske potpore – pitanje P12 (*“Odrasli iz moje obitelji su me pohvalili kad sam bio/bila dobar/-a u školskoj matematici.”*) i P17 (*“Odrasli iz moje obitelji me i dalje pohvale kad postignem dobar rezultat na kolokvijima.”*) – grupiralo i pitanje ME15 (*“Dobilo/-la sam dobre ocjene na zadnjem kolokviju iz matematike.”*) s negativnim koeficijentom korelacije. To pokazuje da je i studentima, a ne samo učenicima važna pohvala i potpora u obitelji te da manje uspješni studenti imaju jače izražen strah od razočaranja roditelja. Dalje, kod uspješne grupe studenata nije se izdvojio faktor obiteljske potpore (pitanja P12 i P17) što ne znači da uspješnim studentima nije važna potpora obitelji, već da na njihovu samoeфикаsnost neće utjecati strah od razočaranja roditelja. Živčić-Bećirević (2003) navodi da uspješniji studenti češće koriste ohrabrujuće misli, imaju jače izraženu motivaciju za učenje i interes za gradivo te manje izražen strah od razočarenja roditelja.

Ovo istraživanje daje korisne informacije o faktorima koji grade samoeфикаsnost kod studenta te nam daju uvid o tome koliko je važna potpora okoline da bi se podigla postignuća u učenju matematike. Informacija o samoeфикаsnosti može pomoći nastavnicima da prilagode svoje strategije poučavanja i načine na koje komuniciraju sa studentima tako da u nastavnom procesu daju najbolju moguću povratnu informaciju studentima. Na kraju, važno je istaknuti ulogu pohvale u podizanju samoeфикаsnosti bez obzira na dob učenika/studenta i poznatu činjenicu da “ništa ne uspijeva kao uspjeh”.

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Istraživačka nastava matematike u osnovnoj školi: prednosti i primjeri (poster)

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Sažetak. Poznata je Konfucijeva izreka: “Čujem i zaboravim. Vidim i zapamtim. Učinim i razumijem.” Istraživačka nastava poznat je oblik nastave, no u matematici se iz raznih razloga dosta rijetko koristi. U nastavi prirodnih znanosti posljednjih se desetljeća sve više razvija pristup u kojem učenici otkrivaju zakonitosti, te se potrebni pojmovi i ostala teorija objašnjavaju tek nakon što su, primjerice temeljem eksperimenta, doneseni određeni zaključci. Matematika je pak više poznata kao teorijska, nego kao eksperimentalna znanstvena disciplina te se stoga čini da je bitno teže razviti nastavne planove koji bi bili prikladni za istraživačku nastavu. S druge strane, matematička formalnost nije sama sebi svrhom, iako se tako čini mnogom djetetu tokom škole i taj stav zadrži i kao odrasla osoba. Kroz istraživačku (kao i primjerice problemsku i projektnu) formule se mogu uvesti postepeno, na kraju, uslijed potrebe za skraćanjem opisa postupka.

U mnogim je radovima isticana prednost učenja otkrivanjem, a posebice za učenje geometrije otkrivanjem postoje i mnogi razrađeni primjeri, vidi npr. Varošanec, S., (2006). Cilj je ovog priopćenja povećati fond ideja za provođenje istraživačke nastave matematike, posebice za negeometrijske teme, te tako potaknuti veće korištenje ovog oblika u redovnoj nastavi matematike u osnovnoj školi. Nadalje, istaknut će se kako često korišteni argument “nemamo vremena da od klasične nastave otkinemo dio vremena za istraživačku/projektnu nastavu” zapravo nije argument, jer se ovim drugim pristupom obrađuje isto gradivo, ali se pritom povećava razumijevanje i ujedno interes učenika za matematiku.

Ključne riječi: istraživački oblik nastave, projektna nastava, matematički eksperimenti, pristup konstruktivističkog procesiranja informacija u učenju i poučavanju, interaktivni konstruktivistički pristup učenju i poučavanju

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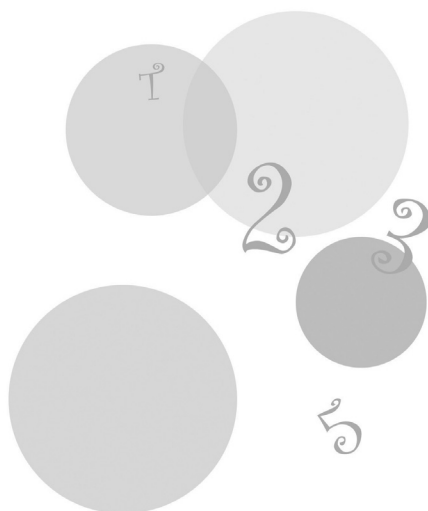
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4.

Učitelj i učenici u nastavnoj praksi



U četvrtom poglavlju govori se o rezultatima istraživanja i/ili dobrim primjerima strategija poučavanja koji rezultiraju poželjnim ishodima učenja djece i učenika. Učitelji iz prakse prednost daju aktivnim metodama učenja i poučavanja, a kod djece mlađe školske dobi, stručno osmišljenoj igri. Neki od autora smatraju da u prvom redu, osobito u školskoj matematici treba, kad god je to moguće, zamijeniti inzistiranje na formalizmima tzv. eksperimentalnim aspektima matematike. Prema njihovom mišljenju, studenti učiteljskih studija trebaju ovladati interaktivnim konstruktivističkim pristupom učenju i poučavanju matematike te biti spremni na popularni pristup u nastavi matematike, kad god se za to pokaže prilika. Autori izvještavaju da se među roditeljima, učiteljima i u društvu općenito sve bolje prepoznaje važnost ranoga matematičkoga obrazovanja. Također, sve je više roditelja zainteresirano za uključivanje njihove djece s teškoćama u redovitu nastavu na što nas obavezuje nacionalna strategija razvoja Republike Hrvatske. Učitelji iz prakse upozoravaju da su u tom segmentu ponijeli nedostatna znanja i vještine s fakulteta te ukazuju na potrebu razvoja kurikula kojim bi se studenti učiteljskih studija osposobljavali za prepoznavanje, izobrazbu i podršku učenika s teškoćama.

Matematički eksperimenti

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Sažetak. Matematika je poznata kao teorijska znanost, a zbog svog formalizma spada u najnepopularnije školske predmete i znanosti. Suvremeni trend popularizacije matematike, koji se intenzivno i sustavno razvija na međunarodnoj razini u posljednjih desetak godina, još ne pokazuje veće efekte na promjenu stavova učenika prema matematici. Neovisno o svim širim aktivnostima, iznimna je uloga učitelja/nastavnika matematike kao popularizatora matematike, jer svojim pristupom direktno doprinosi formiranju stavova učenika koje je kasnije teško mijenjati.

Jedna od glavnih prepreka uspješnoj popularizaciji matematike je u tome što se radi o teorijskoj znanosti, za koju se u usporedbu s drugim prirodnim znanostima čini da je teško i vremenski zahtjevno osmisliti aktivnosti koje povezane sa školskim programima i pritom u njima učenici aktivno, ne samo intelektom i pisanjem, otkrivaju matematičke zakonitosti i ljepotu matematike. Matematiku gotovo nitko, čak ni većina znanstvenika matematičara i nastavnika matematike, ne doživljavaju kao eksperimentalnu znanost. Cilj ovog priopćenja je pokazati da je, posebice na školskoj razini, matematika ima i eksperimentalne aspekte te prikazati konkretne primjere aktivnosti prilagođenih djeci koje ističu taj aspekt matematike.

Gljučne riječi: popularizacija matematike, učenje otkrivanjem, interaktivna matematika, pristup konstruktivističkog procesiranja informacija u učenju i poučavanju, interaktivni konstruktivistički pristup učenju i poučavanju

Što je dobro obrazovanje? Sustavno davanje mogućnosti učeniku da sam otkrije stvari.¹

(George Polya, *How to solve it?*, 1945.)

Uvod

Matematika je među najnepopularnijim školskim predmetima i znanostima. Suvremeni trend popularizacije matematike, koja se nakon Svjetske godine matematike 2000. intenzivno i sustavno razvija na međunarodnoj razini, još nema bitnih

¹ What is good education? Systematically giving opportunity to the student to discover things by himself.

efekata na promjenu stavova učenika prema matematici. Uloga nastavnika matematike kao popularizatora matematike je izuzetna, budući da se stavovi o matematici (i drugim znanostima) razvijeni tokom školovanja teško kasnije mijenjaju. Jedna od glavnih prepreka uspjehu popularizacije matematike je ta da se radi o teorijskoj znanosti. Štoviše, često su njene formalne strane u prvom planu, a stvarni sadržaj (i metode) koje one predstavljaju se teško može otkriti. Gotovo nitko, čak ni znanstvenici matematičari ni nastavnici matematike, ne doživljava matematiku kao eksperimentalnu znanost. U jako strogo shvaćenom značenju “eksperimentalne znanosti” to se može prihvatiti kao točno, no s druge strane matematičko eksperimentiranje – s brojevima, oblicima, idejama – može dovesti do otkrića struktura i svojstava, formulacija hipoteza, čak i do prvih ideja dokaza.

U ovom ćemo se članku ograničiti na uži smisao matematičkog eksperimenta: razmatrat ćemo one aktivnosti koje – uz cilj otkrivanja neke matematičke činjenice logičkim zaključivanjem – imaju i izrazitu interaktivnu komponentu. Kako je dobro poznato, aktivno učenje povećava razumijevanje činjenica, te matematički eksperimenti mogu pomoći boljem razumijevanju matematike, ali i u mijenjanju stavova djece o matematici. Dok u višoj matematici nije lako naći teme koje se zgodno mogu prilagoditi nastavnom satu, a da pritom imaju izrazitu eksperimentalnu i otkrivačku komponentu, osnovno- i srednjoškolski nastavni planovi obiluju temama koje se mogu obrađivati putem eksperimenta i interaktivno. Prilično je očito da se gotovo sve školske geometrijske teme mogu uvesti i objasniti kroz otkrivačku nastavu, a većina istraživanja geometrijskih svojstava se na dosta očigledan način mogu formulirati kao interaktivni eksperimenti. No, manje je očito da je to istina i za većinu aritmetičkih i algebarskih tema.

Cilj je ovog članka dati nekoliko primjera eksperimentalnog pristupa nekim standardnim temama koje se susreću u nastavi matematike. Iako se možda čini napornim i vremenski zahtjevnim da se razviju i pripreme matematičke aktivnosti koje su u skladu s nastavnim planovima, a istovremeno takve da učenici aktivno – pri čemu mislimo ne samo na misaonu aktivnost i pisanje – otkrivaju matematičke zakonitosti i ljepotu matematike, ovih nekoliko primjera zamišljeni su tako da pokažu da za mnoge matematičke teme nije teško naći ideju za preoblikovanja standardnog plana nastavnog sata u onaj temeljen na učenju otkrivanjem.

Eksperiment 1: Otkrivanje smisla znamenaka putem jednog magičnog trika

Sljedeći matematički trik [1] je dobro poznat. Izvođač odabire sudionika i daje mu (ili joj) sljedeće upute:

- (1) da odabere troznamenasti broj kojem su prva i zadnja znamenka različite;
- (2) zatim treba obrnuti redoslijed znamenki;
- (3) slijedi oduzimanje manjeg od većeg od dvaju brojeva iz koraka (1) i (2);
- (4) rezultatu iz koraka (3) treba obrnuti znamenke, uz napomenu da ako je rezultat iz (3) dvoznamenkast, treba ga shvatiti kao troznamenkast stavljajući nulu kao prvu znamenku;
- (5) na kraju treba zbrojiti brojeve iz koraka (3) i (4).

Nakon toga izvođač iznenađuje publiku pogađanjem konačnog rezultata: taj je, naime, uvijek 1089.

Izvođenje ovog trika je jedan od najlakših i najzabavnijih načina započinjanja eksperimentalnog istraživanja koje omogućuje otkrivanje značenja znamenaka i njihovih pozicija. Eksperimentalni se dio sastoji u isprobavanju više različitih početnih brojeva i otkrivanju uzoraka. To bi trebalo dovesti do formulacije hipoteze: “Rezultat je uvijek 1089.”. No, može se postići i puno više – neka će djeca imati ideju isprobati troznamenkasti broj kojem se prva i zadnja znamenka podudaraju i tako doći do preciznije formulacije hipoteze: “Rezultat postupka je 1089 ako i samo ako početni troznamenkasti broj ima različitu prvu i zadnju znamenku.”. Eksperimentalni računi također daju prve ideje dokaza, više ili manje formalnog ovisno o uzrastu djeteta, a koji se svodi na odgovor na očigledno pitanje: “Zašto je hipoteza točna?”. Mogu se postaviti i mnoga potpitanja koja će pomoći učenicima u otkrivanju dokaza, ili bar dobivanju osjećaja za potrebne korake (npr. “Zašto je bitno da se prva i zadnja znamenka razlikuju?”). Moguće su i generalizacije (čak je moguće da će neka djeca predložiti vlastite): Što ako počnemo s dvoznamenkastim brojem? Četvero- ili višeznamenkastim? Na višoj razini, problem se može generalizirati i na račun u drugim bazama. Više pitanja vezanih za ovaj trik, a koja se sva mogu analizirati eksperimentalno, mogu se naći u [2].

Ovaj “Trik 1089” lijepo pokazuje razliku između eksperimentalnog otkrivačkog pristupa učenju i klasičnog pristupa tipa “zadatak-rješenje”. Čak i da se isto svojstvo odabere za ilustraciju smisla znamenaka u dekadskom sustavu, pristup tipa “zadatak-rješenje” u kojem bi đaci kao zadatak dobili dokaza tvrdnje “Ako se počne s troznamenkastim brojem kojem se prva i zadnja znamenka razlikuju i zatim provedu koraci (1)–(4), rezultat će uvijek biti 1089.”, ne samo da bi vjerojatno bio pretežak za većinu đaka, nego bi prikrio veći dio stvarne matematike vezane za ovaj problem. Pritom ovdje pod “stvarna matematika” mislimo na: sposobnost formuliranja i ispitivanja hipoteza, nalaženja primjera i protuprimjera, te otkrivanje mogućih generalizacija (ili ograničenja) matematičkih tvrdnji, a što sve dovodi do otkrivanja potrebe za dokazom i nalaženje istog.

Eksperiment 2: Otkrivanje aritmetičkih nizova putem još jednog magičnog trika

Više magičnih trikova [3], [4] koristi listove kalendara, a matematičku pozadinu funkcioniranja trika čine svojstva aritmetičkih nizova. Jedan takav primjer je sljedeći jednostavni trik: Izvođač traži od sudionika da zaokruži četiri broja u jednom stupcu nekog lista (mjesečnog) kalendara te da zbroji ta četiri broja. Čim sazna zbroj, izvođač može reći o koja se četiri broja radi. Naime, ti brojevi su članovi aritmetičkog niza s razlikom 7, dakle je njihov zbroj 42 uvećan za četverostruki prvi broj, te izvođač od zbroja samo treba oduzeti 42 i podijeliti s 4. Gdje je ovdje eksperiment? Kao prvo, za uočavanje spomenutog svojstva djeca mogu isprobati različite listove kalendara i stupce i tako uočiti pravilo o zbrajanju članova aritmetičkog niza. No, postoje i mnoga druga srodna pitanja na koja se odgovor može potražiti istraživanjem: Što ako se uzmu po tri ili pet brojeva u stupcu? Što s redovima? Funkcionira li trik s drugim pravokutnim tablicama brojeva? Možete li konstruirati neke takve tablice, a da nisu sa (svim) brojevima od 1 do 31? Možete li formulirati opće pravilo? Da bi odgovorila na takva pitanja, djeca moraju isprobati razne mogućnosti: trebaju eksperimentirati.

Eksperiment 3: Otkrivanje zbrajanja korištenjem magičnih kvadrata

Magični kvadrati [5], [6] su fascinirajući matematički objekti i očigledno je da ih se može iskoristiti za uvježbavanje zbrajanja. No, mogu se koristiti i u samim počecima učenja zbrajanja prirodnih brojeva: ako brojke reprezentiramo primjerice točkicama kao na dominama, svojstvo “magičnosti” moći će otkriti čak i mala djeca koja još ne znaju zbrajati, jednostavno prebrajajući točkice. Eksperimentalni dio može se primjerice sastojati u konstrukciji različitih magičnih kvadrata. Pritom je (pogotovu s manjom djecom) najbolje raditi s 3×3 kvadratima, ali dozvoliti ne samo klasičnu varijantu popunjavanja brojevima od 1 do 9, nego dopuštajući djeci da isprobaju i druge brojeve te da mijenjaju uvjete magičnosti (npr. tako da izostave uvjet da dijagonale moraju imati isti zbroj kao i retci i stupci). Kako konstruiraju – u najgorem slučaju prebrajanjem točkica – djeca će, sa ili bez pomoći nastavnika, primijetiti da primjerice redak/stupac kvadratića u kojima su redom 8, 1 i 6 točkica uvijek daju ukupno 15 točkica. Uz odgovarajuće navođenje pitanjima otkrit će i komutativnost – zbroj ne ovisi o tome kojim redom prebrajaju točkice.

Eksperiment se dalje može proširiti na klasičnu notaciju s brojevima umjesto točkica, uvježbavanje oduzimanja (nadopunjavanje nedostajućih brojeva u magičnom kvadratu znajući magični zbroj), nalaženje multiplikativnih magičnih kvadrata kako bi se uvježbalo množenje, te naravno na kvadrate različitih veličina, drugačije tablice, pa čak i na tri dimenzije (“magične kocke”). Napomenimo i da konstrukcija magičnih kvadrata s razlomcima, decimalnim brojevima, općim realnim (čak i kompleksnim) brojevima može pomoći u uvježbavanju aritmetike s navedenim vrstama brojeva. U svim slučajevima postoji koristan dodatni efekt: vrlo je vjerojatno da će djeca otkriti i razna svojstva magičnih kvadrata, primjerice da do na rotacije i zrcaljenja postoji samo jedan magični kvadrat s brojevima 1 do 9.

Eksperiment 4: Otkrivanje razlomaka pomoću popločavanja

Dobro je poznato [7] da djeca (pa čak i odrasli) često imaju problema s razumijevanjem razlomaka i izvođenjem aritmetičkih operacija s istima. Postoje razni načini (neki su navedeni u [7]) da se pomogne riješiti taj problem, a većina njih imaju interaktivni aspekt, npr. kad se kao zadatak daje bojanje dijelova (razlomaka) neke slike u različitim bojama. Varijanta na tu temu može se ostvariti korištenjem popločavanja pravilnim mnogokutima (jednakostraničnim trokutima, kvadratima i pravilnim šesterokutima). Pločice je lako napraviti od kartona. Glavna prednost ovakve varijante u odnosu na korištenje zadane slike podijeljene na dijelove koji se koriste za uvođenje u pojam razlomka je da koristeći popločavanja lako variramo nazivnik (broj pločica) te da popločavanja možemo koristiti i za uvođenje aritmetičkih operacija (posebno jednostavno: dijeljenje razlomka prirodnim brojem). Eksperimentalna verzija se može sastojati u tome da tražimo od djece da konstruiraju primjere zadanih razlomaka, što može dovesti i do otkrića kraćenja (2 od 8 pločica su podjednako primjer razlomka “jedna četvrtina” kao što je 5 od 20 pločica). Također, djeca mogu eksperimentirati s različitim oblicima i tako otkriti nezavisnost razlomka (broja) od objekta na koji se odnosi (primjerice: dva kvadrata popločavanja s pet kvadrata zaslužuju biti opisana istim imenom kao četiri šesterokuta popločavanja s njih deset – razlomak “dvije petine”). Mnoge

druge činjenice o razlomcima i njihova svojstva se također mogu otkriti eksperimentirajući s popločavanjima, primjerice grupiranja u “egipatske razlomke” [8] čime se uvježbava zbrajanje i oduzimanje razlomaka. Drugačiji eksperimentalni pristupi razlomcima mogu se postići koristeći origami [9], kuhanje [7], glazbu [7] te očigledno mjereći različite fizikalne veličine (vrijeme, duljinu, površinu, volumen, ...).

Eksperiment 5: Otkrivanje površina likova pomoću tangrama

Iako je tangram dobro poznato didaktičko pomagalo, nažalost se rijetko koristi u nastavi matematike. To je posebno šteta jer se može koristiti vezano za različite matematičke teme i s različitim uzrastima. Tangram je koristan pri učenju osnovne terminologij (trokut, kvadrat, paralelogram, ...), razvoj konceptata površine i opsega, kuteva, sukladnosti i sličnosti, konveksnosti, simetrije, razlomaka i iracionalnih brojeva (kvadratni korijen iz 2), čak i analitičke geometrije i kombinatorike [10], [11] kao i za neke naprednije teme [12]. Dodatne koristi su razvoj sposobnosti logičkog zaključivanja i otkrivanje da (matematički) problemi mogu imati više točnih rješenja (ili pak uopće nemati rješenje). Izvora za ideje ima mnogo [13], [14], pa ćemo dati samo jednu ideju za eksperiment s tangramom. Pretpostavimo da svako dijete ima svoj kompletni tangram (primjerice tako da je svatko za domaću zadaću trebao napraviti jedan iz kartona) i da želite predavati o površini koristeći jedinični kvadrat kao mjeru. Najmanji kvadrat tangrama može poslužiti kao jedinica površine, a djeca mogu istraživati (eksperimentirajući: koristeći dijelove tangrama i crtajući ih u bilježnice) površine pojedinih dijelova tangrama kao i različitih trokuta i četverokuta (i drugih mnogokuta) koji se mogu složiti iz dijelova tangrama. Pritom je vrlo vjerojatno da neće sva djeca odrediti površine istih likova jer postoji mnogo likova koji se mogu složiti iz dijelova tangrama², tako da će postojati dosta prostora za komunikaciju i razmjenu ideja. Istraživanje se može dodatno proširiti koristeći druge srodne slagalice, bilo već poznate [15], [16] ili vlastite.

Diskusija

Po mišljenju – zapravo, čvrstom uvjerenju – autora, moguće je razviti planove nastavnih sati matematike (na temelju bilo kojeg standardnog kurikulumu), posebno za osnovnu školu, koji su u potpunosti temeljeni na učenju otkrivanjem.

Prethodni su primjeri bili zamišljeni kao izvor inspiracije. Posebice, njihova je svrha bila da posluže kao “dokaz” da nije teško osmisliti eksperimentalne i istraživačke koncepte za korištenje u nastavi matematike. Općenito, ideje za eksperimente lako je naći i doraditi za geometriju, a uz malo kreativnosti nije puno teže oblikovati planove nastavnih sati na temelju eksperimenta i istraživanja za sve, ili bar skoro sve, teme koje uobičajeni matematički kurikulumi predviđaju za osnovnu (i srednju) školu. Također, ne smije se zaboraviti i očigledna mogućnost

² Koristeći svih sedam dijelova tangrama može se složiti trinaest konveksnih poligona: jedan trokut, šest četverokuta i po tri peterokuta i šesterokuta. To su 1942. godine dokazali kineski matematičari Fu Tsiang Wang i Chuan-Chih Hsiung.

matematičkog eksperimenta korištenjem kompjuterskih programa (proračunske tablice, programi za dinamičku geometriju, itd.), iako je njihova mana u odnosu na klasičnije interaktivne metode da se mora pretpostaviti (vjеровати) računalu (npr. da računalo “zna” što je kvadratna funkcija i da pravilno prikazuje njen graf).

N. B. Eksperimentalni pristup *ne* eliminira potrebu za matematičkim dokazom. Iako postoji mogućnost da ovim pristupom djeca otkriju matematička svojstva i uvjere su u njihovu istinitost, a da pritom ne vide potrebu za dokazom, jedan od zadataka nastavnika (osim da bude koordinator eksperimentalnih aktivnosti i nadgleda ih) je da ukaže na to da ideje koje dobijemo iskustvom time još nisu dokazane te da navede učenike na zaključak da je dokaz potreban. No, čak i za otkrivanje potrebe matematičkog dokaza eksperimentalni pristup nerijetko može biti koristan: u mnogim će slučajevima djeca primijetiti potrebu efikasnijeg pristupa nego jednostavnog ispitivanja svih mogućih slučajeva, čak i ako ih je konačno mnogo, a kroz same rezultate svojih istraživanja otkriti i pravilnosti koje im mogu pomoći u tom efikasnijem pristupu i tako na prirodan način doći do osnovnih ideja dokaza.

Mogući problemi

Neki nastavnici – a vjerojatno ne samo oni – sad će reći da je učenje otkrivanjem doduše zgodna ideja, no skupa je, zahtijeva previše truda, i/ili nekompatibilna je s trenutnim kurikulumom te da nemaju vremena ili sredstava da ju primijene izuzev možda povremeno. Dozvolite da probam dati neke odgovore na te prigovore.

Prvi prigovor – troškovi – lako je odbaciti. Naime, sva pomagala za interaktivnu nastavu matematike mogu se napraviti ili nabaviti za vrlo malo novaca [17]. Za većinu takvih aktivnosti dovoljni su papir, karton, škare, ljepilo, pisaljke, mali predmeti poput kamenčića, kocke i karte za igru, dnevne novine i slično – a sve je to lako i jeftino za nabaviti, zar ne?

Drugo, iako je istina da istraživačka nastava matematike zahtijeva veću kreativnost nastavnika nego, primjerice, za kemiju u kojoj je (ili bi bar trebalo biti) mnogo očiglednije kako koncipirati i koristiti eksperiment u nastavi, ipak postoji dovoljno dobrih i prikladnih postojećih ideja, kako u knjigama [18], [19], [20], tako i na internetu [21]. Tako i pet eksperimenata opisanih u ovom članku nisu originalne ideje autora, nego varijante na teme koje već postoje u drugim izvorima. Napor koji je potrebno uložiti za nalaženje ideja i njihovo prilagođavanje upotrebi u nastavi je bitno manji od konačnog efekta (a u krajnjoj liniji, jednom razvijeni planovi nastavnih sati moći će se puno puta iznova koristiti). Također, moglo bi se dogoditi da umorni nastavnik odjednom otkrije koliko je matematika zabavna. Da, čak i za nastavnika matematike. Nikako se ne smije podcijeniti efekt kojeg ima nastavnik koji uživa u onome što podučava. Kako kaže Jerry P. King³ u svojoj

³ Govoreći o školskom satu matematike, King kaže (slobodan prijevod F.M.B.): *Treća grupa u toj prostoriji imala je samo jednog člana: nastavnika. Ta je osoba imala tri karakteristike za koje smo uočili da su više ili manje zajedničke srednjoškolskim nastavnicima matematike koji su slijedili: nije volio matematiku, nije razumio matematiku, nije smatrao da je matematika bitna. ... I dalje, objašnjavajući kako su učenici to zaključili, King kaže: Podučavao je matematiku radnim danom s manje entuzijazma nego što ga je imao subotom za košenje trave u svom vrtu.*

knjizi *The Art of Mathematics* [24]: jedan od važnih razloga zašto učenici ne vole matematiku je da mnogi nastavnici matematike i sami odaju dojam da ju ne vole. Ipak, ovdje je potrebno malo opreza: nije svaki “zabavni” matematički zadatak pogodan za korištenje u nastavi, a mnoge dobre ideje trebaju se pažljivo prilagoditi upotrebi u nastavi.

Treći prigovor je malo osjetljiv, ali čest. Većina koji prigovaraju konceptu istraživačke nastave kažu da je doduše učenje otkrivanjem vrlo dobra, no vremenski prezahtjevna ilustracija određene nastavne teme te oduzima vrijeme potrebno za “objašnjavanje teorije”. Tu je bitno primijetiti da istraživačka nastava nije dodatak, već zamjena za klasični *oblik* – ne radi se o promjeni *sadržaja*. U ovom je pristupu “teorija” zaključak, jasno iskazana tvrdnja o činjenicama koje su otkrili sami učenici. Mnogi [25] (ne samo konstruktivisti) slažu se da dijete bolje uči – u smislu razumijevanja, a ne samo memoriranja činjenica i operiranja različitim formulama – ako je aktivno uključeno u proces učenja. To je posebice točno za mlađu djecu, uzevši u obzir da je poznato da otprilike do svoje dvanaeste godine djeca uče na konkretan način. Dodatna je prednost istraživačke nastave da omogućuje lakše prilagođavanje različitim stilovima učenja i sposobnostima. Ovaj oblik nastave očigledno podržava i opći suvremeni trend [26] u nastavi matematike i znanosti, a to je razvoj kurikuluma i nastavnih sati koji razvijaju sposobnosti rješavanja problema.

Umjesto zaključka...

... pitanje za nastavnike:

Ako imate izbor između oblika nastave

- 1) koji zahtijeva malo truda, jer ga vi i svi drugi odavno koristite, i kako je pokazano da ima mana i ne inspirira ni nastavnika ni učenika ili onog
- 2) koji zahtijeva više truda, uglavnom jer je nov i ne postoji puno pripremljenih nastavnih materijala za njega, ali obećava bolja učenička postignuća kao i veću zanimljivost i za nastavnika i za učenika,

koji oblik biste odabrali?

Članak završavamo kako smo ga i započeli – citatom znamenitog mađarskog matematičara George-a Pólya-e (1887.–1985.)⁴:

Matematika prezentirana u euklidskom stilu čini se sustavnom, deduktivnom znanošću; no, matematika u svom nastajanju čini se eksperimentalnom, induktivnom znanošću. Obje strane su stare koliko i matematika kao znanost. No, u jednom pogledu druga je strana ipak nova: matematika *in statu nascendi*, u nastajanju, nikad dosad nije baš tako predstavljena učeniku, čak ni učitelju, niti široj publici. (George Polya, *How to solve it?*, 1945.).

⁴ Mathematics presented in the Euclidean way appears as a systematic, deductive science; but mathematics in the making appears as an experimental, inductive science. Both aspects are as old as the science of mathematics itself. But the second aspect is new in one respect; mathematics “in statu nascendi,” in the process of being invented, has never before been presented in quite this manner to the student, or to the teacher himself, or to the general public.

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Prirodno učenje matematičkih pojmova u dječjem vrtiću

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Sažetak. Polazeći od teorije socijalnog konstruktivizma, važnosti kvalitete okruženja i samomotiviranosti djece za učenje činjenjem u radu se prikazuje simultani razvoj razumijevanja pojmova: količine, broja, težine i njihovih odnosa kao matematičkih kategorija kod djece predškolske dobi. Rad se utemeljuje na primjeni etnografske metode kojom su se tijekom dvije godine u dječjem vrtiću “Dječji svijet” u Varaždinu prikupljali i interpretirali podaci o dječjem prirodnom, praktičnom učenju početnih matematičkih pojmova u socijalnim interakcijama s drugom djecom i odgojiteljem kroz eksploraciju funkcioniranja vage i spontano nastalih i rješavanih praktičnih problema u svezi s tim. Naknadna zajednička refleksija etnografskih zapisa koju su kontinuirano provodili odgojitelji i pedagog, omogućila je da se putem stručnog dijaloga dogovaraju i primjenjuju promjene u materijalnoj ponudi za eksploraciju i rješavanje problema kod djece kao i promjene u socio-pedagoškom ozračju odgojnih skupina. Pretpostavka da će se u kvalitetnijim uvjetima okruženja za učenje amplificirati postojeća dječja iskustva i stjecati nova u cilju boljeg razumijevanja osnovnih matematičkih pojmova i holistički djelovati na sve aspekte dječjeg razvoja se potvrdila. Dobiveni rezultati pokazuju neposrednu povezanost kvalitete okruženja (socijalnog i fizičkog) s kvalitetom dječjeg učenja i razumijevanja navedenih matematičkih pojmova. Materijalizacija dječjeg mišljenja i razumijevanja matematičkih pojmova putem akcije, opisana i interpretirana u nizu praktičnih egzemplara dječjeg učenja u ovom radu, pokazuje kako okruženje “hrani” učenje.

Ključne riječi: učenje predškolskog djeteta, početni matematički pojmovi, okruženje za učenje, razumijevanje matematike u akciji

Uvod

Socio-konstruktivistička teorija učenja utemeljena na radovima Piageta (1963), Vigotskog (1977) i Brunera (2000) ističe izgradnju razumijevanja svijeta na temelju vlastitog iskustva i prethodnog znanja ljudske jedinice u kolaborativnom procesu s drugima u ovisnosti od kulture kojoj pripada. Tako Piaget za stvaranje

mentalnih struktura u prvi plan stavlja spoznaju o fizičkim svojstvima predmeta kao uvjetu razvoja mišljenja; Vigotski uz značaj dječjeg praktičnog iskustva naglašava važnost socijalnog iskustva u procesu učenja; a Bruner govori o akcijskom, ikoničkom i simboličkom sustavu kroz koji dijete prolazi do najvišeg stupnja reprezentacije stvarnosti. Verbalno-pojmovnom i logičko-matematičkom mišljenju, dakle, prethodi opažajno-praktičko i opažajno-predodžbeno mišljenje. I upravo se na ovim teorijskim spoznajama treba temeljiti osnove metodičkog pristupa učenju osnovnih matematičkih pojmova djece predškolske dobi. Djeca predškolske dobi matematičke pojmove razvijaju postupno, a tu postupnost je Liebeck (1995,11) prikazala kao:

- I** – iskustvo s fizičkim predmetima
- G** – govorni koji opisuje to iskustvo;
- S** – slike koje prikazuju to iskustvo i
- Z** – pismeni znakovi koji generaliziraju to iskustvo.”

Iz gore navedenih razloga mnogi autori učenje matematike u ranoj dobi nazivaju iskustvenom matematikom, situacijskim učenjem ili matematikom u kontekstu (Seefeldt, Barbour, 1994). U takvim uvjetima učenje matematike se ne odvaja od konteksta u kojem dijete stječe iskustvo, jer je učenje sastavni dio okoline pa se situacije učenja mogu nazvati “suprodukcijom znanja kroz aktivnost” (Mortimore 1999, 11).

Kvaliteta okruženja za učenje ima, dakle, presudan utjecaj na proces i učinke učenja. Pritom se pod okruženjem podrazumijeva fizičko okruženje u kojem se socijalna akcija učenja pojavljuje, participirajući u odnosima u kojima se te aktivnosti događaju. Budući da je ono što se uči jako povezano sa situacijom u kojoj se uči, potrebno je da odgajatelj dobro dekodira aktualne dječje interese za učenje kako bi na njima gradio poticajno, stimulativno okruženje, kreirao motivirajući kontekst učenja i osigurao veliki broj raznovrsnih resursa. Oni bi djeci trebali pomoći da sami, pomoću brojnih praktično-manipulativnih i simboličkih aktivnosti otkrivaju matematiku. Pored predmeta svakodnevnog upotrebe, didaktički strukturiranih materijala i tzv. prirodnih, ambalažnih i odloženih materijala značajnu ulogu u aranžiranju poticajnog okruženja za učenje imaju i različiti slikovno-grafički materijali u kojima su kvantitativni, veličinski i težinski odnosi predloženi simbolima.

Jedna od temeljnih teorijskih postavka o nastanku spoznaje je da se sve više psihičke funkcije razvijaju najprije kao socijalne pojave između individua (inherentno), a onda, u daljem razvoju, interiorizacijom prelaze na unutrašnji plan i postaju individualni procesi (intra mentalni). Suvremeno stajalište o učenju predškolskog djeteta, uz poticajno fizičko okruženje, za izgrađivanje znanja sve više ističe važnost suradničkog učenja koje djeci u socijalnoj interakciji omogućuje rasprave, suradnju, konfrontiranje, dogovaranje i sl., naročito ako se radi o uočavanju i rješavanju problemskih situacija. Preduvjet učenja je razumijevanje, kako problemske situacije koja je preduvjet za samo učenje, tako i razumijevanje djece međusobno, ali i djece i odgajatelja. Razumijevanju uvelike pridonosi rad s pedagoškom dokumentacijom i rad na njoj (Taguchi, 2010).

Predškolsko dijete okruženo poticajnim materijalima i vršnjacima, razvojno je sposobno za prakticiranje grupnog iskustva u matematičkim aktivnostima koje

traže višu razinu misaonih sposobnosti uključujući analizu (rastavljanje materijala na njegove dijelove da shvati strukturu, uočli sličnosti i razlike, količinu, veličinu), sintezu (sastavljajući elemente formira novu cjelinu, rearanžira i reorganizira ju) i evaluaciju (procjenjuje vrijednost materijala po definiranim kriterijima).

Predškolska djeca žele i trebaju izražavati ideje i poruke putem mnogih različitih načina ekspresije, pa tako koristiti i simboličke medije. To im pomaže upotrijebiti mentalne slike-predodžbe, reprezentirati ideje drugima i komunicirati sa svijetom kombinirajući razne načine izražavanja. Na taj način razvijaju svoju kompetenciju u svim aspektima razvoja. Razmjenom svojih ideja s idejama i zamislima drugih, dijete se postupno osposobljava za razumijevanje perspektiva druge djece te pod utjecajem toga provjerava i dograđuje svoj rad i proširuje svoju svjesnost o aktualnim matematičkim temama iz okruženja.

Odgojno obrazovni proces u predškolskoj ustanovi zasnovan na "intra-aktivnom pristupu" (Taguchi, 2010) i "pedagogiji slušanja" (Rinaldi i dr., 2002) proizlazi iz aktivnog djelovanja prvenstveno djece, ali i odraslih, na razumijevanju svijeta pomoću izravnih aktivnosti djece – činjenja i praćenja tih aktivnosti putem pedagoške dokumentacije koja služi za zajedničku refleksiju i za međusobno razumijevanje svih sudionika odgojno – obrazovnog procesa.

Uloga odgojitelja u takvom shvaćanju dječjeg učenja ne može biti poučavateljska jer i on sam uči o dječjem načinu unutrašnje konstrukcije vanjskog svijeta te i sam rekonstruira svoju sliku djeteta i njegovih razvojnih mogućnosti. On djeci ne daje svoje, odrasle zamisli, ne vodi dijete k točnim, gotovim i šablon-skim rješenjima, niti ih uvjerava u njihova pogrešna shvaćanja. Suprotno, potiče djecu da originalna rješenja pronalaze sama u diskursu s drugom djecom koristeći raspoloživa oruđa u okruženju, kao što će se vidjeti na primjeru istraživanja u tekstu koji slijedi.

Cilj, metodologija i tijek istraživanja

Cilj ovog interpretativnog, kvalitativnog istraživanja je razumijevanje stvaranja matematičkog značenja kojega djeca konstruiraju u njihovim svakodnevnim praktičnim akcijama, smještenim u socio-kulturni i fizički kontekst vrtića.

Rad se utemeljuje na primjeni etnografske metode kojom su se u razdoblju od 2007.-2010. godine u dječjem vrtiću "Dječji svijet" u Varaždinu prikupljali i interpretirali podaci o dječjem prirodnom, praktičnom učenju početnih matematičkih pojmova u socijalnim interakcijama s drugom djecom i odgojiteljem kroz eksploraciju funkcioniranja vage i spontano nastalih i rješavanih praktičnih problema u svezi s tim. Etnografska metoda činila se najprimjerenijim metodologijskim pristupom s obzirom na specifičnost odgojno obrazovne prakse i ciljeve istraživanja djece rane dobi "čija se ponašanja, aktivnosti i interakcije teško mogu razumjeti i interpretirati primjenom drugih istraživačkih pristupa bez bogatih opisa konteksta i situacija u kojima se pojavljuju, a koji im daju svrhu i značenje." (Petrović-Sočo, 2009, 15).

Etnografski zapisi dječjih aktivnosti (foto materijal uz simultano vođenje anegdotskih bilježaka pedagoga i odgojitelja) bili su polazište za zajedničku refleksiju koju su kontinuirano provodili odgojitelji i pedagog, što je omogućilo da se putem stručnoga dijaloga dogovaraju, primjenjuju i valoriziraju promjene u materijalnoj ponudi za eksploraciju i rješavanje problema kod djece kao i promjene u socio-pedagoškom ozračju odgojne skupine i postupcima odgojitelja. Iz tako nastalog obimnog istraživačkog materijala za potrebe ovoga rada izdvojene su i analizirane samoinicirane matematičke aktivnosti djece u centru vaganja nastale etnografskim zapisima u prirodnom, svakodnevnom okruženju djece u vrtiću.

Opremanje centra aktivnosti za vaganje započeto je (11.2.2008.) na inicijativu odgojiteljice koja je u tom području uočila zainteresiranost djece mješovite dobne skupine od 3 do 7 godina. Centar aktivnosti opremljen je s četiri improvizirane vage koje su se međusobno razlikovale po temeljnom načinu vaganja: jednokraka vaga - vaga koja omogućuje iskustvo nejednakog kraka, vaga za uspostavljanje ravnoteže s jezičcem na sredini i vaga za uspostavljanje ravnoteže krakova bez oznake sredine.

S obzirom da je u ovoj prvoj etapi uz vage u centru bila ponuđena relativno mala količina materijala za vaganje, analiza etnografskih zapisa pokazuje da su djeca u dobi od 6 do 7 godina istraživala puko funkcioniranje same vage kao sredstva za mjerenje težine (istraživala su što mogu činiti s vagama, promatrala i proučavala kako se one pokreću, ujednačuju, vrši otklon u jednu ili drugu stranu i sl.). Zajedničkim zapažanjima i raspravama odgojitelja i pedagoga kroz refleksiju ovih dječjih aktivnosti zaključeno je da u centru vaganja djeca nemaju dovoljno materijala koji bi stavljala u međuodnose s vagom te da ga je potrebno dopuniti materijalima koji su raznovrsni po veličini i težini (npr. kore drva i drveni materijali, ploče od gipsa otežane umetnutim metalima i dr.). Odgojiteljica je pretpostavila da će dječji interes potaknuti novi materijali većih i manjih dimenzija, međusobno različitih po težini, koji će ih "zavesti" vizualnim obilježjima veličine (Liebeck, 1995, 62).

Etnografskim zapisima u daljnjoj etapi istraživanja smo dokumentirali kako je djecu u dobi od 6 do 7 godina i nadalje najviše zanimalo uravnotežavanje vage. Igrajući se, djeca su vaganjem otkrivala i potvrđivala iskustvo kako oba kraka vage mogu biti u ravnoteži ukoliko se dodaju ili oduzimaju predmeti na pojedinoj strani vage. Analizirajući ove aktivnosti djece, tijekom zajedničke rasprave smo se složili da djeca još uvijek ne mogu pronaći složenije akcije u ovoj aktivnosti jer nemaju dovoljno različitih materijala u centru koji bi im omogućili proširivanje iskustva o razlikama u težini neovisno od veličine, oblika ili količine predmeta. Zato smo centar vaganja dodatno dopunili različitim novim materijalima (drvenim oblicima različite težine, kamenčićima, gipsanim pločicama, jabukama, kestenima, orasima, bočicama napunjenim različitim materijalima, vatom, spužvom, kukuruzom i sl.). Naknadno smo reflektirali učinke raznovrsnosti materijala na dječje aktivnosti i konstatali da su zahvaljujući bogatijoj ponudi djeca od 6 do 7 godina starosti sada bila dulje angažirana oko uravnotežavanja vage i da su mlađa djeca profitirala istraživanjem svojstava pojedinačnih materijala i njihovim stavljanjem u međuodnos s drugim materijalima (punila su i praznila posudice, izdvajala i klasificirala materijale i dr.)

Putem rasprave se konstatiralo da unatoč namjere odgojitelja i pedagoga da se stariju djecu potakne na složenije aktivnosti vagom uz pomoć novih materijala, ona su i dalje ostala na prijašnjoj razini, konkretnije, na istraživanju i rješavanju problema kako dovesti vagu u ravnotežu! Drugo ih nije zanimalo, a pogotovo ne namjere i očekivanja odraslih! Djeca i dalje nisu obraćala pažnju na količinu, ni veličinu materijala, nego samo na ravnotežu, na pitanje i provjeravanje: koji raspored materijala će održati vagu u ravnoteži? Ovom razinom dječjih aktivnosti odgojiteljica nije bila zadovoljna, pa je odustala od svog angažiranja u centru vaganja i u želji da djecu usmjeri na složenije djelovanje ponudila im je igru u pješčaniku na način da su djeca različitim autićima prevozila posudice iz centra vaganja napunjene materijalima različite težine. Odgojiteljica je, naime, imala hipotezu da će djeca uočiti različitu dubinu tragova u pijesku koju različito teški materijali ostavljaju u prevoženju! Međutim, djeca su u tako opremljenoj aktivnosti bila zainteresirana za brzinu kojom su se ona sama kretala “vozeći” automobile-igračke po pijesku, a nakon toga su započela gradnju cesta i mostova pa im težina na način, kako ju je odgojiteljica zamislila, uopće nije bila aktualna. To samo govori u prilog sintetičkom, holističkom modelu dječjeg (samo)učenja i njihovoj otpornosti na potrebu odraslog da ih poučava, jer “svaki pojedinac mora aktivno izgraditi svoje znanje” (Krsnik, 2001, 14) i to onda kad je spreman sam stvoriti nove mentalne konstrukcije, a ne transferiranjem znanja od odraslog. U prilog tome Liebeck (1995) također govori o važnosti aktivnog uključivanja djeteta u svakodnevne životne situacije jer na taj način djeca istovremeno misle na više stvari, i tako ohrabreni na razmišljanje stvaraju matematičke koncepte.

Daljnja etapa istraživanja nastavljena je uočavanjem, fotodokumentiranjem i bilježenjem dječje spontane aktivnosti u trenutku kada su vage i materijali iz centra aktivnosti vaganja bili odloženi na stol radi sređivanja, čišćenja i premještanja centra vaganja na drugo mjesto u vrtiću. Tada je zabilježena aktivnost jednog dječaka (u dobi od 6 godina) koji se igrao vaganjem na vazi za uspostavljanje ravnoteže s jezičcem na sredini te na drugoj vazi koja omogućuje iskustvo vaganja na vazi nejednakih krakova. Na tim vagama dječak je raspoređivao različita geometrijska tijela: valjke, kocke, kugle jednake veličine, ali različite težine. Promatranjem ovih zapisa te raspravom o njima uočeno je kako je dječak bio zaokupljen neravnotežom vage nastalom raspodjelom elemenata istog oblika, ali nejednake težine. Bilo je to sasvim novo iskustvo, kako njemu samome tako i nama kao promatračima njegova praktično-iskustvenog učenja. Dječaka više nije interesiralo uravnotežavanje vage, već obrnuto – neravnoteža, to jest pitanje: što se događa kad vaga nije u ravnoteži i o čemu ovisi dubina otklona od središta vage?

I ne samo da nam je promatranje etnografskog zapisa dječakove aktivnosti pomoglo da uočimo kako razmišlja i uči u praktičnoj akciji – postavljajući sebi novi problem – nego nam se to i potvrdilo i u nastavku aktivnosti kada je provjeru istog problema izvršio na drugačijoj vazi (vazi jednoga kraka koja omogućuje vaganje raspoređivanjem predmeta po dužini kraka različito udaljenih od centra vage). Dječak je “proizveo” problem, stavljajući na vagu materijale na onu stranu koja je već bila nagnuta, koja je, dakle, u neravnoteži. Sad je bilo evidentno da je dijete pošlo od nove hipoteze tražeći odgovor na pitanje što uzrokuje neravnotežu, kako se ona manifestira i o čemu ovisi.

U zajedničkoj refleksiji na temelju ovog zapisa konstatirali smo da bi dječak vjerojatno ranije došao na ideju istraživanja opisanog fenomena da smo mu otpočeka stvorili uvjete u centru vaganja koji bi bili u skladu s njegovim potrebama učenja. Podsjećamo da je u početku odgojiteljica opremila centar vaganja s nekoliko različitih vaga, ali ne i s dovoljno raznovrsnog materijala za vaganje pa dijete nije imalo mogućnosti djelovati u tom kutiću na složenijoj razini aktivnosti. Tek u ovoj etapi su odgojiteljice, promatrajući dječaka kojeg je interesirala neravnoteža, u zajedničkoj refleksiji s pedagoginjom osvijestile zašto je djeci potrebno omogućiti učenje na problemskim situacijama, a ne na tradicionalnom nuđenju “polugotovih” rješenja odraslog. Od tog trenutka je i praćenje dječjih aktivnosti za odgojiteljice dobilo novi, dublji smisao. Shvatile su da materijalno okruženje u kojem se odvija učenje uistinu potiče ili ograničava razinu složenosti dječjih aktivnosti putem kojih uče ili kao što piše Krsnik (2001), tradicionalni pristup učenju i poučavanju sad su zamijenile stavljanjem djeteta u prvi plan na način da u odgojno – obrazovnom procesu ono postaje aktivno.

Iz dosad prezentiranog moguće je zaključiti kako djeci treba omogućiti da se aktivnošću, koju su oni izabrali, bave na način koji im omogućuje stvaranje problemskih situacija, a ne “brzi” dolazak do rješenja te da im se pritom omogući onoliko vremena koliko im je potrebno, bez izravnog interveniranja odgojitelja u načine rješavanja problema. Nije potrebno nuditi rješenje, nego suprotno, omogućiti im materijale koji potiču nastanak različitih problemskih situacija te na osnovi promatranja dječjih aktivnosti zaključivati koja im je problemska situacija zanimljiva i koju su razinu razumijevanja problema o određenoj problematici postigli. Nakon ovoga iskustva centar vaganja opremili smo novim raznovrsnim materijalima, vagama koje mogu lako pokazati neravnotežu i ravnotežu, te prepustili djeci da potpuno samostalno organiziraju aktivnost.

Sljedeći etnografski zapis nastao je u centru vaganja nakon što je odgojiteljica “očišćene materijale odložila” u prostor centra vaganja (istog dana kada je dječak spontano započeo igru neravnoteže (6.2.2009.), kad je drugi dječak (u dobi od 6,5 godina) započeo aktivnost “uravnotežavanja” vage. Pridružio mu se dječak iste dobi te su zajednički, vrlo brzo zapazili problemsku situaciju u kojoj ista količina materijala (jedan valjak s jedne strane vage i drugi s druge strane vage) nije u ravnoteži, a nejednaka količina materijala (2 valjka s jedne strane vage i 3 valjka s druge strane vage) jest u ravnoteži (didaktička igračka: memory težine). Dječaci su dosad poistovjećivali količinu i težinu i ovo im je bila nova problemska situacija koju je omogućio novi materijal. Etnografskim zapisom je zabilježena aktivnost iz koje je vidljivo da dječaci uočavaju razlike u količini drvenih valjčića. Međutim, premještajući ih s jedne strane vage na drugu, mijenjajući broj valjčića na pojedinoj strani, dječaci uočavaju da jednaka količina valjčića na obje strane vage ne uspostavlja ravnotežu. Isto tako uviđaju da je jedan valjčić teži od dva. Zbunjeni su, ne razumiju ovu neočekivanu situaciju, ali ne odustaju od daljnjeg istraživanja. Dapače, njihova aktivnost trajala je neprekidno sat i četrdesetpet minuta, što pokazuje velik interes i potrebu da istraže, otkriju i spoznaju odnos težine i količine. Tijekom aktivnosti dječaci su u jednom trenutku, slučajnim razmještajem elemenata, proizveli situaciju u kojoj je 6 valjčića bilo teže od 4, što je jednog dječaka jako razveselilo – jer napokon je “točno da je više (veća količina istog) i

teže"! To nas je podsjetilo na Piagetovu tzv. situacijsku inteligenciju prema kojoj je dijete u ovom periodu pod snažnim utjecajem vizualnog doživljaja stvarnosti, kad je njegova misao "zarobljena" perceptivnim mehanizmima (Donaldson,1982) pa se dječak nije mogao izdići iz realnog doživljaja.

Međutim, dječak time nije završio aktivnost, nije zaboravio ono što je prethodno zapazio – da su količine elemenata na vazi bile u nejednakim odnosima težine. Sumnja i znatiželja, potreba da se slučajni "rezultat" potvrdi ili opovrgne bili su veći od postignutog "uspjeha" pa su dječaci nastavili aktivnost. Htjeli su istražiti, otkriti i shvatiti o čemu ovisi ravnoteža vage, ako ne o jednakoj količini predmeta na obje strane? Sad su utvrdili spoznaju, kao što navodi Liebeck, (1995,13) "isti su, ali različiti." .Znaju da broj predmeta ne određuje ravnotežu vage. Ali što ju određuje? Svjesni su da su otkrili problem, problem koji je njihov – ne problem koji im je netko (odgojitelj) nametnuo ili podmetnuo – kao slučajno je tu! Djeca žele biti aktivna u stvaranju problema – jer kad ga uoče – onda imaju potrebu i istražiti ga, postavljati hipoteze, propitkivati, varirati uvjete istraživanja i tragati za rješenjima. Problem koji nisu sami konstruirali – nije problem za njih jer, ili mu nije vrijeme, ili mjesto! Znači, ili nije za njihov stupanj aktualnog razvoja (Vigotski,1977), ili pak nije nastao u prirodnoj situaciji – nego je izmišljen, a izmišljene probleme ne treba rješavati jer ne zadovoljavaju prirodnu potrebu za rješavanjem. U svezi s tim Liebeck (1995,8) također navodi kada "djeca uče matematiku, trebaju se igrati stvarnim predmetima i proučavati probleme koji ih zanimaju".

U sljedećoj etapi istraživanja uz pomoć etnografskog zapisa analizirali smo situaciju u kojoj djeca raspoređivanjem jabuka na klasičnoj vazi nisu otkrila u čemu je tajna uravnotežavanja pa su uzela drugu vagu za nastavak svog istraživanja. Vaganje jabuka na vazi jednakih krakova zamijenili su vaganjem geometrijskih tijela (kocke, valjci, kugle) na vazi jednoga kraka. Odmah su ih rasporedili tako da potvrde svoje novo znanje kako količina predmeta nije povezana s njihovom težinom. Na jednu stranu vage su stavili tri predmeta koja su bila teža od četiri predmeta s druge strane vage. Zatim mijenjaju vagu, uzimaju onu jednakih krakova na kojoj važu otpočeka, ali sada sa sasvim novim materijalima - sitnim komadićima gline. Zbog vrlo različitih oblika ti su djelići gline bili nejednake težine i toliko sitni da ih dječaci na vagu nisu stavljali jedan po jedan, nego su ih grabili rukama, ne brojeći ih, kako bi izjednačili težinu i uspostavili ravnotežu vage. U jednom trenutku dječak je zaključio: "Vidiš, jednako je kada ne brojimo, jer teško se ne broji!", želeći pritom iskazati svoju novu spoznaju da izjednačavanje težine ne ovisi o količini. Ova sekvenca dječje socijalne interakcije možda najbolje pokazuje kako im je za mišljenje potrebno senzorno uporište.

Dječaci su ponovo uzeli vagu jednog kraka i na njoj odmah, bez pogreške, razmjestili različita geometrijska tijela na međusobnu udaljenost od centra tako da je vaga odmah bila u ravnoteži. Nastavkom aktivnosti, načinom na koji su rasporedili predmete na vazi, putem praktičnih akcija su pokazali da razumiju princip vaganja i uravnotežavanja vage putem ravnomjernog raspoređivanja težine neovisno o količini.

Naše razumijevanje dječjih aktivnosti u centru vaganja nastavilo se radom na dokumentaciji na način da smo djeci prikazali etnografske zapise na računalu kako

bi nam ih ona objasnila i komentirala. Nakon uzastopnog ponavljanja zajedničkog gledanja etnografskih zapisa, (što djeca vole raditi), bez procjenjivanja i uplitanja odgojiteljica, postigla se meta-razina dječjeg spoznavanja, ili kako Bruner navodi meta – zona. Tako je dječak, koji je i ranije u samoaktivnosti proizveo problemsku situaciju neravnoteže, uzeo vagu kao potporanj za verbalno izražavanje procesa vaganja i pokazivao što je i kako napravio da bi uravnotežio vagu. Na taj je način pokazao sposobnost upravljanja vlastitim spoznavanjem. Drugi dječak je pomoću crteža objašnjavao što su i kako djeca vagala i komentirao kako je na jednom kraku vage “više dijelova jer su lakši” od onog drugog kraka gdje je “manje dijelova koji su teži – pa zato sve stoji ravno!”

Međutim, kada smo djeci predložili da nacrtaju što je bilo na jednom, a što na drugom dijelu vage oni su i dalje crtali ravnotežu ili neravnutežu vage ovisno o količini elemenata koji su na pojedinoj strani. Tom prilikom su brojala količinu dijelova pa treba razmisliti je li uzrok tome teškoća i nesnalaženje djece u načinu kako da tehnikom crtanja izraze razlike u težini, ili je vjerojatnije da je perceptivna vezanost djece za stvarnost i u predodžbi snažnija od “labavo” i netom formiranog pojma težine kao misaonog konstrukta, neovisno od količine i veličine? Sigurno je ovo drugo.

U jednoj od narednih analiza etnografskih zapisa zamijetili smo situaciju u kojoj su dva dječaka (u dobi od 6 godina) uspoređivala na vazi samo 2 elementa drva – približno jednaka po veličini, ali različita po težini – ovdje držimo bitnim naglasiti da se radilo o istom materijalu približne veličine (volumena) -drvu-, ali različite vrste (jelovina i bukovina), dakle različitih po težini. Sve do aktivnosti s drvenim materijalom pri čemu su dva dijela bila slična po obliku a različita po težini – djeca su u uspoređivanju vaganih materijala kao objašnjenje težine koristila pojmove: više ih je, tanji su, veći su i slično. Prekretnica u dječjem shvaćanju težine nastala je tek u trenutku kada je dječak slučajno uzeo u ruke dva drva slična po veličini, ali bitno različita po težini i dok je drugi dječak – koji je bio suigrač i partner prvome, crtajući što se događa s vagom rekao: “Valjda je unutra neka kaj je različito”.

Analizirajući i raspravljajući o ovoj aktivnosti prilikom razgledavanja njezina etnografskog zapisa pedagoginja je zapitala prisutne odgojiteljice hoće li djeca isto ili različito pristupiti postavljanju i rješavanju problema u uvjetima materijala koji im je bio dostupan na početku i na kraju ovog istraživanja zato da ih potakne na samouvid o nužnoj međuovisnosti složenosti dječjih aktivnosti i onoga putem čega uče, a to je izbor, dostupnost, količina i stupanj složenosti ponuđenog materijala.

Rezultati istraživanja i rasprava

Analiziranjem i interpretiranjem gore opisanih etnografskih zapisa uočili smo šest etapa složenosti u razvoju dječjih samoiniciranih matematičkih aktivnosti u centru vaganja i to kako slijedi:

1. istraživanje funkcioniranja vage (kada i kako se vaga pokreće)

2. istraživanje uravnotežavanja vage (uspostavljanje ravnoteže na vazi s jezičcem na sredini)
3. istraživanje neuravnoteženosti vage i pokušaj razumijevanja uzroka
4. postavljanje i rješavanje problemskih situacija na temelju dječje hipoteze da su težina i količina iste kategorije (težina ovisi o količini predmeta)
5. rješavanje problemske situacije u kojoj djeca pretpostavljaju da su količina i veličina iste kategorije (težinu poistovjećuju s veličinom predmeta)
6. rješavanje problemske situacije zaključivanjem da je težina drugačija, neovisna kategorija i od količine i od veličine predmeta

Na temelju praćenja samoiniciranih matematičkih aktivnosti i načina na koji djeca u njima proizvode, pristupaju i rješavaju praktične problemske situacije zaključili smo da dostupnost, preglednost, količina, raznovrsnost i složenost ponuđenih sredstava imaju odlučujuće djelovanje na kvalitetu i dosege dječjeg učenja matematike. Iz tih razloga centar vaganja obavezno bi trebao biti opremljen materijalima:

- jednakima po obliku i volumenu, a različitima po težini (u našem slučaju valjčići – memory težine)
- različitih težina i veličina (npr. teško a malo, a lako i veliko, isto navodi i Liebeck, 1995)
- sitnim materijalima neodređenog oblika koji nije brojiv (u našem slučaju komadići gline).

Također je važno da se u centru vaganja nalazi vaga koja omogućuje iskustvo nejednakog kraka, vaga za uspostavljanje ravnoteže s jezičcem na sredini i vaga za uspostavljanje ravnoteže krakova bez oznake sredine. Istraživanje pokazuje da je to minimum opreme neophodan djeci za stjecanje iskustva o pojmovima težine koje može osigurati formiranje matematičkih pojmova: teže, lakše i razumijevanje djece da je težina neovisna od količine i veličine.

Krsnik (2001,16) navodi kako “Za nastavnika koji želi prijeći s tradicionalne nastave na konstruktivističku velik je problem kako u razredu organizirati odgovarajuće problemske situacije.” Promišljena ponuda materijala primjerena aktualnom stupnju dječjeg razvoja koja se utemeljuje na odgojiteljevom pažljivom promatranju djece, uz višu razinu razumijevanja djece ostvarenu pomoću etnografskih zapisa i pedagoške dokumentacije, a provedene u pozitivnom emocionalnom ozračju, sudeći po ovom istraživanju, za nas je na osnovi ovog istraživanja i u ovom trenutku, djelomičan odgovor na pitanje kako “prijeći s tradicionalne nastave na konstruktivističku”, odnosno kako prijeći s poučavanja na samostalno stjecanje znanja.

Umjesto zaključka

Polazeći od socio-konstruktivističke teorije dječjeg učenja istraživanjem smo željeli odrediti na koji način je materijalno poticajno okruženje povezano s kvalitetom dječjeg iskustva za razumijevanje količinskih, veličinskih i težinskih odnosa

u centru vaganja, to jest koliko su materijalni poticaji važni za sukonstruiranje znanja kvantitativnih odnosa i relacija kod djece predškolske dobi u vrtiću. Na temelju proučavanja i metodički primjerenog poticanja dječjeg učenja određenih matematičkih pojmova u određenom institucijskom kontekstu u ovisnosti od unapređivanja tog istog okruženja može se zaključiti da dječje prirodno, praktično učenje matematike u velikoj mjeri ovisi o ponudi konkretnih materijala i njihovog potencijala za stavljanje u različite međuodnose, ispitivanje, istraživanje, pretpostavljanje i provjeravanje jer upravo ponuda materijala, između ostalog, određuje složenost aktivnosti, to jest kvalitetu dječjeg učenja. Učenje činjenjem (Miljak, 2009), putem praktičnog djelovanja u socijalnoj interakciji djeteta s drugom djecom i odgojiteljem u vrtiću omogućuje stjecanje, variranje, dograđivanje i revidiranje iskustva u spiralnoj progresiji iz kojega dijete postupno derivira svoje znanje na razinu misaonog konstrukta ili pojma. Iz toga kao i iz opisa dječjega ponašanja na izravne odgojno-obrazovne intervencije odgojitelja zaključujemo da se matematika ne može poučavati, nego se uči onda kada djeca konstruiraju vlastito matematičko razumijevanje. Zbog toga se od odgojitelja ne očekuje poučavanje, nego poznavanje prirode dječjeg učenja utemeljenog na promatranju i praćenju djeteta u svakodnevnim aktivnostima. To će mu pomoći da prepozna razvojni trenutak u kojem se dijete nalazi i da na osnovi toga projektira bogaćenje okruženja određenim materijalima za daljnje učenje. Stoga područje razvoja početnih matematičkih pojmova kao i svih ostalih područja u predškolskim ustanovama zahtjeva visok stupanj profesionalne osposobljenosti odgojitelja i kontinuirano jačanje njegove stručne kompetencije utemeljene na autorefleksiji i grupnoj refleksiji vlastite prakse.

Važno je istaknuti da rezultati dobiveni ovim etnografskim istraživanjem u znanstvenom smislu nisu generabilni na širi skup jer je istraživanje provedeno na malom, hotimičnom uzorku djece. No i takvi mogu pomoći i usmjeriti daljnja istraživanja u području učenja matematike u vrtiću i doprinijeti primjerenom metodičkom oblikovanju aktivnosti za učenje matematičkih pojmova predškolske djece utemeljenom na teoriji socijalnog konstruktivizma i usmjerenim na postavljanje i rješavanje matematičkih problema, a ne na poučavanje o njima. Te se aktivnosti "grade" na dječjim praktičnim akcijama predmetima, objektima i pojavama u realnom okruženju koje dijete dovodi u veze i odnose na mentalnom planu i izgrađuje simboličke strukture: mišljenje, govor i pisane znakove.

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Poticanje razvoja matematičkih vještina kod djece predškolske dobi kroz projekte

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Sažetak. U razvojno primjerenim programima, matematika zauzima važno mjesto. Vrijednost matematike očituje se u njezinoj prisutnosti u svakodnevnom životu. Matematički koncepti, poput brojeva, mjerenja, prostornih odnosa i sl. djeci predškolske dobi su bliski i dostupni. U radu predstavljamo aktivnosti poticanja matematičkih vještina u okviru dječjeg projekta "Svemir" provedenog koncem 2009. godine u Osijeku. Pokazalo se kako je projekt, koji je trajao tri mjeseca, bio prilika za poticanje cjelokupnog razvoja djeteta, a time i razvoja matematičkih vještina. Cilj ovog rada je prikaz aktivnosti, sastavnog dijela projekta, kojima se poticao razvoj matematičkih vještina. Djeca su praćena metodom promatranja i bilježenja u sklopu čega su im nuđeni zadaci u skladu s njihovim razvojnim mogućnostima. Matematičke pojmove djeca su stjecala kroz igru, različite aktivnosti i zadatke te samostalna istraživanja.

Gljučne riječi: matematika, matematičke vještine, djeca predškolske dobi, dječji projekti

Uvod

Matematika je sastavni dio svakodnevnog života. U razvojno primjerenim programima ona zauzima važno mjesto. Slunjski (2006) navodi kako razvoj početnih matematičkih pojmova kod djece treba biti u skladu s razvojnim karakteristikama pojedinoga djeteta te karakteristikama procesa učenja predškolskog djeteta. Matematički koncepti, poput brojeva, mjerenja, prostornih odnosa i sl. djeci predškolske dobi su bliski i dostupni. U praksi, to znači kako će se u poučavanju matematike polaziti od dječjeg iskustva uz primjenu konkretnih materijala koje će djeca upoznati (u prvom redu taktilno i vizualno), igrati se s njima i onda ih uspoređivati, brojati, klasificirati... Čudina-Obradović (2002), Čudina-Obradović i Brajković (2009) navode da je u projektu osobito izražena djetetova aktivnost. Tijekom projekta djeca odlučuju, raspravljaju, objašnjavaju i obrazlažu svoje ideje vršnjacima, predviđaju rezultate neke aktivnosti, donose pretpostavke, provjeravaju činjenice, zaključuju iz podataka, bilježe opažanja, nude prijedloge i primjedbe

i prihvaćaju odgovornost za svoj rad i za postignute rezultate. Obzirom da se matematičke aktivnosti mogu provoditi tijekom predškolskog perioda, osmišljene su aktivnosti poticanja matematičkih vještina u okviru dječjeg projekta “Svemir”. Tijekom tri mjeseca, koliko je projekt trajao, djeca su osnovne matematičke pojmove usvajala kroz igru i samostalne istraživačke zadatke.

Nastanak i cilj projekta “Svemir”

Ovaj projekt počeo je u starijoj odgojnoj skupini kada je jedan dječak od roditelja na poklon dobio teleskop. Svaki dan dječak je pričao o zvijezdama, mjesecu, suncu s velikim zanimanjem i oduševljenjem. Donio je u skupinu i razne enciklopedije o svemiru, a u njihovo proučavanje uključivala su se i druga djeca. Ubrzo smo svi zajedno otišli u knjižnicu posuditi još slikovnica i knjiga, a uključili su se i roditelji s plakatima Sunčevog sustava i zvijezda. Soba se dnevnog boravka ubrzo pretvorila u “svemir”. Prateći interese djece, osmišljen je projekt “Svemir” kojemu je glavni cilj bio zadovoljiti dječji interes za svemirom, planetima sunčevog sistema, svemirskim letjelicama, astronautima i potaknuti ih da promatraju, postavljaju pitanja, istražuju, samostalno razmišljaju, zaključuju i proširuju svoja znanja o svemiru. U ovom radu osvrnula sam se na matematičke vještine i sposobnosti koje su djeca stjecala tijekom projekta. Kirsten i sur. (2004) navode da matematika predstavlja temelj tehnološkog napretka koji se događa u svijetu, počevši od industrijske revolucije, preko svemirskih istraživanja pa sve do doba telekomunikacije i internetizacije. Djeci stoga moramo pružiti matematičku pismenost. Odgajatelji trebaju pomoći djeci da postanu svjesni funkcije matematičkih pojmova i da razviju svoju matematičku kompetenciju u direktnoj interakciji sa svijetom oko sebe. Kirsten i sur. (2004) navode da će rana iskustva s matematičkim pojmovima učvrstiti temelje za matematičko razmišljanje, a dobri temelji omogućiti će djeci da primjene matematičko znanje u praktičnim situacijama i da se uspješno ukllope u tehnološki složenu globalnu zajednicu.

Prikupljanje materijala i početne aktivnosti

Razvojem projekta rasla je i dječja znatiželja o svemiru. Osim u knjigama odgovore su tražili i kroz samostalne zadatke pa su djeca pratila mjesečeve mijene te su uspoređivala planete po boji, veličini, udaljenosti od Sunca i Zemlje. Tijekom projekta nastala je i scenska igra “Sunčeva obitelj” te kazalište sjena (za što je bilo potrebno načiniti lutke — planete te pribaviti ploču od pleksiglasa i reflektor). Sa scenskom igrom “Sunčeva obitelj” bili smo na međunarodnom susretu kazališnih grupa dječjih vrtića u Podgorici, a uz pomoć jednog tate organizirali smo i posjet Planetariju i Tehničkom muzeju u Zagrebu. U projekt su se postepeno uključivale obitelji djece te nam je djed jednoga dječaka pomogao izraditi raketu. Djeca su u nju donosila telefone, digitrone, razne kutije od kojih su pravili upravljačku ploču te pisali brojeve i slova. Načinili su robota od kutija, različite letjelice od kartona i

maketu sunčevog sustava. Djeca su tijekom ovih aktivnosti ovladavala prostornim odnosima, razvijala i uočavala smjerove u prostoru i pojam udaljenosti, povezivala predmete i pojave te uzročno-posljedične veze među njima, razvijala načela brojanja, pridruživanja broja, imenovanja i prepoznavanja slike broja i geometrijskih likova, istraživala, pitala, uočavala odnose, samostalno rješavala probleme i stjecala znanja.

Razvoj (tijek) projekta

Kako se projekt razvijao, zapazila sam kako se aktivnosti mogu razvrstavati na istraživačko-spoznajne, aktivnosti za razvoj govora i pismenosti, likovno-kreativne aktivnosti, glazbene, matematičke, tjelesne, manipulativne i društvene igre. Često je bilo teško jednu aktivnost svrstati u samo jedno područje jer su se aktivnosti ispreplitala. Što se tiče matematike, aktivnosti u ovom projektu su bile slijedeće: brojanje, uspoređivanje brojeva, upoznavanje prostornih odnosa, uspoređivanje predmeta i klasifikacija predmeta.

I. Brojanje:

1. Mehaničko brojenje

- a) Prepoznavanje brojevnik riječi i njihova redoslijeda

2. Brojenje pridruživanjem

- a) Razumijevanje smisla brojenja

(Primjer igre: *Tko će skupiti više zvjezdica?* — Na različita mjesta u sobi staviti fluorescentne zvijezde. Zamračiti sobu dnevnog boravka. Djeca sjede u krugu i svako dijete baca veliku kocku te skuplja onoliko zvijezda koliko je pokazala bačena kocka. Na kraju kada su sve zvijezde prikupljene, svako dijete prebroji svoje zvijezde. Dijete koje ima najviše zvijezda je pobjednik.)

3. Prepoznavanje brojaka i pridruživanje količini

- a) Brojevi kao količina premeta

(Primjeri aktivnosti i igara: *Zvijezde* — pravljenje kolača što uključuje mjerenje, vaganje, miješanje, oblikovanje. Djeca imaju na stolu ponuđen recept, sastojke, digitalnu stolnu vagu, posudu, valjak i oblike zvijezda. Jedno dijete čita recept, drugo važe, treće miješa sastojke, te svi zajedno od tijesta prave zvijezde i nose ih u kuhinju peći, te ih kasnije i jedu i nude prijateljima.

Robot za mjerenje — aktivnost mjerenja, ponuđene su igračke robota različite veličine, od manjih, srednjih do većih, Lego kockice za mjerenje, papir, boje, olovke. Pokraj robota djeca naslažu Lego kockice pa zajedno izbroje koliki je broj kockica visok robot. Broj kocaka dijete upiše pored crteža robota.

Koliko je zvijezda na nebu? — radne liste, pridruživanje broja, prostorni odnosi. Nacrtni su skupovi sa različitim brojem zvijezda. Dijete treba pored skupa napisati broj.

Nacrtaj onoliko zvijezda koliko piše! — ponuđene su boje, olovke, dijete treba nacrtati točan broj zvijezda u oblačić.)

II. Uspoređivanje brojeva

a) Jednako, za jedan više, za jedan manje

Uspoređivanje veličine skupina, djeca postepeno uočavaju kad se skupine razlikuju.

(Primjeri igara: *Svemir-domino* — kartice u boji sa sličicama, na karticama je jedna raketa ili dvije planete ili tri zvijezde ili jedno sunce. Igra se kao domino.

Memory — zvijezde različitih boja i s različitim brojem krakova; dijete treba pronaći parove, igra se u paru.)

b) Usporedba brojeva “u glavi”

Bez brojanja prstiju i predmeta reći koji je broj veći ili manji.

Sve ove vještine i znanja djeca su stjecala tijekom projekta krećući se prostorom, baratajući predmetima, uspoređujući ih, otkrivajući njihova svojstva i uspoređujući količine. Roditelji su se uključivali u ovo dječje istraživanje i nudili nam nove sadržaje, aktivnosti, svoju pomoć i podršku. Izrađivanjem radnih listova bio mi je cilj usmjeravati i održavati dječju pozornost, razumijevanje odnosa među predmetima, razumijevanje crteža i brojeva kao zamjene za stvarne predmete i odnose u okolini, prepoznavanje i pamćenje brojaka kao i njihove povezanosti s količinom, postepeno privikavanje na samostalnost i ustrajnost. Radni listovi poticali su i međusobno dogovaranje djece i rasprave o rješenju zadatka. Nastojala sam uvijek pohvaliti dječji rad i trud, te napor i upornost u izradi zadatka.

III. Upoznavanje prostornih odnosa

Tijekom projekta djeca su učila o odnosima u prostoru, o odnosima predmeta, o svojstvima predmeta.

(Primjeri aktivnosti i igara; *Planete su gore iznad nas* — promatranje teleskopom, doživljaj u Planetariju. *Planete bliže i dalje od Sunca i Zemlje* — uočavanje udaljenosti u slikovnicama, knjigama, enciklopedijama, plakatima, mjerenje prstom, ravnalom, papirnatim trakama, djeca sama smišljaju načine kako da točno utvrde kolika je udaljenost planeta, primjećuju da neke viša sjaje a neke manje, da su neke žute, druge crvene, pitaju jesu crvene toplije. *Putanja planeta* — planete se zavrtje na crnom krugu oko Sunca i gleda se njihova putanja, napravljena maketa od kolaža i tvrdog papira sa čavličem u sredini.)

IV. Uspoređivanje predmeta

(Primjeri igara i aktivnosti; *Planete* – izrezane slike planeta, planete od kugli od stiropora, uspoređivanje njihove veličine te slaganje od manje do veće i obrnuto, stiropor djeca bojaju odgovarajućim bojama planeta a papirne planete lijepe na papir, prave makete, slažu ih po udaljenosti i slijedu.

Dan i noć — pokretna igra, jedno dijete govori upute a druga djeca kad je dan stanu, a kad je noć, čučnu.

Djeca na plakatima i u knjigama uočavaju svojstva planeta; tople i hladne, velike i male, crvene i plave, s prstenom i bez prstena.

Sunčev sustav — suprotnosti — potaknuti djecu na nabranje suprotnosti vezane uz sunčev sustav (mali/veliki, blizu/daleko, toplo/hladno, lijevo/desno, gore/dolje, visoko/nisko, dan/noć. . .)

Zvijezde različitih veličina — djeca prave rastuće i padajuće nizove na temelju minimalnih razlika u pogledu veličine po kojoj se predmeti nižu. Zvijezde su od papira ili plastične.)

V. Klasifikacija predmeta

a) Svrstavanje i razvrstavanje

Djeca su pronalazila zajednička svojstva različitih predmeta i načine kako se ona mogu razvrstati i svrstati prema zajedničkim bitnim svojstvima (planete, rakete, svemirci) a razvrstavala su i geometrijske likove.

(Primjeri aktivnosti; *Raketa* — radni list sa zadatkom da se oboji trokute žuto, krugove plavo a pravokutnike zeleno, raketa je nacrtana od dijelova geometrijskih likova.

Zvijezde — pronađi par, zvijezde su različitih boja i s različitim brojem krakova, djeca pronalaze parove, igra se u paru, tko više skupi je pobjednik.)

b) Sparivanje i pridruživanje

(Primjeri igara i aktivnosti; *Strujni krug* — povezivanje predmeta s istovrsnim i pripadajućim (astronauta s raketom, zvijezde s nebom, astronom s teleskopom. . . i svemirske pojmove sa slovima — dijete poveže sliku i riječ, lampica zasvijetli ili poveže sliku broja sa brojem zvijezda ili planeta.)).

Zaključak

Kirsten i sur. (2004) navode kako matematika prožima cjelokupno iskustvo čovjeka. U svakoj kulturi matematički pojmovi koriste se u svakodnevnom životu. Matematika predstavlja temelj zapanjujućeg tehnološkog napretka koji se događa u svijetu, zato našoj djeci trebamo pružiti matematičku pismenost i dobre temelje za matematičko razmišljanje, a dobri temelji omogućiti će im da primjene matematičko znanje u praktičnim situacijama i da se uspješno uklape u tehnološki složenu globalnu zajednicu. Osnovna pretpostavka projekta je da djeca promatraju, pitaju, istražuju i poduzimaju određene aktivnosti u vezi s temom, te samostalno razmišljaju i zaključuju te šire svoja znanja aktivirajući pritom svoje potencijale i razvijajući mnoge sposobnosti. Radeći na projektu o svemiru, učila sam i istraživala zajedno s djecom. Često su me iznenadila njihova pitanja i putovi pomoću kojih su dolazili do znanja te njihova uronjenost u problem i istraživanje. Tek kada sam sistematizirala aktivnosti uvidjela sam koliko područja djetetovog razvoja može obuhvatiti jedan projekt i koliko mogu biti zastupljene matematičke aktivnosti te kolika je njihova vrijednost u cjelokupnom razvoju djeteta.

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A valódi tárgyak szerepe a matematikatanításban

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Összefoglaló. A manipulációs eszközök nagyon fontosak, mert konkrét tevékenységen keresztül teszik lehetővé az elvont matematikai fogalmak megértését.

Először Bruner és Dienes egyetemi kollégák gyerekein próbálta ki olyan eszközök alkalmazását amelyekkel algebrai összefüggések fedezhetők fel. Sikereik és kutatásaik révén ma már általánosan elfogadottá vált a manipulációs eszközök alkalmazása.

Hipotézis:

Vannak gyerekek, közöttük igen tehetségesek is, akiknek alapvető hiányosságaik miatt a matematikai játékok, az azokban megtestesülő matematikai tartalmak elérhetetlenek.

Hipotézisünket kvalitatív módszerekkel vizsgáltuk.

Fejlesztő foglalkozás keretében feltártuk a hiányosságokat és didaktikai módszereket konstruáltunk azok megszüntetésére.

1. Nem matematikai játékok a dobókockával, a kártyák értékével összefüggő tapasztalatok megszerzésére
2. Tárgyakkal végzendő, matematikailag releváns, feladatok

A a matematikatanulást segítő, kifejezetten erre a célra készült eszközök (Dienes készlet, színesrudak, logikai készlet) használata a tanulóktól matematikai előismereteket valóban nem igényel, viszont szükséges a szimbolikus gondolkodás elég magas szintje. Ezt a gyerekek a szokásos családi, óvodai játékok során el is érik, vannak azonban kivételek is, elősorban a társadalmilag hátrányos helyzetű gyerekek.

A vizsgálatainkat 8-10 éves tanulók körében végeztük.

Manipulációs eszközök és a matematikatanítás

Az eszközök használatának hosszú története van a matematikatanításban. Most már virtuális eszközöket is használhatunk, ennek egyik szép gyűjteményét találhatjuk az http://www.ct4me.net/math_manipulatives.htm lapon. Ennek ellenére a tárgyi manipulációs eszközök nem veszítették el jelentőségüket.

Hogyan használhatjuk a manipulációs eszközöket a matematikai fogalmak megértésének és a problémamegoldó képességek fejlesztésének segítésére? Piaget (1999) a matematikatanulásban a tárgyak szerepét emelte ki, Vigotszkij (1966) pedig a társas kapcsolatokat. Cole (é. n.) elemezte, hogy a gyakorlatban e két nézet nem ellentétes, jól kombinálható.

Bruner tanulásmélete (1968) két fontos új gondolattal gazdagította a matematikadidaktikát is. Az egyik szerint az új ismeretek a tudatunkban kétféle módon rendeződnek el (Bruner, 1991). Az egyik a narratív mód, amely megtartja az ismeretek megszerzésének történetiségét, a másik pedig a logikai mód, amely a logikai kategorizáláson alapul. Hasonlóképpen fontos a reprezentációs módok elmélete, amely szerint a tárgyi reprezentációból a képin keresztül jutunk el a szimbolikus reprezentációhoz (Bruner, 1974). A tanulás párhuzamosan zajló folyamatait fogalmazta meg egy új rendszerben Paivio és Clark (1991) duális kódolási elmélete, amely szerint agyunk a verbális és a képi-tárgyi információkat párhuzamosan dolgozza föl. Az iskolai matematikatanulás pontosabb leírhatósága érdekében Nakahara (2008) tovább finomította Bruner reprezentációs módjait.

- S2 Szimbolikus reprezentáció, amely számokat, betűket és egyéb szimbólumokat alkalmaz
- S1 Nyelvi reprezentáció, amely a mindennapi, beszélt nyelvet alkalmazza
- I Illusztratív (képi) reprezentáció
- E2 Manipulációs reprezentáció, amelyben a tanulók modelleket és egyéb oktatási eszközöket használnak
- E1 Realisztikus reprezentáció, amely a valódi tárgyakon alapul.

A Nakahara által megállapított szintek, akárcsak Bruner reprezentációs módjai a szimbólumhasználat jellegére utalnak, nem a problémamegoldás szintjeire. A realisztikus reprezentáció segítségével is adhatunk a tanulóknak nehéz megoldandó problémákat. Alan Bishop, aki a matematikatanítás szociológiája elnevezésű kutatási irány egyik megalapítója a modellek irányából közelítette meg a kérdést. Bishop szerint a valódi világ jelenségeinek létezik matematikai modellje és a matematikai fogalmaknak létezik fizikai modellje, a matematikai fogalmak valódi tárgyakkal modellezhetőek.

Hipotézis

Feltételezzük, hogy vannak olyan tanulók, közöttük igen tehetségesek is, akiknek a matematikai játékok és a játékok matematikai vonatkozásai elérhetetlenek hiányos előismereteik és szegényes tapasztalataik miatt. Számukra a realisztikus reprezentáció nagy segítséget jelent, és ez független értelmi képességeiktől.

Oktatási tapasztalataink összevont tanulócsoportos magyar, valamint kenyai iskolákban

Hipotézisünket kvalitatív módszerekkel vizsgáltuk. A munka során készült fényképek és oktatási eszközök, valamint a pedagógusok beszámolóí és a tanulók

írásbeli reflexiói a kutatás részét képezik, azok a konferencia honlapján tekinthetők meg.

Célunk az volt, hogy a hátrányos helyzetű tanulók képessé váljanak matematikai játékok játszására, ennek érdekében olyan tapasztalatokhoz akartuk juttatni őket, amelyeket a többi tanulók saját, középosztálybeli családjukban megszerezhetnek.

Vizsgálatainkat 8-9 éves tanulók körében végeztük.

A legfontosabb módszertani megoldás a valódi, más célokra is használt tárgyak oktatási eszközként való alkalmazása volt.

- Megmutattuk néhány tárgy szimbolikus jelentőségét, dobókockát használtunk, bátorítottuk a kártyajátékokat. A gyerekek sok rajzos és írásos feladatot kaptak, elsősorban azért, hogy ezáltal is a szimbólumhasználatot gyakorolják.
- Sok eszközt használtunk a matematikai fogalmak és összefüggések megértésének elősegítésére. Nem használtuk a standard oktatási eszközöket, bár azokat igen fontosnak tartjuk, de véleményünk szerint használatukhoz a szimbolikus gondolkodás viszonylag magas szintje szükséges, mi pedig éppen ennek a képességnek az alapjait terveztük kiépíteni. Az eszközeink a gyerekek számára ismerős, pl. a konyhákban is megtalálható tárgyak voltak.

A kísérleti órák szokásos menete a következő volt:

- A diasorozatok bemutatása számítógépek segítségével
- Problémamegoldás valódi tárgyak segítségével
- A gyerekek történetmesélése Mi történt az órán? címmel

Öt témában készült diasorozatot próbáltunk ki az iskolákban

1. Poharak, mérés
2. Kirándulás, térbeli tájékozódás
3. Egyiptomi számírás, a bizonyítások első lépései
4. Utazás, adatkezelés
5. Poliéderek

Poharak, mérés, mértékegységek

Cél: A mérés és a mértékegység fogalmának kialakítása, elmélyítése az űrtartalom alkalmilag választott egységekkel történő mérése révén, annak a hétköznapi tapasztalatnak beépítése a matematikai ismeretek körébe, hogy ha kisebb edénnyel töltünk meg egy tartályt, akkor többször kell fordulnunk, mintha ugyanezt nagyobb edénnyel tennénk.

Szükséges eszközök: Víz, kisebb és nagyobb poharak, kancsó.

Követelmények: Minimális követelmény: A tapasztalatok közvetlen megfogalmazása, pl.: „Kisebb edénnyel tovább tart a kancsó megtöltése.” Optimális

követelmény: Más mennyiségek mérésére és általában a mérésre vonatkozó összefüggések megfogalmazása, a váltószámok ismerete, a mértékegységek adott feladathoz illeszkedő megválasztása. A témához kapcsolódó szöveges feladatok megfogalmazása és megoldása.

Kirándulás, térbeli tájékozódás

Cél: A három térbeli irány szavainak, le-föl, előre-hátra, jobbra-balra használatával a térbeli tájékozódás tudatossá tétele, a térfogalom kialakítása első lépéseinek megtétele. Térbeli alakzatok építése, a párhuzamosság és a merőlegesség, az azonos élhossz reprodukálása különböző modellekkel. A bal és jobb oldal, a balra és jobbra kanyarodás fogalmának gyakorlása.

Szükséges eszközök: Gyurma, pálcikák; hely és eszközök szituációs játékokhoz, a „jobbra kanyarodó autó”.

Követelmények: Minimális követelmény: A tapasztalatok közvetlen megfogalmazása, a bal és jobb oldal biztos ismerete, pl. bal kéz, jobb kéz. Optimális követelmény: Az irányok biztos alkalmazása mozgásban is.

Időkerék, egyiptomi számírás és az első lépések a matematikai bizonyítás felé

Cél: A történelmi keret segítségével elősegíteni két, didaktikailag nagyon különböző jellegű fogalom mélyebb megértését. Az egyik a mi számírásunk, ami annyira egyszerűnek, nyilvánvalónak tűnik, hogy egy másik kultúra alapvetően más megoldásának ismerete segíthet megérteni a benne rejlő tartalmat, a helyiértékes számírás lényegét. A másik fogalom a bizonyítás, amely éppen hogy nagyon távolinak, idegennek látszik. Nehéz észrevenni, hogy mindennapi életünkhöz valójában milyen közel áll, hiszen szinte minden döntésünk feltételezéseken, hipotéziseken alapul.

Szükséges eszközök: Rajzeszközök a hieroglifák leegyszerűsített megrajzolásához, papírmodellek a „csokoládékhoz”, a négyzetek területének összehasonlításához.

Követelmények: Minimális követelmény: Az egyiptomi, szigorúan additív számírás írása, olvasása. Próbálkozás a megismert probléma „Melyik terület nagyobb? A két kis négyzeté együtt, vagy a nagyé?” kísérleti megoldására. Optimális követelmény: Az egyiptomi számírás biztos ismerete, a tanuló legyen képes összehasonlítani a kétféle számírást néhány szempont szerint, szorzás egyiptomi módra. A terület-egyenlőség átdarabolásos bizonyításának megértése.

Utazás, adatkezelés

Cél: Egyszerű példán megismertetni a tanulókkal az alkalmazott matematika néhány sajátosságát, gyakorlatszerzés az adatok gyűjtésében és elrendezésében, feladatok megfogalmazása és megoldása az adatok segítségével.

Szükséges eszközök: Menetrendek, árlisták. Dobókocka a feladatok generálásához.

Követelmények: Minimális követelmény: A kapott adatok beírása a kapott táblázat megfelelő helyére, egyszerű számítások elvégzése. Optimális követelmény: Annak ismerete, hogy az adatok elrendezésének és bemutatásának többféle módja lehet. Az adatok elemzése, kérdések felvetése és válaszok keresése, becslés és számolás

Poliéderek

Cél: A poliéder szemléletes fogalmának kialakítása, a poliéderek fajtái, tulajdonságai, néhány példa bemutatása nem poliéder testekre. A poliéderek összehasonlítása, megkülönböztetése az élek, lapok, csúcsok száma alapján.

Szükséges eszközök: Gyurma, pálcikák, dobozok, építőkockák.

Követelmények: Minimális követelmény: A kocka és a téglatest fogalma, ráismerés, modellek előállítása. Optimális követelmény: Problémamegoldás a poliéderekkel kapcsolatban.

Következtetések

Az órákról kapott beszámolók alapján a tanulók szívesen végezték feladataikat. Számunkra meglepő volt, hogy néhányan korábban nem ismerték a dobókockát, semmilyen táblás játékkal még nem játszottak. A dobókockás, az ügyességi és a szituációs játékok során a tanulók sok tapasztalatot szereztek a szimbólumok használatában. A valódi, korábban is ismert tárgyakkal szerzett tapasztalatokra építve kezdtünk el egyszerű matematikai játékokat játszani matematikai oktató eszközökkel is, például a színes rudakkal és a logikai készlettel. Minden tanuló sokat fejlődött, közülük a tehetségesek képesek voltak nehéz problémák megoldására is.

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Prilagodbe nastavnih jedinica iz matematike za učenike s teškoćama u redovnom školskom sustavu

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Osnovna škola Ljudevita Gaja, Osijek, Hrvatska

Sažetak. Dječja prava i inkluzivno obrazovanje predstavljaju temu koja ima mogućnost povećati značenje, moć i prava u obrazovanju (Hart i sur., 2001).

Današnje vlade preuzimaju odgovornost za obrazovanje sve djece. Posebna pozornost usmjerena je na sposobnosti i potrebe djece kroz individualizirane pristupe, prilagođene ili posebne programe poučavanja uz neizostavnu ulogu roditelja sve djece i aktivno sudjelovanje učenika u školi radi promicanja prava na kurikulum (um) usmjeren na učenika (Garner i Dietz, 1996). U hrvatskim se školama posljednjih tridesetak godina na različite načine izrađuju i primjenjuju prilagođeni programi i individualizirani postupci. "Prilagođeni program" je program primjeren osnovnim karakteristikama teškoće u razvoju djeteta, a u pravilu podrazumijeva smanjivanje intenziteta i ekstenziteta pri odabiru nastavnih sadržaja obogaćenih specifičnim metodama, sredstvima i pomagalicama. Prilagođeni program izrađuje učitelj u suradnji s defektologom odgovarajuće specijalnosti s trajnim ili povremenim uključivanjem u rehabilitacijske programe specijaliziranih organizacija (Pravilnik o osnovnoškolskom odgoju i obrazovanju učenika s teškoćama u razvoju, NN, 23/1991).

Svaka prilagodba koja služi svojoj svrsi, a to je da učenik ili učenica s teškoćama u inkluzivnom odjelu sa svojim vršnjacima ima priliku i mogućnost kroz individualni pristup razviti svoje vještine i znanja i osposobiti se za sljedeći stupanj školovanja ili zaposlenje, jest dobra prilagodba. U ovom radu biti će opisano devet načina prilagodbi (Deschenes, Ebeling i Sprague, 1994). Objasnit ćemo na što se svaka pojedina prilagodba odnosi, a zatim navesti primjere za svaku vrstu prilagodbe koja se može provesti u matematici.

Planiranje u odgojno obrazovnom sustavu nije doseglo zadovoljavajuću razinu te su potrebne promjene u planiranju rada za djecu s posebnim obrazovnim potrebama.

Ključne riječi: devet načina prilagodbe, matematika, učenici s teškoćama

Uvod

Edukacijsko uključivanje djece s teškoćama u redovni sustav odgoja i obrazovanja predstavlja važan indikator kvalitete života ove skupine djece (Igrić, Kobetić, Lisak, 2008; prema Krampač-Grljušić, Lisak, Žic Ralić, 2010). Prethodna istraživanja ukazala su kako je za sustavnu podršku potrebna aktivna uloga učitelja, pozitivni stavovi prema integraciji (Kiš-Glavaš, 2000; prema Krampač-Grljušić, Lisak, Žic Ralić, 2010), stručno usavršavanje učitelja (Stančić, Kiš-Glavaš, Igrić, 2001; prema Krampač-Grljušić, Lisak, Žic Ralić, 2010) te njegova dobra suradnja sa stručnim suradnikom, roditeljima i samim djetetom (Hirsto, 2010; prema Krampač-Grljušić, Lisak, Žic Ralić, 2010).

Današnje vlade preuzimaju odgovornost za obrazovanje sve djece. Posebna pozornost usmjerena je na sposobnosti i potrebe djece kroz individualizirane pristupe, prilagođene ili posebne programe poučavanja uz neizostavnu ulogu roditelja sve djece i aktivno sudjelovanje suučenika u školi radi promicanja prava na kurikulum¹ usmjeren na učenika (Garner i Dietz, 1996). Hrvatski nacionalni obrazovni standard (Ministarstvo znanosti, obrazovanja i športa, 2006) naglašava zajedničku aktivnost i odgovornost svih sudionika odgojno-obrazovnog procesa, roditelja, izvanškolskih institucija i cijele lokalne zajednice. Preporučuje izradu individualiziranog odgojno – obrazovnog programa.

Prilagođeni programi i individualizirani postupci

U hrvatskim se školama posljednjih tridesetak godina na različite načine izrađuju i primjenjuju prilagođeni programi i individualizirani postupci. “Prilagođeni program” je program primjeren osnovnim karakteristikama teškoće u razvoju djeteta, a u pravilu podrazumijeva smanjivanje intenziteta i ekstenziteta pri odabiru nastavnih sadržaja obogaćenih specifičnim metodama, sredstvima i pomagalicama. Prilagođeni program izrađuje učitelj u suradnji sa stručnjakom edukacijsko rehabilitacijskog profila odgovarajuće specijalnosti s trajnim ili povremenim uključivanjem u rehabilitacijske programe specijaliziranih organizacija (Pravilnik o osnovnoškolskom odgoju i obrazovanju učenika s teškoćama u razvoju, NN, 23/1991).

Individualizirani odgojno – obrazovni program izrađuje se za svakog pojedinog učenika koji ima Rješenje Ureda državne uprave temeljem Odluke Učiteljskog vijeća škole. Mjerilo za određivanje primjerenog modela školovanja, oblika i razine podrške jest vrsta i stupanj teškoća. Individualne posebnosti svakog djeteta i prepreke koje proizlaze iz okoline se zanemaruju. Metode, sredstva i primjeren program školovanja planira se temeljem utvrđene vrste i stupnja teškoća (Krampač-Grljušić, Mihanović, 2010).

¹ Kurikul(um) znači organizirani aranžman procesa učenja i sadržaja obzirom na određene svrhe i ciljeve (Knoll, 1989; prema Pastuović, 1999). Kurikulum je razvojni odgojno-obrazovni ciklus koji ima svoje zakonitosti i određuje konkretne nastavne aktivnosti odgovarajući na pitanja: Što, S kime, S čime, Kako, U kojem vremenu, Koliko efektivno, Koliko efikasno za za djecu.

Individualizirani odgojno – obrazovni program nije i ne može biti isti za sve učenike. Za svakog pojedinog učenika izrađuje se program različite razine ovisno o prethodnim znanjima, vještinama i sposobnostima učenika. Izrađuje se za sve ili određene nastavne predmete temeljem Rješenja Ureda državne uprave. Treba sadržavati sve važne podatke: ciljeve, sadržaje, metode, vrijeme, sudionike, suodnose i vrjednovanje. U njegovoj izradi trebaju sudjelovati svi sudionici odgojno – obrazovnog procesa koji su zaduženi za njegovu provedbu, dakle učitelji koji predaju pojedinom učeniku za kojega se izrađuje program i stručni suradnici. Učitelji i stručni suradnici trebaju pri izradi programa krenuti od svojih jakih strana: pedagog, stručnjak edukacijsko – rehabilitacijskog profila ili psiholog imaju mnogo više znanja o samoj prirodi učenikove teškoće, dok su učitelji stručnjaci za ciljeve, metode i oblike rada. Oni bolje poznaju razredno ozračje, ali i načine na koje određeni učenik reagira na postavljene zadatke. Upravo je iz tih razloga suradnja učitelja i stručnih suradnika nužna za izradu kvalitetnog i realnog individualnog učeničkog programa.

Postoje različiti pristupi u izradi individualnog učeničkog programa. **Tradicionalni**, tzv. **model deficita** kojeg napuštaju sve zemlje u kojima se uvidjela važnost razvoja obrazovanja za djecu s teškoćama temelji se na onome što učenik ne zna, ne može i nije u stanju učiniti. U takvim ćemo programima naići na izraze poput “Ne zna dijeliti troznamenkaste brojeve”, “Ne čita na razini svoga razreda” i slično. Pri uporabi takvog deficitarnog modela teško je razviti konkretne i dostižne ciljeve. Ovakve je tvrdnje teško pretvoriti u ciljeve jer su vrlo općenite i ne govore mnogo o trenutačnom postojanju ili nepostojanju određenih sposobnosti i vještina. One se temelje na slabim stranama djeteta, što se nikako ne preporučuje pri izradi individualiziranog odgojno – obrazovnog programa.

Suvremeniji pristup izradi individualiziranog odgojno – obrazovnog programa je tzv. **individualizirani model**. On se temelji na djetetovim jakim stranama, interesima i potrebama. Obrasci kojima se učitelji služe za izradu programa razlikuju se od zemlje do zemlje, pa čak i od škole do škole, no ono što je svim kvalitetnim programima zajedničko jest stavljanje naglaska na sposobnosti i mogućnosti djeteta. Uvijek trebamo imati na pameti da je čovjek biopsihosocijalna struktura i da njegov postojanje, kvaliteta života i kvaliteta učenja ne smiju biti određeni njegovim nedostacima, već upravo suprotno – njegovim jakim stranama (Krampač-Grljušić, Marinić, 2007).

Individualizirani odgojno-obrazovni programa učitelji mogu izraditi u suradnji sa stručnim suradnikom i/ili timski u suradnji s drugim učiteljima. Plan se izrađuje u četiri koraka²:

- 1) Razvoj djetetova profila;
- 2) Razvoj ciljeva i zadataka;
- 3) Identifikacija modela prilagodbe, potrebne potpore i pomoći stručno-razvojne službe;
- 4) Osiguravanje potpore i praćenje ciljeva i zadataka.

² U sklopu OECD-ovog projekta “Razvoj obrazovanja za djecu s teškoćama i djecu u riziku” od 2003. do 2007. voditeljica edukacija za učitelje i ravnatelje model škola Darlene Prener sa Sveučilišta u Bloomsbergu predlaže uvođenje ovog Individualiziranog odgojno – obrazovnog programa.

Identifikacija modela prilagodbe, te potrebne potpore i pomoći

Kada smo odredili ciljeve i zadatke za učenika ili učenicu s teškoćama, bit će potrebno uvesti određene postupke prilagođavanja kako bi učeniku ili učenici kroz osmišljene zadatke osigurali usvajanje znanja i vještina koje će dovesti do željenih ciljeva. S obzirom na to da se dijete s teškoćama nalazi u razredu s vršnjacima različitih mogućnosti, pred učitelja je stavljena nimalo laka zadaća. Prilagođavanjem učitelj osigurava učeniku razumijevanje i usvajanje sadržaja, mogućnost da se osjeća ravnopravnim dijelom razreda, a ujedno olakšava i sebi rad u razredu s učenicima različitih mogućnosti.

Pri odabiru postupaka prilagođavanja vraćamo se na sam početak planiranja, proučavamo interese, jake strane i dosadašnja iskustva učenika/učenice s teškoćama kako bi dijete što jednostavniji i njemu prihvatljiv način usvojilo potrebna znanja i vještine.

Devet načina prilagodbi u matematici

Općenito govoreći u radu s učenicima različitih mogućnosti moguće je prilagoditi same nastavne sadržaje, ciljeve i zadatke, ali i vrijeme usvajanja ili rada na zadatku, količinu pojmova, strategije i metode poučavanja i sl.

Svaka prilagodba koja služi svojoj svrsi, a to je da učenik ili učenica s teškoćama u inkluzivnom odjelu sa svojim vršnjacima ima priliku i mogućnost kroz individualni pristup razviti svoje vještine i znanja i osposobiti se za sljedeći stupanj školovanja ili zaposlenje, jest dobra prilagodba. U nastavku će biti opisano devet načina prilagodbi (Deschenes, Ebeling i Sprague, 1994) koje možemo koristiti pri prilagodbi nastavnih tema/jedinica u nastavnom predmetu matematike.

Prilagodba količine

- a) Smanjiti količinu (broj zadataka) koje učenik mora izvršiti i razvijati osnove matematičke pismenosti težeći uvježbavanju prve skupine matematičkih kompetencija: reprodukcije, definicije i računanje prema mogućnostima djeteta.
- b) Učenik sudjeluje u skupnom radu ili projektu, izražujući samo neke zadatke (npr. baca kockice i zadaje ostalim učenicima zadatke ili pri mjerenju opsega igrališta drži metar za mjerenje i označava polje).
- c) Smanjiti broj zadataka za domaću zadaću.
- d) Smanjiti broj zadataka u matematičkom diktatu i pri pisanim provjerama.

Prilagodba vremena

- a) Produljiti vrijeme za izvršavanje zadatka.
- b) Pri učenju novih nastavnih jedinica produljiti vrijeme za učenje (prvo usvojiti zbrajanje brojeva do 100 bez prijelaza u drugu deseticu, a zatim s prijelazom u drugu deseticu)
- c) Dati učeniku više vremena za realizaciju nekih projekata ili skupnog rada.

Stupanj pomoći

- a) Pomoć učiteljice – dok drugi učenici ponavljaju i vježbaju, učiteljica prati učenikov radi i po potrebi pomaže.
- b) Pomoć vršnjaka – učenik iz klupe čita tekstualni zadatak svom prijatelju iz klupe ili kad je gotov sa svojim zadatkom pomaže i usmjerava svog prijatelja iz klupe. Osigurati pomoć vršnjaka u radu kod kuće (vježba).
- c) Pomoć asistenta u nastavi – pomaže djetetu u radu na satu i na dopunskoj nastavi
- d) Pomoć starijeg djeteta i roditelja – prema naputcima učiteljice pomaže djetetu pri ponavljanju i utvrđivanju te vježbanju gradiva.
- e) Pomoć stručne službe (pedagoga, stručnjak edukacijsko-rehabilitacijskog profila, psihologa i logopeda) koji s djetetom rade individualno na rješavanju teškoća.
- f) Pomoć računalnih programa – koji kroz edukativnu igru pomažu u kontroli točnosti rješenja pri vježbanju.

Prezentacija nastavnih sadržaja

- a) Čitati tekstualne zadatke naglas;
- b) Uz usmene upute dati i pismene upute (u zadacima riječima dozvoliti bojanje ili podvlačenje bitnih i sličnih dijelova zadatka);
- c) Prezentacija na računalu (konstrukcija usporednih i okomitih pravaca);
- d) Praktične aktivnosti (učenik sastavlja geometrijske likove izrezane od kolaž, mjeri masu nekog tijela dok drugi pretvaraju mjerne jedinice, kombinira novčanice kojima može platiti neki predmet);
- e) Vizualno označavanje bitnih dijelova, bojanje, isticanje, uvećavanje dijelova zadatka.

Težina

- a) Zadatke riječima zadati kao problem iz stvarnog života (trgovina, popločavanje kupatila, police s knjigama);
- b) Podijeliti zadatak na korake npr. razlici brojeva 629 i 358 dodaj broj 154.

Prvi korak: Izračunaj razliku brojeva 629 i 358

Drugi korak: Razlici brojeva (gore dobiveni rezultat) dodaj broj 154.

- c) Pri računanju dozvoliti upotrebu didaktičkih materijala (olovke, kuglice, klikeri, kockice);
- d) Prilagoditi zadatak: dok drugi računaju opseg trokuta uvrštavanjem u formulu, učenik vrpcom mjeri opseg trokuta oblikovanog na ploči od pluta i zapisuje rezultat. Ne računa površinu kvadrata i pravokutnika, već crta u

računalnom programu koji sam računa površinu. Dok drugi učenici rastavljaju brojeve na D i J, učenik broji loptice u boji i slaže ih po D i J.

- e) Prilagoditi metode: pisano oduzimanje brojeva do 1000 računati nadopunjavanjem umanjitelja do umanjenika. Crtati usporedne i okomite pravce u geometrijskom programu umjesto rukom. Uporaba kalkulatora pri rješavanju složenijih zadataka. Dozvoliti pomoćne radnje, npr. crtanje kružića pri pisanom dijeljenju ako učenik nije svladao tablicu množenja.

Iskazivanje znanja

- a) usmeno odgovaranje umjesto pisane provjere;
- b) provjeravanje po koracima kako je dijete učilo;
- c) Kratki diktati – sporijim tempom, veći broj ponavljanja;
- d) Konstrukcije u računalnim programima;
- e) Dijete smije koristiti slike i didaktičke materijale, kartončić s pretvorenim mjernim jedinicama (m, dm, cm, mm);
- f) Pri složenijem zadatku neki su koraci već napravljeni (u zadatku riječima zadatak je već postavljen, učenik treba samo izračunati);
- g) Dozvoliti ponovno pisanje provjere s tipovima zadataka koje nije točno riješio.

Stupanj sudjelovanja

- a) Brisanje ploče;
- b) Dijeljenje pisanih radova drugim učenicima;
- c) Asistencija učitelju pri power point prezentaciji (klik mišem);
- d) Zadavanje zadataka drugim učenicima (igra dremuckanje);
- e) Sudjelovanje u kvizu znanja s prilagođenim zadacima pri ponavljanju nastavne cjeline;
- f) Sudjelovanje u skupnom projektu s lakšim zadatkom;
- g) Aktivno uključivanje u rad (zadatci na ploči, sudjelovanje u dijelu skupnog projekta).

Zamjenski cilj

- a) Igra igru “Čovječe ne ljuti se” (broji) sa skupinom učenika, dok drugi vježbaju;
- b) Mjeri zadane duljine dužina u cm dok drugi računaju u svim mjernim jedinicama;
- c) Mjeri volumen nepravilnog tijela uz pomoć menzure dok drugi izražavaju volumen u različitim mjernim jedinicama;
- d) Dok drugi uče konstrukciju pravokutnog trokuta, učenik na računalu slaže likove od gotovih konstrukcija;
- e) Zapisuje zadatke (sastavlja) množenja gledajući slike i zapisuje rezultate dok drugi vježbaju tablicu množenja.

Zamjenski kurikulum

- a) Dok djeca uče zbrajati brojeve do 1000, učenik sa slušalicama sluša zadatke tablice množenja, a zatim ih rješava u bilježnicu;
- b) Učenik odlazi kod članova stručne službe na vježbe;
- c) Učenik na računalu u razredu igra didaktičke igrice;
- d) U nekim slučajevima zamjenski kurikulum nije potreban.

Zaključak

Individualni učenički plan koji počiva na individualiziranom pristupu samo je jedan od načina planiranja za djecu s teškoćama. Isti bi mogao poslužiti kao pozitivni modeli prema kojima bi se postojeći Nastavni planovi i programi odgoja i osnovnog školovanja za učenike s teškoćama u razvoju (Ministarstvo prosvjete i športa Republike Hrvatske, 1996) mijenjali i prilagođavali u skladu s Konvencijom o pravima djeteta (Ujedinjeni narodi, 1992.) i Konvencijom o pravima osoba s invaliditetom (Ujedinjeni narodi, 2006.). Planiranje u kojemu je dijete s teškoćama u središtu pozornosti, a njegove jake strane su polazište i temelj njegovom napredovanju uz uključivanje svih dionika važnih za djetetov život, pridonijet će ponajprije kvalitetnom provođenju inkluzije u školskom sustavu, a zatim i rušenju predrasuda koje okolina ima prema djeci s teškoćama. U želji za ostvarivanjem obrazovnih ciljeva zaboravljamo ono najvažnije: tko je dijete za koje izrađujemo plan, tko su važne osobe u njegovom životu. Na kraju postavlja se i najključnije pitanje: je li dijete za koje planiramo sretno i ostvaruje li ciljeve i napredak bez prisile kroz kvalitetne prilagodbe koje smo predvidjeli planiranjem.

Svako dijete ima drukčije potrebe. Odgovoriti na potrebe sve djece ne znači odgovoriti na potrebe pojedinog djeteta. Potrebno je poštovati različitosti i raznovrsna iskustva djece (Krampač-Grljušić, Mihanović, 2010.)

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Igra – put prema matematičkoj kompetenciji

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Sažetak. Metodika početne nastave matematike je znanstvena disciplina koja proučava matematičko odgajanje i obrazovanje u prva četiri razreda osnovne škole i u njoj učenici stječu početnu matematičku pismenost. Obilježavaju je nastavni sadržaji visoke apstrakcije i učenici je najčešće ne mogu iskustveno spoznati. Dijete u ovoj dobi može logički misliti, ali samo ako je misao utemeljena na radnjama s konkretnim objektima. Stoga se u našim školama pronalaze načini smišljenog i učinkovitog sjedinjavanja učenja i matematičke igre jer igra učenike privlači, motivira, te razvija njihovo apstraktno mišljenje i logičko zaključivanje. Učeći matematiku kroz igru, dijete stječe znanja i sposobnosti o matematičkom sadržaju, a pritom se zabavlja. U didaktičkim matematičkim igrama sjedinjava se više operacija iz više područja, ali prednost treba dati misaonim operacijama tako da u igri dođe do izražaja učenička dosjetljivost, oštroumnost, kombinatorika i kreativnost. Sadržaj igre izvodi se iz nastavnog programa matematike, pa možemo zaključiti da je njen učinak pokazatelj uspješnosti ostvarenja tog istog nastavnog programa u početnoj nastavi matematike. U osmišljavanju i provođenju didaktičke igre u nastavi matematike značajna je uloga učitelja, pri čemu do izražaja dolazi njegova kreativnost, sustavnost i inovativnost.

U radu su prikazane didaktičke igre koje se najčešće koriste u nastavi matematike, te stavovi učitelja razredne nastave o primjeni didaktičke igre u nastavi matematike.

Ključne riječi: početna nastava matematike, uloga didaktičke igre, učitelj razredne nastave

Uloga didaktičke igre u početnoj nastavi matematike

Metodika početne nastave matematike je disciplina koja proučava matematičko obrazovanje i odgajanje u *prva četiri razreda osnovne škole* i posjeduje obilježja po kojima se razlikuje od idućih stupnjeva matematičkog školovanja. Nastavni sadržaji

se u početnoj nastavi matematike usvajaju planirano i organizirano te često bez neposrednog dodira sa stvarnošću, ali, premda dijete u ovoj dobi može logički misliti, ta misao ipak treba biti utemeljena na aktivnostima s konkretnim objektima. U početnoj nastavi matematike obrazovna postignuća odnose se na procese uočavanja, prepoznavanja, početnog razumijevanja, razlikovanja i imenovanja temeljnih pojmova koji ulaze u sadržaj predmeta matematike i na kojima će se temeljiti nastava matematike na višim razinama. Nastava matematike je uglavnom usmjerena na sposobnosti operiranja i pritom se zanemaruju “krucijalne sposobnosti kao apstraktno mišljenje, logičko zaključivanje, rješavanje problemskih zadataka, sposobnost interpretacije grafičkih prikaza” (Đuranović, Klasnić, 2009.). Učeći matematiku na konkretnim predmetima i rješavajući stvarne probleme učenici će usvojiti znanja i vještine koje omogućavaju uspješnu primjenu matematike u svakodnevnom životu, pa će time uvidjeti važnost matematike u svojim životima. Upravo se matematička kompetencija (NOK, 2010.) odnosi na osposobljenost učenika na razvijanje i primjenu matematičkog mišljenja u rješavanju problema u nizu svakodnevnih situacija, te će tako postati aktivni sudionici u procesu učenja i pripremljeni za cjeloživotno učenje. Nadamo se da će se u hrvatskoj suvremenoj školi voditi briga o razvijanju matematičkih kompetencija jer istraživanje provedeno 2008. godine (Baranović i sur., 2008.) pokazalo je da se u nastavi matematike najveći naglasak stavlja na rješavanje matematičkih zadataka i razvoj logičkog načina razmišljanja, a manje na uporabu matematike u svakodnevnom životu i razvoj kritičkog promišljanja o matematičkim konceptima i postupcima.

Prema postavkama Glasserove *Kvalitetne škole*, uspješan rad u školi je onaj koji zadovoljava četiri osnovne ljudske potrebe – znanje, moć, ljubav i zabavu (Glasser, 2005.). Upravo je zabava učenicima ugodna, radosna i privlačna, a u nastavi nam je lako ostvariva kroz igru jer stvara pozitivan emocionalni doživljaj. Marzollo i Lloyd (1972., prema Sharma, 2001., 124. str.) smatraju da učenik u igri uči “koncentrirati pažnju, vježba vizualizaciju, iskušava ideje, prakticira odraslo ponašanje i razvija osjećaj kontrole nad vlastitim svijetom”. Svaka igra kao zabavna djelatnost ima svoj sadržaj i po njemu najčešće dobiva naziv. Budući da se u nastavi uglavnom proučava realitet, didaktičke igre se mogu podijeliti po nastavnim predmetima tako da razlikujemo jezične, glazbene, tehničke, povijesne, zemljopisne i matematičke igre. Matematičke igre potiču matematičko razmišljanje i mentalnu aktivnost. “Iako matematika nije igra, mnoge igre imaju veze s matematikom, a i mnogo matematičkih tema može se opisati i objasniti na zabavan način, često i kroz igru” (Bruckler, 2010., 57. str.).

Prema područjima aktivnosti – senzornim, manualnim, izražajnim ili misaonim može se složiti igra. Prema Jursić (2010.), neke matematičke igre potiču razvoj logičko-matematičke inteligencije (brojevi), a neke razvoj vizualno-spacijalne inteligencije (geometrija). U didaktičkoj igri sjedinjava se više operacija iz više područja, ali prednost treba dati misaonim operacijama tako da u igri dođe do izražaja učenikova dosjetljivost, oštroumnost, kombinatorika i kreativnost. Učenik “u igri stvara modele koje razumije i voli i koje kasnije može efikasno upotrijebiti za razumijevanje apstraktnih matematičkih ideja” (Sharma, 2001., 124. str.). Upravo u tome je posebnost didaktičkog značenja igre. S obzirom da se didaktička igra izvodi iz nastavnog sadržaja stoga se njezin zadatak i cilj podređuju ciljevima

i zadatcima konkretnog nastavnog programa – stjecanju određenih znanja i razvijanju određenih sposobnosti. Neki didaktičari savjetuju da se igra ugrađuje u teže dijelove nastavnog procesa. U kojem dijelu nastavnog procesa će učitelj primijeniti igru ovisi o procijeni samog učitelja, te njegovoj kreativnosti, sustavnosti i inovativnosti. Prije prijelaza na samu igru učitelj odgovarajućim metodičkim postupcima upoznaje učenike sa sadržajem igre, objašnjava njene zadatke, imenuje i opisuje upotrijebljena pomagala, demonstrira izvođenje igre u cjelini i u pojedinostima, obrazlaže pravilnost pojedinih postupaka te upozorava na pravila igre. Didaktičke igre će svojom zanimljivošću i natjecanjem motivirati učenike da verbaliziraju ideje, smišljaju odgovarajuća rješenja i strategije u igri tako da na taj način provode puno više vremena ('svojom voljom') družeći se s matematikom i matematičkim pojmovima.

Svaka igra, pa tako i didaktička, izvodi se po unaprijed utvrđenim pravilima. Tako se uređuje i prilagođava odnos prema predmetu igre i drugim suigračima, te korektnost, tolerantnost, poštenje, pravednost. Poštivanje pravila igre stoga pridonosi kvalitetnoj socijalizaciji u komunikaciji. Učenici moraju "kombinirati i umno se naprezati kako bi igru odigrali do kraja, moraju se znati svladati, moraju naučiti pobijediti u igri, ali i gubiti" (Đurović i Đurović, 1984., 25. str.).

Konačan učinak didaktičke igre je stečeno znanje i sposobnost o sadržaju igre s popratnim priznanjem, pohvalom, pa i visokom ocjenom najboljima. Kako je sadržaj igre izveden iz nastavnog programa to je i njen učinak pokazatelj uspješnosti realizacije nastavnog programa.

Uloga učitelja

Primjena igre je veliki izazov i zahtjev za učenika i učitelja. Od učenika se očekuje pažljivost i strpljivost, pamćenje pravila igre i rješavanje problema. Velika je i uloga učitelja pri uporabi igre jer didaktička igra treba imati svoje opravdanje u nastavnom procesu – za uvježbavanje i primjenu obrađenih matematičkih sadržaja te automatizaciju određenih matematičkih operacija. Didaktička igra se ne uvodi u početnu nastavu matematike zbog igranja, već radi uspješnijeg, trajnijeg ali i lakšeg učenja. Međutim, prema Sharma (2001.), većina učitelja uvodi igru u nastavu samo kao dodatnu aktivnost koja služi većem motiviranju i opuštanju učenika. Osim što treba isplanirati igru i znati u kojem trenutku će je i s kojim ciljem primijeniti, učitelj treba pripremiti potreban materijal i dobro isplanirati vrijeme trajanja igre.

Uporabom igre u nastavi matematike, učitelj omogućava učenicima odmor od rješavanja zadataka kroz kinestetičko-matematičku aktivnost. Igre su često natjecateljske, što dodatno osvježava nastavni sat i motivira učenike na logičko razmišljanje i brzo zaključivanje. Planirajući i pripremajući didaktičke igre za nastavu matematike učitelj treba učenicima prilagoditi nastavne metode, oblike rada i nastavna sredstva koja će ih potaknuti na stvaralački i suradnički rad. Rezultati provedenih istraživanja (Arambašić i sur., 2005.) pokazali su da se učiteljima treba osigurati podrška kako bi se kod njih razvili pozitivni stavovi i uvjerenja o matematici i trebalo bi ih uputiti na odabir metoda poučavanja koje su naprimjerene njihovim učenicima.

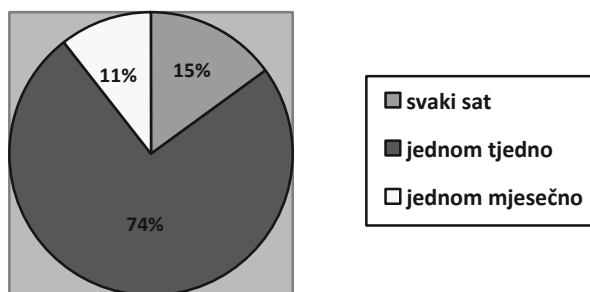
Prema Sharma (2001.), sve vrste didaktičkih igara možemo podijeliti na tri velike kategorije: manipulativne, reprezentativne i strukturirane igre s pravilima. U manipulativnoj igri dijete razvija logičko-matematička iskustva koja postaju temeljem smislenog učenja i intenziviraju njegov kognitivni rast. U reprezentativnoj igri učenik uvodi maštu koja je važna komponenta razvijanja apstraktnog mišljenja u starijoj dobi jer “osmišljavanje igara, slaganje modela, taktilno i vizualno doživljavanje stvorit će neophodno polazište za prijelaz k apstraktnome” (Đuranović, Klasnić, 2009.). U strukturiranim igrama učenik “uči slijediti upute u nizu, neposredno razvija matematičke vještine, matematičko i prostorno mišljenje, učvršćuje naučene matematičke koncepte, postupke i činjenice” (Sharma, 2001., 124. str.).

Pri konstruiranju kvalitetne didaktičke igre učitelj se koristi dostupnom literaturom, različitim medijima (za pripremu i izvođenje igre), informacijskim centrima, ali ipak najviše pokazuje (uz metodičko i didaktičko znanje) svoju dosjetljivost, kreativnost i iskustvo.

Mišljenja učitelja o primjeni didaktičke igre u nastavi matematike

U istraživanju provedenom u studenom 2010. godine na stručnom skupu Županijskog stručnog vijeća grada Siska sudjelovala su 94 učitelja razredne nastave. Prema stručnoj spremi, većina anketiranih učitelja ima visoku stručnu spremu (56 učitelja – 60%).

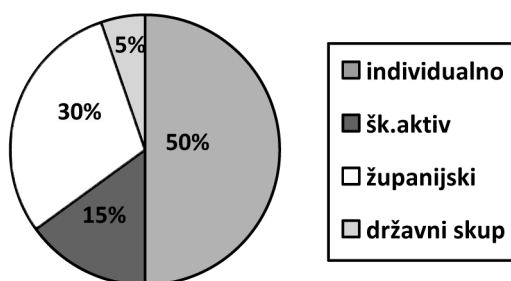
Na Slici 1 vidljivo je da svi anketirani učitelji primjenjuju igru u nastavi matematike, pri čemu je 70 učitelja (74%) primjenjuje jednom tjedno.



Slika 1. Koliko često primjenjujete igru u nastavi matematike?

Didaktičke igre učitelji u jednakom postotku primjenjuju u uvodnom i završnom dijelu sata, pri ponavljanju ili vježbanju. Uočeno je da niti jedan anketirani učitelj igru ne primjenjuje u glavnom dijelu sata. 65% učitelja češće koriste matematičke igre bez pomagala i materijala, dok 35% učitelja navode da koriste kocke i kockice, karte, domino ili neke druge društvene igre.

Najčešće primjenjivane didaktičke igre u nastavi matematike, prema učestalosti pojavljivanja u anketnim upitnicima su: igre s kockicama, dremuckanje, domino, memory, izvlačenje brojeva iz šešira, pikado, tombola, bingo i slagalica.



Slika 2. Mišljenje učitelja o razini stručnog usavršavanja na kojoj smatraju da će ojačati svoje matematičke kompetencije

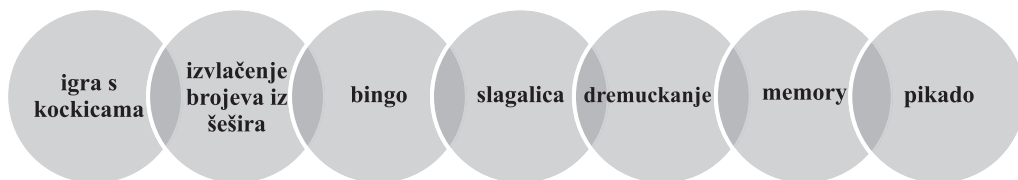
Prema Slici 2 učitelji smatraju da će im pri jačanju matematičkih kompetencija i primjeni didaktičke igre u nastavi najviše pomoći individualno stručno usavršavanje (47 učitelja), a zatim stručno usavršavanje na županijskoj razini (28 učitelja).

U anketnom upitniku su učitelji imali mogućnost napisati što misle o primjeni didaktičke igre u nastavi. Slijede neka njihova razmišljanja:

- ✓ Matematičke igre su korisne jer razvijaju kreativnost kod učenika, uz igre učenici brže i lakše pamte.
- ✓ DA za igre jer davno je dokazano kako se kroz igru najkvalitetnije uči i dugotrajno pamti.
- ✓ Zanimljive igre motiviraju učenike za (novo i staro) gradivo, daju samopouzdanje i mogućnost aktivnog sudjelovanja svakog učenika.
- ✓ Didaktičke matematičke igre nisu preporučljive – one su POTREBNE!
- ✓ Matematičke igre osvježavaju nastavu i približavaju matematiku stvarnom životu.
- ✓ Didaktičke igre pomažu učeniku prihvatiti matematiku kao nešto stvarno i korisno.

Primjeri odabranih didaktičkih igara

U nastavku rada opisane su didaktičke igre koje učitelji razredne nastave Grada Siska najčešće koriste u nastavi matematike Slika 3:



Slika 3. Didaktičke igre koje učitelji razredne nastave grada Siska najčešće koriste u početnoj nastavi matematike

1. Igra s kockicama

Učenici bacaju kockicu i glasno izgovaraju broj točkica na kockici. Ako imaju dvije kocke mogu uvježbavati zbrajanje i oduzimanje, ako imaju tri ili više kocaka uvježbavaju zbrajanje ili sa dvije kocke množenje brojeva.

2. Izvlačenje brojeva iz šešira

Učenici glasno izgovaraju izvučene brojeve. Ako su brojevi trodimenzionalni najprije ih prepoznaju zatvorenih očiju (taktilnost). Brojeve zatim pričvršćuju na ploču. U kombinaciji sa slijedećim izvučenim brojevima nastaje matematički zadatak (brojevni izraz ili matematička priča).

3. Bingo

Bingo je igra slična lotu. Primjenjuje se najčešće u 1. ili 2. razredu zbog brojevnog niza do 20, a kasnije s brojevima do 50 ili 100. Može se igrati individualno ili grupno.

Individualno – materijal: bilježnica, olovka, kartice sa brojevima

Tijek igre: svaki učenik nacrtava tablicu sa 6 polja i u svako polje upiše broj. Brojevi se ne smiju dva puta pojavljivati u istoj tablici. Učitelj izvlači kartice s brojevima i glasno izgovara broj, a učenici koji imaju taj broj u tablici moraju ga prekrižiti. Učenik koji prvi prekriži svih 6 brojeva vikne "BINGO!" i on je pobjednik.

Grupno – materijal: ploča, kreda, kartice s brojevima

Tijek igre: razred je podijeljen u dvije grupe. Crtaju se dvije tablice s po 6 polja na ploči. Učenici iz svake grupe govore brojeve i učitelj ih upisuje u tablice. Učenici iz jedne i iz druge grupe naizmjenice izvlače brojeve, a izvučeni brojevi se precrtavaju u obje tablice na ploči. Pobjednik je ona grupa koja prva prekriži sve brojeve.

4. Slagalice

U svakom kvadratiću nalazi se po jedan zadatak. Zadatke treba izračunati, položiti na karton sa rješenjima, tako da je rezultat okrenut prema dolje. Na poleđini svakog crteža je neka slika. Ako se svi zadatci pravilno izračunaju, dobije se pravilan crtež.

5. Dremuckanje

Učenici su spuštene glave prema klupi ('dremuckaju'). Učitelj šeće učionicom i izgovara brojevni izraz. Učenik kojeg dodirne po ramenu treba ponoviti brojevni izraz i reći rezultat. Ako je točno riješio brojevni izraz podiže glavu (budi se).

6. Memory

Na kartone u dvije različite boje napišu se zadatci – na jednu boju zadatak, a na drugu rješenje zadatka. Kartoni se poslažu na stol licem prema dolje. Učenik okreće kartone različitih boja. Ako okrene kartone na kojima su zadatak i rješenje uzima ih sebi, a zatim otvara novi par kartona. Ako ne otvori zadatak i točno rješenje, vraća kartone na mjesto licem prema stolu. Igra se u skupinama po 3-4 igrača, pobjednik je igrač koji skupi najviše parova.

7. Pikado

Na ploči ili plakatu je nacrtano 5 koncentričnih krugova. (Krugovi mogu biti podijeljeni okomitim crtama tako da se dobije više različitih polja.) U svaki krug (polje) upiše se jedan broj. Učenici stoje u dvije kolone ispred krugova na istoj udaljenosti i bacaju kedu u metu. Brojeve koje pogode zbrajaju/množe. Pobjednik je kolona koja sakupi najviše bodova.

Zaključak

Osnovni cilj nastave matematike je stvoriti matematički pismenog, kompetentnog pojedinca koji je sposoban primjenjivati matematiku u svakodnevnom životu. S obzirom da su nastavni sadržaji od samog početka apstraktni, važna je kvalitetna metodička obrada i stalna povezanost matematičkih sadržaja sa životnom stvarnošću. Za usvajanje matematičkih sadržaja u nastavi često se koristi didaktička igra jer povezuje učenje i zabavu, motivira učenike, razvija apstraktno mišljenje i logičko zaključivanje. Pritom je velika uloga učitelja u osmišljavanju i provođenju igre u početnoj nastavi matematike jer didaktička igra treba imati opravdanje u nastavnom procesu. Obogaćivanje učenja didaktičkom igrom i poticanje učenika na matematičke igre razvija njihove matematičke sposobnosti. U nastavi matematike češće se koriste igre s pravilima jer one od učenika zahtijevaju pamćenje i sadržaja i pravila igre, ali i mnoge socijalne vještine vezane uz pobjeđivanje i poraz u igri.

Istraživanje provedeno među učiteljima razredne nastave na Županijskim stručnim vijećima grada Siska pokazalo je da učitelji barem jednom tjedno primjenjuju didaktičku igru u nastavi matematike i to pri ponavljanju i vježbanju u uvodnom ili završnom dijelu sata. Niti jedan učitelj didaktičku igru ne primjenjuje pri obradi u glavnom dijelu sata i stoga je potrebno poticati učitelje na pronalaženje, osmišljavanje i korištenje didaktičkih igara koje će pomoći učeniku pri usvajanju novih matematičkih sadržaja, a time i razvijanju matematičkih kompetencija. Od didaktičkih igara koje učitelji najčešće primjenjuju u nastavi autorice rada izdvojile su i opisale 7 igara: igra s kockicama, izvlačenje brojeva iz šešira, bingo, slagalica, dremuckanje, memory i pikado.

Anketirani učitelji većinom smatraju da će im pri jačanju matematičkih kompetencija i primjenjivanju didaktičke igre u početnoj nastavi matematike pomoći individualno usavršavanje pri čemu će do izražaja doći njihova kreativnost, iskustvo i inovativnost.

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Didaktičke igre u nastavi matematike

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Sažetak. Igra kao osnovna dječja aktivnost i djetetova potreba utječe na cjelokupan djetetov razvoj. Didaktičke igre su skupina igara koje se u odgojno-obrazovnom procesu rabe s ciljem ostvarivanja unaprijed postavljenih odgojno obrazovnih zadataka. Pozitivno utječu na rezultate učenja, posebno na trajnost znanja, motivaciju, pažnju i aktivnost učenika, a učenje čine zanimljivijim. Suvremena škola upotrebom didaktičkih igara kao nastavne metode stavlja naglasak na potrebe i prirodu učenika-djeteta. U nastavi matematike didaktičke igre primjenjuju se u upoznavanju, vježbanju i ponavljanju matematičkih sadržaja.

Ključne riječi: igra, didaktička igra, matematika, nastava

Uvod

Igra je osnovna dječja potreba. Ona je spontana i slobodna. U ranom djetinjstvu ona je najvažnija djetetova aktivnost jer kroz igru dijete uči, razvija se, izgrađuje svoje stavove prema svijetu koji ga okružuje. Stoga je prirodno djetetu omogućiti da se što više igra.

Igra se odvija uglavnom zbog zadovoljstva koje pruža. Prihvaćamo je iz vlastitih potreba, bez vanjske prisile. Dijete se u igri osjeća nesputano i otvoreno i slijedi vlastitu koncepciju. Nije dobro prekidati igru nepoželjnim intervencijama. Dijete u igri istražuje, kombinira, isprobava i koristi različite strategije, a odrasla osoba može biti poželjan suigrač ako i sama tako doživljava igru, uvažava zamisli i ideje djeteta i nenametljivo mu nudi nove mogućnosti.

Igru kao neizostavan dio djetinjstva zbog toga ne smijemo isključiti iz nastavnog procesa. Dapače, uvođenjem igre kao nastavne metode kad god je to moguće i što je više moguće poboljšat ćemo rezultate učenja i povećati djetetovo zanimanje za školu.

Igra

Igra je optimalno sredstvo za učenje u ranom djetinjstvu. Omogućiti djetetu da se igra znači dopustiti mu da bude sretno, zdravo i kreativno dijete.

Igru određuju slijedeće karakteristike:

1. Igra je svojevoljno i intrinzično motivirana.
2. Igra je simbolična, puna značenja i transformativna.
3. Igra je aktivna.
4. Igra može biti ograničena pravilima.
5. Igra pruža zadovoljstvo.

Igra je važna u djetetovu razvoju jer:

- omogućuje djetetu da osmisli svoj svijet;
- razvija društveno i kulturno razumijevanje;
- omogućuje djetetu izražavanje svojih misli i osjećaja;
- potiče fleksibilno i divergentno mišljenje;
- pruža mogućnost za suočavanje s problemom i njegovo rješavanje;
- razvija jezične i pismene sposobnosti te proširuje poznavanje pojmova.

Osim toga, dijete kroz igru razvija motoriku, intelektualne vještine, kreativnost, samopouzdanje, smisao za humor, uči govor, socijalizira se i priprema za ulazak u svijet odraslih.

Vrste igara

Teoretičari su napravili razne podjele prema vrstama igre. Ovdje ću navesti podjelu koju je dao Friedrich Fröbel (1782.-1852.), njemački pedagog i osnivač dječjeg vrtića u Njemačkoj 1837. Fröbel je koristio prve radne materijale i igračke za djecu. Cilj njegove didaktičke opreme je poticanje djetetove samokreativnosti i osvještavanje procesa učenja koje se, prema njemu, odvija po određenim stupnjevima. Prema Fröbelu, odgoj počinje od rođenja i taj period koji traje do polaska u školu smatrao je značajnim za razvoj i formiranje čovjeka. Osnova odgoja prema Fröbelu su igre i razne aktivnosti s odgovarajućim didaktičkim materijalom. Fröbel igre dijeli na:

1. majčinsko pjevanje i milovanje,
2. igre predmetima,
3. igre kretanjem.

Pritom *igre predmetima* dijeli na:

- a) igre tijelima (loptom, elementima za građenje, kuglom kockom, valjkom stošcem. . .);
- b) igre plohami (pločom, listom papira. . .);
- c) igre crtama (drvenim štapićima, drvcima, papirnatim trakama, nacrtanim linijama. . .);
- d) igre točkama (kamenjem, sitnim voćem, perlama. . .).

S matematičkog stanovišta u toj podjeli možemo uočiti važnost razlikovanja geometrijskih i matematičkih svojstava predmeta već u najranijoj fazi odgoja djeteta.

S obzirom na djetetovu spoznajnu razinu djeteta i razvoj igre tijekom djetinjstva, također razlikujemo nekoliko vrsta igre. To su:

1. funkcionalna igra,
2. konstruktivna igra,
3. igra uloga,
4. igra s pravilima.

Funkcionalna igra započinje već u dojenačkoj dobi. U tom razdoblju igra se sastoji od jednostavnih mišićnih pokreta koji se ponavljaju, bilo da dijete pri tome koristi ili ne koristi neke predmete. Npr. djeca tresu zvečkom, skaču, povlače igračke koje sviraju, bacaju predmete na pod.

Konstruktivna igra počinje se javljati ulaskom u predškolsku dob, te traje i kroz ranu školsku dob. U ovom tipu igre djeca koriste različite predmete i materijale u namjeri da od njih nešto stvore. Tu spadaju igre poput gradnje kula od kocaka, rezanje i lijepljenje slika, izrada figurica od plastelina. Iako je konstruktivna igra često individualna, u njoj mogu sudjelovati i odrasli (roditelji, odgajatelji). Pritom ne treba pokušavati ukalupiti djetetove uratke u estetske kriterije odraslih, već treba dozvoliti da se djetetova mašta oslobodi. Time potičemo djetetovu originalnost i kreativnost.

Igra uloga počinje se javljati već oko druge godine, paralelno sa smanjenjem funkcionalne igre. Ovo razvojno razdoblje je izrazito maštovito i originalno. Igra uloga uključuje upotrebu predmeta ili osoba kao simbola za nešto što oni inače nisu. Najviše je zastupljena u dobi do 4. ili 5. godine. Igra uloga je važna jer kroz pretvaranje djeca uče kako se uživjeti u tuđu perspektivu, razvijaju vještine u rješavanju socijalnih problema i postaju kreativnija. U gotovo svim takvim igrama mogu se prepoznati tri elementa: zaplet ili priča, uloge i različita pomagala. Djeca u svojoj igri koriste predmete kao pomagala, npr. pretvaraju se da piju čaj iz šalica. Obično postoji neka priča, iako ona može biti jako jednostavna. Teme u igri često odražavaju ono što se događa u dječjim životima pa se igraju mame i tate, liječnika, trgovine, škole i sl. Djeca u predškolskoj dobi često teško uspijevaju riječima opisati što osjećaju ili što im se dogodilo. Igra je lakši način da se izraze mnoge stvari.

Nakon pete godine djeca sve češće pokazuju interes za sudjelovanje u različitim **igramama s pravilima**. Tu spadaju igre s unaprijed poznatim pravilima i ograničenjima u kojima postoji unaprijed određen cilj. Primjer takve igre su igre poput skrivača, graničara te društvene igre poput čovječe ne ljuti se. Djeca kroz ovakve igre uče da su u životu nužna i neka pravila, uče kako ih poštivati, kako se nositi s uspjehom, ali i neuspjehom. Igra s pravilima nije nužno natjecateljska, nego se može učiniti i kooperativnom, pri čemu se djecu uči timskom uspjehu i naporu, pomaganju i dijeljenju s drugima. Kooperativne igre, npr. što duže dodavanje lopte tako da lopta samo jednom udari o zemlju između igrača, uče djecu koja se boje neuspjeha koliko je važno ustrajati te ih uče da je uspjeh dio timskog procesa. Timski sportovi također su primjer kooperativne igre, ali unutar jednog tima.

U igre s pravilima spadaju i društvene igre koje se mogu napraviti ili kupiti u trgovini, a za njih je obično potrebna kartonska ploča ili/i neki drugi rekviziti. Društvene igre možemo podijeliti na *strategijske igre* (kao što su šah, domino,

memory. . .) i *igre na sreću* (igre u kojima se koristi kockica ili više njih, igre u kojima se nasumično izvlače karte ili brojevi, npr. tombola). Strategijske igre obično se igraju u paru, i u njima djeca između ostalog uče gubiti i nositi se s porazom. Neke su igre kombinacija strategijskih igara i igara na sreću, npr. monopoly.

S obzirom na vrstu aktivnosti tijekom igre, igre dijelimo na:

1. senzorne igre,
2. igre izražavanja,
3. motorne igre,
4. misaone igre.

Didaktičke igre

Bognar i Matijević (1993) didaktičku igru definiraju kao složenu pedagošku aktivnost koja omogućuje stjecanje znanja, razvoj sposobnosti i doživljavanje posljedica vlastitih postupaka.

Mnogi autori slažu se u tome da je didaktička igra drugačija od slobodne dječje igre jer je određena pravilima i opterećena nastavnim ciljevima, dakle, nije u potpunosti slobodna. Ipak, djeca u nastavi lako prihvaćaju didaktičku igru samo kao igru, ne razmišljajući previše o njenoj namjeni, naravno uz uvjet da je ta didaktička igra pravilno odabrana. Pravilno odabrana didaktička igra mora biti primjerena dobi učenika, ne smije biti prelagana niti preteška, ne smije predugo trajati i učenicima mora biti zanimljiva.

U raznim istraživanjima znanstvenici koji su proučavali odnos učinka primjene didaktičkih igara kao nastavne metode i prema učinku klasičnog poučavanja dolaze do zaključka:

1. primjenom didaktičkih igara postiže se podjednak učinak kao primjenom nastavnih listića u vježbanju i ponavljanju gradiva;
2. pažnja učenika bolje je usmjerena kod primjene didaktičkih igara;
3. učenici slabijih i prosječnih mogućnosti postižu bolje rezultate vježbajući uz didaktičke igre;
4. didaktičkim igrama postiže se bolje i trajnije zapamćivanje;
5. didaktičke igre omogućavaju djeci s posebnim potrebama i povučenoj djeci da otkriju i istaknu svoje sposobnosti.

Složenije matematičke igre pogodnije su za rad s učenicima natprosječnih matematičkih sposobnosti i trebalo bi ih izostaviti iz redovite nastave (jer bi mogle obeshrabriti prosječne i ispodprosječne učenike). S takvim zadacima možemo motivirati učenike na dodatnoj nastavi ili ih koristiti u individualnom, mentorskom radu s darovitim učenicima.

Primjena u nastavi matematike

U nastavi matematike od većeg značenja su konstruktivne igre i igre s pravilima. Mogu se provoditi i neke igre uloga, npr. igrajući se trgovine možemo

vježbati zbrajanje. Matematičke igre su misaone igre, djelomično neke od njih mogu biti i motoričke igre (rješavanje matematičkih zadataka može se kombinirati s utrkvanjem do ploče i sl.).

Međutim, obzirom na zadovoljstvo i užitek koje pruža pronalaženje rješenja, te motiviranost učenika takvom vrstom zadataka, u didaktičke igre bismo mogli ubrojiti i niz zagonetki čije rješavanje zahtijeva uključivanje matematičkog načina razmišljanja i koje pretpostavljaju određenu razinu poznavanja i prepoznavanja matematičkih pojmova i odnosa. Takve matematičke zagonetke učenici obično rješavaju individualno, a njima se radije oduševljavaju djeca natprosječnih sposobnosti. U novije vrijeme pojavljuju se i računalne igrice kojima je svrha utvrđivanje, vježbanje i ponavljanje nastavnih, između ostalih i matematičkih sadržaja (npr. *Učilica, Sunčica...*).

Matematičke igre učenici mogu igrati:

1. individualno,
2. u paru,
3. u grupi.

Igra u kojoj sudjeluje čitav razred može se organizirati kao natjecanje, pojedinačno ili grupno, a u nekim zahtjevnijim zadacima bolje je da učenik problem rješava sam, bez natjecanja i vremenskih ograničenja, kako bi njegova kreativnost i originalnost došla do punog izražaja.

Primjeri

Slijedi nekoliko karakterističnih didaktičkih igara koje se mogu koristiti u nastavi matematike u osnovnoj školi.

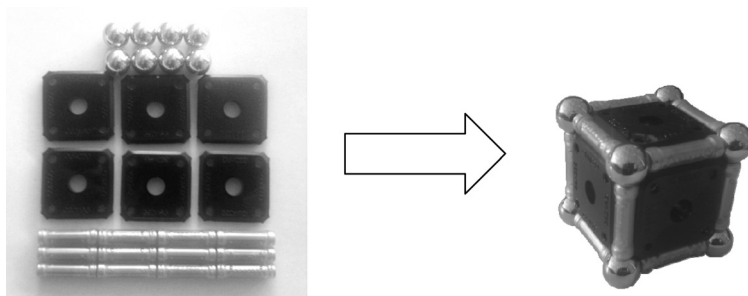
Primjer 1. Igra s magnetima (konstruktivna igra)

Za ovu igru potreban je set konstruktivnih elemenata¹ koji se sastoji od metalnih kuglica, plastificiranih štapičastih magneta i plastičnih pločica u obliku trokuta, kvadrata, romba i pravilnog peterokuta. Od navedenih dijelova mogu se složiti geometrijska tijela - poliedri. Pritom kuglice imaju ulogu vrhova (točaka), štapići predstavljaju bridove (crte), dok su pločice plohe poliedra. Iako su moguće različite konstrukcije, skup tih konstrukcija je ograničen količinom, dimenzijama i oblikom elemenata. Moguće je složiti svih pet pravilnih poliedara, različite piramide, prizme... Metalne kuglice su nužne jer povezuju magnete, a pločice konstrukciju podupiru i održavaju čvrstom. Konstruiranim poliedrima lako je prebrojati vrhove, bridove i plohe, jer su oni konkretni, opipljivi.

Ova igra potiče kreativnost i maštu, razvija spretnost i finu motoriku, usmjerava djetetovu pažnju i uči ga na njemu prihvatljiv i zanimljiv način geometrijskim oblicima i pojmovima kao što su ploha, brid i vrh.

¹ U primjeru je korišten set magneta za igru *Geomag*

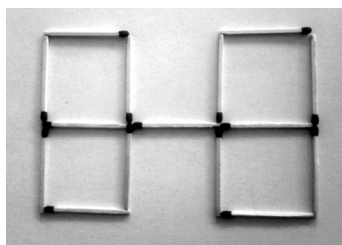
Zadatak: Od zadanih elemenata složite kocku.



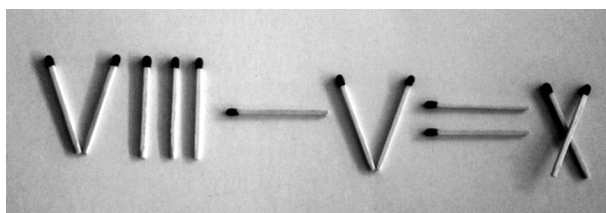
Primjer 2. Zadaci sa šibicama

Pri zadavanju ovih zadataka možemo se poslužiti šibicama ili nekim drugim štapićima jednake duljine, ili se šibice mogu jednostavno nacrtati. Razlikujemo dvije vrste ovakvih zadataka. To su geometrijski zadaci gdje su šibice raspoređene tako da tvore razne poligone i geometrijske figure, i aritmetički zadaci koji su obično zadani s rimskim brojevima, a nešto rjeđe s arapskim. U aritmetičkim zadacima šibice formiraju rimske (arapske) znamenke i matematičke znakove. Osim upoznavanja i usvajanja geometrijskih pojmova, vježbanja aritmetičkih operacija i zapisivanja rimskih brojeva, ovim zadacima razvija se logičko mišljenje i kombinatorika.

Zadatak A: Premjestite dvije šibice tako da dobijete pet jednakih kvadrata.



Zadatak B: Premjestite dvije šibice tako da račun bude točan.

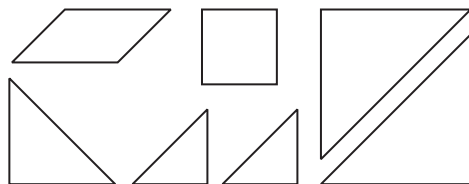


Primjer 3. Tangram

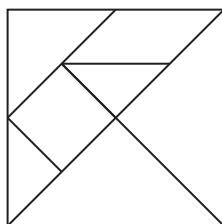
Kineska igra-slagalica sastoji se od 7 elemenata koji se mogu složiti u kvadrat, ali i u mnoge druge zadane likove. Ti elementi mogu se lako napraviti od kartona, a mogu se kupiti i drveni modeli u specijaliziranim trgovinama. Ova igra

pogodna je za rad u grupama natjecateljskog tipa (po principu *tko će prije*), gdje se osim usvajanja geometrijskih pojmova i vježbanja kombinatornog razmišljanja uči i suradničko ponašanje unutar grupe. Mogu biti zadani i drugi načini i oblici razrezivanja kvadrata, ili nekog drugog lika.

Zadatak: Zadane elemente složite tako da tvore kvadrat.



Rješenje:



Primjer 4. Igre s brojevima

S brojevima se možemo igrati na razne načine. Jedan od njih je formiranje različitih nizova brojeva i uočavanje pravilnosti (uzoraka). Takvi su npr. slijedeći nizovi jednakosti:

Tablica 1:

$$\begin{array}{rcl}
 1 \cdot 1 & = & 1 \\
 11 \cdot 11 & = & 121 \\
 111 \cdot 111 & = & 12321 \\
 1111 \cdot 1111 & = & 1234321 \\
 \vdots & & \vdots \\
 11111111 \cdot 11111111 & = & 12345678987654321
 \end{array}$$

Tablica 2:

$$\begin{array}{rcl}
 7 \cdot 7 & = & 49 \\
 67 \cdot 67 & = & 4489 \\
 667 \cdot 667 & = & 444889 \\
 6667 \cdot 6667 & = & 44448889 \\
 \vdots & & \vdots
 \end{array}$$

Uočimo da je druga tablica beskonačna, za razliku od prve. Uz ovakve primjere učenicima se mogu zadati slijedeći zadaci:

Zadatak A: Nastavite niz.

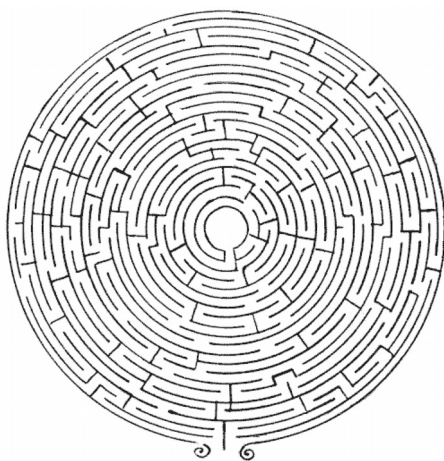
Zadatak B: Provjerite računanjem.

Zadatak C: Izračunajte prvih nekoliko umnožaka, a onda uočite pravilnost i nastavite niz bez računanja.

Primjer 5. Labirinti

Labirinti mogu biti jednostavni i vrlo složeni. Jednostavniji labirinti pogodni su u početnoj nastavi matematike, preciznije u prvom razredu, za uvježbavanje crtanja zakrivljenih linija. Pored toga, pogoduju uvježbavanju kombinatornog i logičkog razmišljanja, kao i koncentracije. Mogu biti osmišljeni zajedno s pričom koja dodatno motivira, npr. 'pomozite Crvenkapici da dođe do bakine kuće, ali tako da ne susretne vuka'. Složeniji labirinti su namijenjeni starijoj djeci, ali i odraslima, mogu iziskivati dosta vremena i truda, pa su izazov za naprednije učenike.

Zadatak: Nađite najkraći put od ulaza do središta labirinta.²



Primjer 6. Magični kvadrati

Magični kvadrat je prastara matematička zagonetka kojoj se u povijesti davalo i mistično značenje. U tablicu dimenzija 3×3 treba upisati brojeve od 1 do 9 tako da zbroj brojeva u svakom retku i stupcu, te po dijagonalama, bude konstantan. Jedno od osam rješenja prikazuje slika:

6	7	2
1	5	9
8	3	4

Preostala rješenja dobiju se rotacijom kvadrata za 90° i zrcaljenjem obzirom na dijagonale.

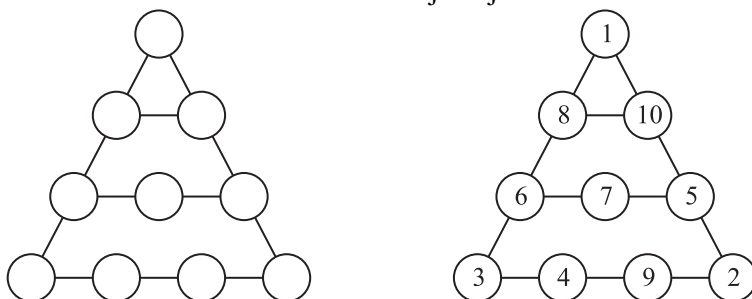
Postoje, dakako, magični kvadrati većih dimenzija, kao i drugi 'magični' geometrijski likovi. Ova mozgalica može se upotrijebiti za uvježbavanje zbrajanja,

² Labirint je preuzet iz čuvene zbirke zagonetki *Amusements in Mathematics* (H. E. Dudeney).

ali zahtijeva i visok stupanj logičkog i kombinatornog matematičkog mišljenja. Stoga je pogodnija za dodatnu nastavu matematike. Igre sličnog tipa su japanske mozgalice *sudoku*, *kakuro* i sl.

Zadatak: U prazne kružice upišite brojeve od 1 do 10 tako da na svakoj liniji zbroj brojeva bude jednak.

Rješenje:



Primjer 7. Zagonetke

Zagonetke se mogu rješavati individualno ili u grupi, gdje se može koristiti tehnika *brainstorming*-a ('oluja ideja'). Postoje dvije vrste zagonetki prema načinu rješavanja – to su *logičke* i *lateralne* zagonetke. Kod logičkih zagonetki rješenje proizlazi iz niza logičkih zaključaka (vertikalno), a kod lateralnih je rješenje potrebno potražiti zaobilaznim putem (do njega dolazimo tzv. divergentnim razmišljanjem, lateralno). Neki psiholozi uspješno koriste obje vrste zagonetki u terapijama za djecu s hiperaktivnim poremećajem i ADHD-om.

Zadatak A (logička zagonetka): Anton promatra patke u seoskom ribnjaku. Jedna patka pliva ispred dviju pataka, a jedna patka pliva iza dviju pataka. 'Toliko pataka nikada nismo imali na seoskom ribnjaku', pomisli Anton. Koliko pataka vidi Anton?

Zadatak B (lateralna zagonetka): Čovjek uđe u gostionicu. Na šanku naruči čašu obične vode. Poslužitelj izvuče revolver i uperi ga u gosta. Ovaj mu zahvali i napusti gostionicu. Što bi to trebalo značiti?³

Primjer 8. Origami

Japanska vještina savijanja papira origami pogodna je kao vježba spretnosti, preciznosti, koncentracije, te kao metoda vizualizacije i usvajanja geometrijskih pojmova. Dijete uz origami uči slijediti niz uputa, a tehniku može iskoristiti i za istraživanje vlastite kreativnosti. Može se provoditi s učenicima različitih uzrasta, od jednostavnijih modela u početnoj nastavi matematike, do težih modela u višim razredima osnovne škole, uz koje učenici moraju slijediti složenije matematičke upute te ih povezivati s matematičkim svojstvima modela.

³ Ovo su zagonetke iz knjige *IQ trening*.

Primjer 9. Matematički rebusi

Matematički rebusi su zadaci sa zadanim pravilima: treba zamijeniti znak u tablici odgovarajućom znamenkom, tako da račun bude točan. Različitim znakovima uvijek odgovaraju različite znamenke. Znakovi predstavljaju znamenke broja, tako da prva znamenka broja nikad nije jednaka nuli. Ponekad je zadano i pravilo da rebus ima jedinstveno rješenje (u suprotnom, ako ima više rješenja, to treba naglasiti kako bi se odgonetaču olakšao posao). Znakovi mogu biti sličice ili slova. Rebusi zadani slovima u kojima se pojavljuju riječi koje imaju smisla ili čak šalju neku poruku, posebno su zanimljivi. Na engleskom jeziku takvi rebusi nazivaju se *alphametics*, dok se obični rebusi nazivaju *cryptarithms*, skraćeno od *crypt-arithmetics* (taj termin se prvi put pojavljuje 1931. godine u belgijskom enigmatskom časopisu *Sphinx*). U početnoj nastavi matematike mogu se zadati i jednostavniji rebusi u kojima su neke znamenke poznate, a neke su zamijenjene zvjezdicama. Takvi zadaci obično nisu preteški i imaju svrhu vježbanja izvođenja računskih operacija, ali kroz njih također indirektno uvodimo početno rješavanje jednostavnih algebarskih jednadžbi.

Zadatak A: Umjesto zvjezdica upišite odgovarajuće znamenke:

$$\begin{array}{r} * \ 0 \ * \ * \\ - \ 2 \ * \ 0 \ 5 \\ \hline 4 \ 1 \ 2 \ 3 \end{array}$$

Zadatak B: Simbole zamijenite znamenkama:

$$\begin{array}{r} \heartsuit \times \diamondsuit \smiley = \spadesuit \smiley \clubsuit \\ : \qquad \qquad \qquad + \qquad \qquad \qquad - \\ \smiley + \heartsuit \blacktriangle = \heartsuit \clubsuit \\ \hline \smiley + \frown \clubsuit = \clubsuit \text{☀} \end{array}$$

Zadatak C: Nađite znamenke reprezentirane slovima:

$$\begin{array}{r} A \ B \ C \\ A \ B \ C \\ + \ A \ B \ C \\ \hline B \ B \ B \end{array}$$

Zadatak D: Nađite znamenke reprezentirane slovima:⁴

$$\begin{array}{r} S \ E \ N \ D \\ + \ M \ O \ R \ E \\ \hline M \ O \ N \ E \ Y \end{array}$$

⁴ Zagonetka koju je 1924. godine objavio H. E. Dudeney.

Primjer 10. Društvene igre

U razredu se mogu igrati razne društvene igre kao što su domino, bingo, čovječe ne ljuti se i slično, modificirane tako da se tijekom igre trebaju rješavati matematički zadaci. Npr. u igri memory na nekim karticama mogu biti kratki zadaci, a na drugima rješenja tih zadataka, a cilj igre je upariti zadatke s rješenjima.

U početnoj nastavi matematike ovakvim igrama mogu se uvježbavati osnovne računske operacije.

Primjer 11. Matematička križaljka

U matematičkoj križaljci u samu tablicu upisuju se znamenke, a ne slova. Zadaci su zadani pod uobičajenim rubrikama vodoravno i okomito i prilagođeni su nastavnim sadržajima i uzrastu učenika.

Zadatak: Riješite matematičku križaljku⁵:

1		2		3		4
		5				
6	7			8	9	
			10			
11			12	13		
		14				15
16				17		

Vodoravno:

- 1** $49 + 98$
3 $313 + 313$
5 $197 + 4$
6 $586 + 399$
8 $290 + 220$
11 $79 + 11$
12 $209 + 51$
14 $240 + 560$
16 $832 + 136$
17 $821 + 101$

Okomito:

- 1** $102 + 67$
2 $525 + 145 + 55$
3 $418 + 197$
4 $470 + 130$
7 $853 + 7$
9 $45 + 55$
10 $626 + 94$
11 $333 + 666$
13 $596 + 13$
14 $33 + 22 + 33$
15 $0 + 12$

Primjer 12. Računalne igre

U suvremeno doba djeca su sve više zaokupljena tehnologijom, obožavaju računalne i razne druge elektroničke igrice. Dobro je ponekad tu njihovu strast iskoristiti i u obrazovne svrhe. Danas postoji mnogo obrazovnih računalnih igrica na CD-ROM-u ili na Internet-u. Od igara na hrvatskom jeziku najpoznatije su Učilica i Sunčica, koje, među ostalim, pokrivaju i matematičke sadržaje primjerene djeci odgovarajućeg uzrasta. Mnoge od ranije navedenih matematičkih igara mogu se naći u obliku računalne igre na internetskim stranicama. U primjeni suvremene tehnologije ipak ne treba ići u krajnost, već je najbolje kombinirati klasične metode igre s ovim novim, suvremenim oblicima igre.

⁵ Mrkonjić, Salamon, Vlatka & Matka 3

Zaključak

Dijete kroz igru stječe nove spoznaje o svijetu oko sebe i sebi samom. Što je dijete starije, to je njegova igra složenija i njegova znatiželja postaje sve veća. Dijete kroz igru uči.

Igra i istraživanje su pokretač spoznajnog razvoja djeteta. Igrajući se s predmetima i oblicima djeca uče o prostoru, svladavaju koncepte težine, visine, obujma. . .

Igra se u školi može primijeniti kao nastavna metoda. U nastavi matematike kroz igru se mogu ponavljati matematički sadržaji, uvježbavati i automatizirati matematičke radnje, vježbati logično mišljenje, razvijati afinitet prema predmetu.

Matematičkim igrama možemo učenike više zainteresirati i približiti matematici.

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Učenje matematike otkrivanjem uz pomoć programa dinamičke geometrije *GeoGebra* – akcijsko istraživanje

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Sažetak. Aktivno uključivanje u proces stjecanja znanja jedan je od preduvjeta uspješnog svladavanja matematičkih sadržaja. Teorija konstruktivizma zalaže se da učenik samostalnom aktivnošću na temelju vlastitog iskustva dolazi do vlastitih spoznaja, dok nastavnik oblikuje okolinu za učenje te potiče i usmjerava učenike u njihovom samostalnom učenju. Cilj takvog pristupa je da se učenici s vremenom osamostale i nauče kako sami mogu učiti, a Pólyin model rješavanja problema (Pólya, 2004) nudi odgovor kako to izvesti u praksi – u 4 koraka: razumijevanje problema, stvaranje plana, izvođenje plana i osvrt na učinjeno.

U nastavnoj praksi to je moguće postići metodom učenja otkrivanjem ili istraživački usmjerenom nastavom, a u matematici od velike pomoći mogu biti programi dinamičke geometrije koji zbog dinamičnosti prikaza i interaktivnosti matematičkih objekata pružaju dinamičko virtualno okruženje za istraživanje. Među takvim programima izdvajam *GeoGebra* zbog toga što je besplatna i otvorenog koda, prevedena na hrvatski jezik, intuitivna i lako se svladava rad u programu, objedinjuje elemente geometrije, algebre, analize i statistike, te je zajednica njenih korisnika na internetu brojna i izuzetno aktivna tako da svakim danom ima sve više gotovih primjera koji se mogu koristiti u nastavi.

Zbog raznih problema na koje je autorica nailazila u svojoj nastavnoj praksi pokušavajući implementirati gore opisane ideje, odlučila se za akcijsko istraživanje kojim nastoji odgovoriti na problem: kako pomoći učenicima da samostalno usvajaju nove matematičke pojmove, koncepte i ideje učenjem otkrivanjem uz pomoć računala i programa *GeoGebra*. U ovom radu je opisan tijek istraživanja te prikazan izvještaj o ostvarenom akcijskom istraživanju s učenicima 2. razreda opće gimnazije u Srednjoj školi Čazma u Čazmi.

Ključne riječi: akcijsko istraživanje, *GeoGebra*, Pólyin model rješavanja problema, konstruktivizam, učenje otkrivanjem, virtualno dinamično okruženje za učenje

Uvod

Jedan od ciljeva poučavanja matematike jest naučiti učenike misliti te ih osposobiti za rješavanje problema i donošenje odluka u budućem životu. Kurnik (2008, str. 53) naglašava da su “*osnovne smjernice za osuvremenjivanje nastave pobuđivanje i pokretanje mišljenja učenika i nastojanje da dobar dio novih znanja stječu vlastitim snagama i sposobnostima*”. Za ovakav pristup zalaže se teorija konstruktivizma koja se zasniva na činjenici da znanje nastaje aktivnošću učenika pa je stoga uloga nastavnika kao izvora informacija znatno smanjena u odnosu na ulogu nastavnika koji će voditi i usmjeravati učenike na putu stjecanja novih znanja. Znanje se ne može prenositi od nastavnika prema učeniku na način da se informacije koje odašilje nastavnik “preslikaju” u glave učenika. Učenik se mora aktivno uključiti u nastavni proces te samostalnim radom doći do vlastitih spoznaja. Način učenja je jedinstven za svakog učenika jer (upravo i samo) on vlastitom aktivnošću “konstruira” svoje znanje na temelju vlastitog iskustva. A uloga nastavnika je da odabere prikladne oblike rada, nastavne metode, izvore znanja i na taj način oblikuje okolinu za učenje te da potiče i usmjerava učenike u samostalnom otkrivanju novih pojmova, koncepata i zakonitosti u sadržajima koje uče.

Nastavna strategija učenja otkrivanjem i istraživački usmjerena nastava su modeli nastave koji odgovaraju na gore postavljene zahtjeve (Bognar, Matijević, 2002). **Učenje otkrivanjem** (*discovery learning*) je iskustveno učenje koje se odvija u stvarnosti ili se koristi simulacija. U nastavi matematike sve više se koristi simulacija u programima dinamičke geometrije koji zbog dinamičnosti prikaza i interaktivnosti matematičkih objekata pružaju dinamičko virtualno okruženje za istraživanje. Među takvim programima izdvajam *GeoGebru* zbog toga što je besplatna i otvorenog koda, prevedena na hrvatski jezik, intuitivna i lako se svladava rad u programu, objedinjuje elemente geometrije, algebre, analize i statistike, te je zajednica njenih korisnika na internetu brojna i izuzetno aktivna tako da svakim danom ima sve više gotovih primjera koji se mogu koristiti u nastavi.

U **istraživački usmjerenoj nastavi** (*inquiry based learning*) učenike se potiče da samostalnim istraživanjem dolaze do određenih spoznaja uz odgovarajuću pomoć nastavnika. Ovakav način rada omogućuje visok stupanj diferencijacije nastave jer će nastavnik svakom učeniku pomoći onoliko koliko mu je potrebno. Učenik će samostalno otkriti smisao i važnost informacija što će ga dovesti do zaključka ili razmišljanja o novostečenom znanju. Cilj takvog pristupa je da se učenici s vremenom osamostale i nauče kako sami mogu učiti, a Pólyin model rješavanja problema nudi odgovor kako to izvesti u praksi – u 4 koraka:

1. **Razumijevanje zadatka** – zadatak treba pročitati s razumijevanjem te pokušati odgovoriti na pitanja: *Što je nepoznato? Što je zadano? Kako glasi uvjet zadatka?*
2. **Stvaranje plana** – uočiti zakonitost (pravilo, formula, teorem) koja povezuje poznate i nepoznate varijable, složeniji zadatak raščlaniti na nekoliko lakših.
3. **Izvršavanje plana** – paziti da se točno izvede svaki korak.
4. **Osvrt** – provjera rezultata, odgovoriti na pitanja: *Može li se rezultat dobiti na drugačiji način? Može li se rezultat uočiti na prvi pogled? Može li se*

rezultat ili metoda rješavanja upotrijebiti za neki drugi zadatak? (Pólya, 2004).

O tome kako Pólyin model rješavanja problema ugraditi u *GeoGebra*no dinamično okruženje za učenje te istraživanje i eksperimentalan rad učenika više se može pročitati kod Bjelanović Dijanić (2010) te Karadag i McDougall (2009). U nastavku rada slijedi izvadak iz izvještaja akcijskog istraživanja koje je autorica provela sa svojim učenicima dok su učili prema prethodno opisanim idejama.

Kontekst istraživanja

Sudionici akcijskog istraživanja su učenici 2.c razreda opće gimnazije u Srednjoj školi Čazma (ima ih 20 u razredu), a u ulozi istraživača i nastavnika sam ja – njihova nastavnica matematike i informatike Željka Bjelanović Dijanić. U dva razreda opće gimnazije predajem matematiku, a svim razredima u školi informatiku ili računalstvo te mi je informatička učionica uvijek na raspolaganju. U učionici imamo jedno glavno računalo (server) koje opslužuje 16 tankih klijenata, LCD projektor i platno te je pristup internetu moguć sa svih 16 radnih mjesta. Ponekad organiziram nastavu matematike da učenici samostalno ili u paru rade na interaktivnim digitalnim nastavnim materijalima.

Tijek akcijskog istraživanja pratila su i tri kritička prijatelja:

1. dr. sc. Branko Bogar – docent na Filozofskom fakultetu u Osijeku, bavi se temama vezanima uz akcijska istraživanja¹, također mentor za metodološki aspekt istraživanja;
2. Šime Šuljić – profesor matematike na Gimnaziji i strukovnoj školi Jurja Dobrile u Pazinu, intenzivno se bavi primjenom računala u nastavi matematike, izrađuje interaktivne digitalne materijale, redovito prati razvoj kognitivnih alata u svijetu;²
3. Željko Kralj – ravnatelj Srednje škole Čazma i nastavnik matematike s 30-godišnjim iskustvom koji pokazuje veliki interes za promjene koje pokušavam uvesti u nastavu.

Problem i plan istraživanja

Odgojne vrijednosti od kojih u ovom istraživanju polazimo su **samostalnost** pri učenju, **aktivnost** svih učenika, **suradnja** učenika, **motivacija** za učenje matematike te **učenje uz pomoć računala**. Cjelovit izvještaj akcijskog istraživanja prelazi okvire rada pa će ovdje biti prikazan samo dio istraživanja vezan uz samostalnost i aktivnost u računalom potpomognutom učenju.

¹ Tema doktorske disertacije: *Mogućnost ostvarivanja uloge učitelja – akcijskog istraživača posredstvom elektroničkog učenja* (mentor prof. dr. sc. Ana Sekulić-Majurec, Filozofski fakultet u Zagrebu, 2008.)

² Šime Šuljić je jedan od prevoditelja *GeoGebre* na hrvatski jezik, autor nekoliko desetaka stručnih članaka te predavanja o korištenju računala u nastavi matematike.

Ciljevi		
1. Osposobljavanje učenika za samostalno učenje metodom otkrivanja uz pomoć računalnog programa dinamičke geometrije <i>GeoGebra</i> . 2. Izrada novih i prilagodba postojećih digitalnih materijala potrebama samostalnog učenja otkrivanjem uz pomoć računala.		
Kriteriji		
• Učenici pokazuju veću samostalnost pri usvajanju novih nastavnih sadržaja. • Učenici samostalno dolaze do zaključaka. • Gotovo svi učenici su aktivni na satu. • Učenici samoinicijativno kod kuće posjećuju web stranice s digitalnim materijalima. • Učenici su zadovoljniji na nastavi matematike jer češće rade na računalima.		
Aktivnosti		
Za 1. cilj predviđa se da učenici u informatičkoj učionici samostalno uče otkrivanjem dolje navedene nastavne teme (po dva nastavna sata za svaku temu) kroz tri ciklusa:		
1. ciklus	Nastavna tema	Translacija grafa kvadratne funkcije $f(x)=ax^2$
	Digitalni materijal	http://www.normala.hr/interaktivna_matematika/kvadratna.htm
	Prikupljanje podataka	• upitnik 2 (skala procjene, pitanja otvorenog tipa) • radovi učenika – radni list i kratka pisana provjera • fotografije nastave • istraživački dnevnik nastavnice • bilješke kritičkih prijatelja, e-mail poruke
2. ciklus	Nastavna tema	Nultočke polinoma drugog stupnja i njegov graf
	Digitalni materijal	izraditi novi digitalni materijal uz pomoć kritičkog prijatelja kolege Šime Šuljića
	Prikupljanje podataka	• video snimka i fotografije nastave • bilješke na temelju sustavnog promatranja (video snimka) • radovi učenika – pisana provjera znanja • grupni intervju • istraživački dnevnik nastavnice • bilješke kritičkih prijatelja, e-mail poruke
3. ciklus	Nastavna tema	Definicije trigonometrijskih funkcija šiljastog kuta
	Digitalni materijal	uz dozvolu autora A. Meiera prevesti i prilagoditi materijal na http://www.realmath.de/Neues/10zwo/trigo/sinuseinf.html
	Prikupljanje podataka	• video snimka i fotografije nastave • bilješke na temelju sustavnog promatranja (video snimka) • upitnik 3 (Likertova skala procjene) • istraživački dnevnik nastavnice • bilješke kritičkih prijatelja, e-mail poruke
Vrijeme: tijekom studenog i prosinca 2010.		
Za 2. cilj predviđa se dodatna aktivnost koju nastavnica provodi samostalno i kontinuirano: proučavanje literature vezano uz kreiranje interaktivnih digitalnih obrazovnih materijala koji odgovaraju specifičnostima nastave matematike (Bjelanović Dijanić, 2009, Hohenwarter, Preiner, 2007).		

Tablica 1. Plan istraživanja.

U tradicionalnoj nastavi u kojoj dominira aktivnost učitelja i frontalni oblik nastave gotovo sve navedene vrijednosti su narušene. Učenici se previše oslanjaju

na nastavnika te veći dio njih nije u stanju kod kuće riješiti zadatak ako sličan nije riješen u razredu. Na satu su aktivni samo zainteresirani učenici dok ostali čekaju da prepišu s ploče. A računalo i internet su medij kojim su učenici svakodnevno okruženi pa se kao alternativa nudi učenje uz pomoć računala (*CAL – Computer Assisted Learning*). Međutim, neće računalo samo po sebi poboljšati trenutnu situaciju. I kod prijašnjih generacija učenika provodila sam nastavu učenjem otkrivanjem uz pomoć računala pa smo nailazili na poteškoće, a rezultati se nisu bitno poboljšali. Zbog toga sam odlučila uvesti promjene u svojoj pedagoškoj praksi te provesti akcijsko istraživanje u 2.c razredu. Svih 20 učenika izjasnilo se da žele sudjelovati, a njihovi roditelji potpisali su izjavu da se s time slažu. Očekujem da ćemo ustanoviti zbog čega učenici imaju poteškoća u samostalnom učenju otkrivanjem uz pomoć računala i programa *GeoGebra* na temelju sustavnog praćenja i kritičkog propitivanja te da ćemo zajednički prevladati poteškoće pri usvajanju matematičkih sadržaja novim oblicima i metodama rada.

Stoga smo problem ovog akcijskog istraživanja iskazali pitanjem:

Kako pomoći učenicima da samostalno usvajaju nove matematičke pojmove, koncepte i ideje učenjem otkrivanjem uz pomoć računala i računalnog programa dinamičke geometrije *GeoGebra*?

Potvrdu za ispravnost odluke da prethodno definirani problem istražim koristeći akcijsko istraživanje našla sam u knjizi “Metode istraživanja u obrazovanju” (Cohen, Manion i Morrison, 2007, str. 226) gdje se navodi da se akcijska istraživanja mogu koristiti za istraživanje metoda poučavanja – zamjena tradicionalne metode metodom otkrivanja te pri razvijanju novih metoda učenja. Prema podjeli Zuber-Skerritt (vidi Cohen i dr, 2007, str. 232) ovo istraživanje je emancipacijsko jer se sudionici (nastavnica i učenici) osjećaju odgovornima za rješavanje iskazanog problema kojeg će nastojati riješiti kroz ciklički proces: plan – akcija – promatranje – refleksija. Kako bi u tome bili što uspješniji planira se provođenje aktivnosti kroz tri ciklusa što je vidljivo u tablici 1.

Proces ostvarivanja akcijskog istraživanja

1. ciklus: Translacija grafa kvadratne funkcije

Nastavni materijal koji smo koristili nalazi se na *Interaktivnoj matematici* udruge Normala³ i već su ga koristile prethodne dvije generacije učenika. Autor materijala je Šime Šuljić. Učenici su sami formirali parove prema tome s kime vole učiti. Primjećujem da svaki par čine učenici podjednakih sposobnosti i predznanja te da napreduju tempom koji odgovara njihovim sposobnostima. Rijetko zovu za pomoć, upute su im jasne. Međutim, na 3. vježbi većina učenika je imala poteškoća: “*Pročitali uputu, ali zovu jer ne znaju što treba raditi (bolji učenici). Neki su shvatili kako odabrati točku tjemena, ali ne razumiju što s drugim točkama.*” (istraživački dnevnik, 15.11.2010.) Na satu je bio prisutan ravnatelj i kritički prijatelj Željko Kralj koji je pažljivo pratio što se događa. Nakon sata rekao je da se njemu ovakav način rada čini jako zahtijevan za nastavnika jer treba cijelo

³ <http://www.normala.hr/interaktivna-matematika/kvadratna.htm>

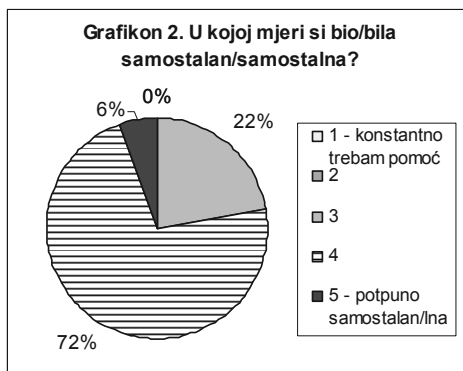
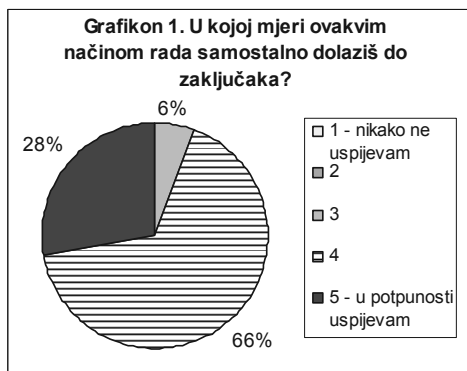
vrijeme hodati oko učenika koji ne napreduju istom brzinom pa u istom trenutku nastavnik treba vladati svim lekcijama kroz koje oni prolaze. Također je primijetio da nekim učenicima jako dobro ide dok uočava da neki drugi uopće ne razumiju što trebaju raditi (istraživački dnevnik, 15.11.2010.).



Slika 1. i 2. Računalo u nastavi – frontalno ili samostalno (u paru).

Slika 1. prikazuje kako smo računalo i projektor koristili u frontalnoj nastavi, dok je slika 2. snimljena za vrijeme dok su učenici učili u paru koristeći digitalne materijale. Branko Bognar dao je u e-mail poruci od 14.12.2010. svoj osvrt na ove dvije fotografije: *“Na prvoj fotografiji je moguće vidjeti frontalnu nastavu u kojoj je dominantan učitelj, dok su na drugoj fotografiji aktivni učenici. Na licima učenika (druga fotografija) moguće je uočiti interes i koncentriranost na aktivnosti kojima se bave.”*

Upitnikom 2. smo utvrdili u kojoj mjeri su učenici bili samostalni u učenju te u dolaženju do zaključaka i jesu li posjećivali te web stranice kod kuće:

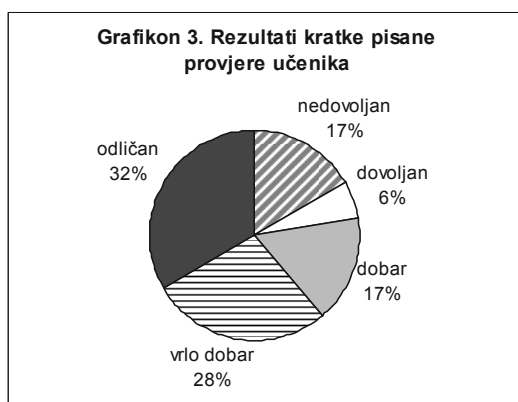


Učenici znaju da im je ovaj digitalni materijal putem interneta dostupan i kod kuće, ali na satu im nisam govorila kako bi bilo dobro da još koji put svrate na te stranice. Ipak, 11 od 18 učenika posjetilo je web stranice od kuće.

Što se radnog lista tiče, zadatke koji su zahtijevali kraće odgovore točno su riješili svi učenici, a zadatke u kojima je trebalo riječima opisati utjecaj parametara

na graf riješila je oko polovica učenika. Većina učenika imala je problema sa zadatkom 19. (odrediti jednadžbu kvadratne funkcije sa skice) te sa zadatkom 20. (crtanje grafa kvadratne funkcije) pa smo ova rješenja analizirali i riješili jedan primjer na ploči. U razgovoru s učenicima nekolicina je uočila da je problem u tome što su učili uz pomoć računala, a ispituje ih se na papiru (istraživački dnevnik, 23.11.2010.)

Analiza kratke pisane provjere koja je sadržavala dva zadatka crtanja grafa (kao u zad. 20.) te tri zadatka očitavanja jednadžbe funkcije sa slike (kao u zad. 19.) pokazala je jako dobre rezultate i napredak kod većine učenika:



Nakon refleksije prvog ciklusa, rezultate sam dostavila kritičkim prijateljima. Željko Kralj bio je u kontaktu sa mnom i razredom vrlo često, pregledao je učeničke radove i nije imao primjedbi osim da bi neki učenici trebali biti uredniji kod crtanja grafova. Branko Bognar je predložio da pokušam sakupiti više kvalitativnih podataka.

2. ciklus: Nultočke polinoma drugog stupnja i njegov graf

Digitalni nastavni materijal za ovu temu pokušala sam napraviti sama (dva dinamična apleta i radni list). Učenici su na prvom satu u informatičkoj učionici (23.11.2010.) imali poteškoća sa shvaćanjem uputa pa je Šime Šuljić pomogao da se materijal doradi i prepravi tako da smo na idućem satu (25.11.2010.) krenuli po novom materijalu⁴ te sam taj sat snimala kamerom. Nakon pregledavanja video snimke u istraživački dnevnik 30.11. sam zapisala: *“Profesorica šeće razredom, nastoji biti nenametljiva, gleda iza leđa učenika što oni rade te se upliće ako primijeti da nešto nisu dobro shvatili. Učenici rade, rijetko ju zovu, eventualno ako je u blizini, češće se obrate susjedu za pomoć.”* Na tom satu zabilježeno je nekoliko fotografija:

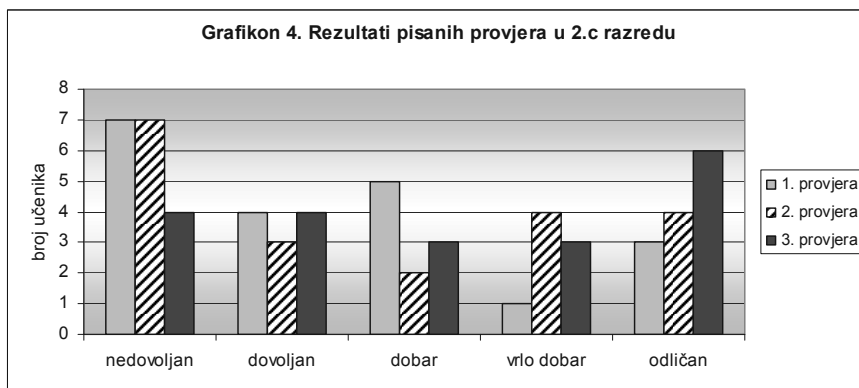
⁴ <http://free-bj.t-com.hr/zbjelanovic/apleti/nultočke.html>



Slike 3. i 4. Rad učenika na digitalnom materijalu i radnom listu.

Šime Šuljić dao je svoje viđenje fotografija: *“Prva fotografija prikazuje radni materijal – aplet i rad učenika u paru. Očito je da raspravljaju svoje ideje. Učenica u redu nasuprot udubljena u problem. Na drugoj slici učenici ispunjavaju radni list. Pritom se oslanjaju na interaktivni sadržaj na računalu. Izgleda da povremeno provjeravaju rezultat na kalkulatoru. Fotografija mi govori da je radna temperatura visoka.”*

Nakon ovog sata preostale nastavne teme obrađivali smo u razredu, a dio i s *GeoGebrom* frontalno na LCD projektoru te se trebalo pripremiti za pisanu provjeru znanja iz ove cjeline. Rezultate 3. pisane provjere možemo usporediti s rezultatima prethodnih dviju:



U obje prethodne pisane provjere prolaznost je bila 65%, dok je u 3. pisanoj provjeri 80%. Osim opadanja broja negativnih ocjena primjećuje se rast broja odličnih ocjena. Željko Kralj je pregledao učeničke radove te razgovarao s učenicima. Vrlo je zadovoljan postignutim rezultatima (ističe prolaznost od 80%), ali je zamjerku uputio onim učenicima koji pišu i crtaju nezašiljenom olovkom te koordinatni sustav crtaju bez ravnala.

Kako bi upotpunili refleksiju 2. ciklusa i dobili viđenja učenika o ostvarivanju kritičko-refleksivne prakse, organizirali smo grupni polustrukturirani intervju.

	Odgovori učenika u grupnom polustrukturiranom intervjuu
Prednosti	• zanimljivije • lakše se usvaja gradivo jer ima puno primjera • samostalno dolaženje do zaključka je poticajno • ostaje dulje u sjećanju • možemo si vizualizirati to što radimo, vizualno više zapamtimo • možemo ići svojim tempom • možemo više puta proći istu stvar ako nam nije jasno da si razjasnimo • materijal na webu, možemo pogledati kod kuće
Nedostatci	• nije nam objašnjeno kako vi objasnite • problem s razumijevanjem uputa • kompjuter nacrtava automatski, a mi to trebamo postepeno • trebalo bi najvažnije zapisati u bilježnicu
Prijedlozi za poboljšanje	• bolje je u paru nego kad smo sami, možemo diskutirati s partnerom • da imamo radni list na koji pišemo • ovo je bolje za vježbu nego za učenje, da na kompjuteru vježbaš tek kad shvatiš • kad bi klizači bili precizniji, da ne preskaču, da možemo provjeriti zadaću, • povratna informacija, ako smo dali krivi odgovor da kompjuter izbaci točan

Tablica 2. Grupni intervju (izdvojeno iz transkripta i kategorizirano).

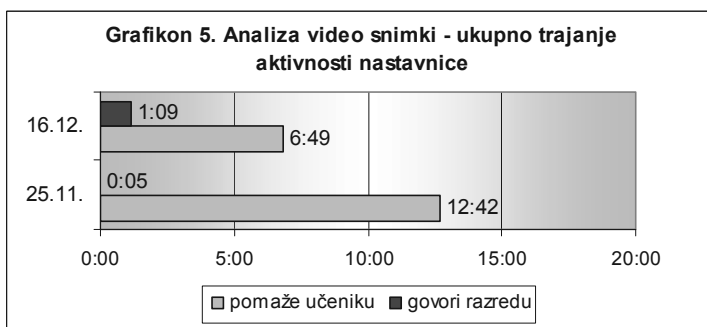
3. ciklus: Definicije trigonometrijskih funkcija šiljastog kuta

Za ovu temu Šime Šuljić i ja zajednički smo prilagodili materijal na njemačkom Andreasa Meiera⁵. Pisala sam mu da nam dozvoli korištenje i prilagodbu materijala na hrvatski, a pošto nije odgovorio odlučili smo postupiti prema Creative Common licenci (dijeli pod istim uvjetima). Prilagodili smo materijal⁶ te naveli tko je izvorni autor. Meier je pripremio dosta zanimljivih interaktivnih vježbi, a pošto bi njihova prilagodba zahtijevala puno vremena, odlučili smo da učenici vježbaju na originalnim njemačkim vježbama.

Taj sat održan je 16.12.2010. u informatičkoj učionici. Učenici su mogli birati žele li raditi u paru ili samostalno. Opet smo snimali video kamerom te fotoaparatom. U istraživačkom dnevniku 16.12. je zabilježeno: “*Vidi se poboljšanje u snalaženju učenika, vježbe su motivirajuće, a sakupljanje bodova ih posebno veseli. Učenici s lakoćom rješavaju radni list. U nekim Meierovim apletima je bilo bugova – upozorila učenike kako da to izbjegnu. U 45 minuta većina učenika odradila posao, dalje nastavljamo po udžbeniku.*” Nakon pregledavanja video snimke u istraživački dnevnik 18.12. sam zapisala: “*Profesorica ovog puta više govori cijelom razredu pa ju zbog toga manje zovu na pojedinačne intervencije. Atmosfera je ležernija, jedan drugome puno više pomažu, vesele se sakupljanju bodova.*” Pažljivo sam pregledala i analizirala prvih 20 minuta na obje snimke (do tada se učenici još nisu počeli javljati da su gotovi) te izdvojila dvije kategorije: nastavnica pomaže učenicima (individualno ili paru) te nastavnica glasno govori razredu. Bilježila sam koliko puta se pojavila koja kategorija te koliko je trajala.

⁵ <http://www.realmath.de/Neues/10zwo/trigo/sinuseinf.html>

⁶ <http://free-bj.t-com.hr/zbjelanovic/apleti/sinus.html>



Na prvoj snimci (25.11.) može se prebrojati da je nastavnica pomagala učenicima (pritom se misli na tihi razgovor između nastavnice i dva učenika u paru) ukupno 28 puta u ukupnom trajanju od 12:42 minute (63, 5%) što bi značilo da je u prosjeku svaki par samo 6, 35% vremena imao pomoć nastavnice, dok ostalo otpada na samostalnu aktivnost. Na drugoj snimci (16.12.) nastavnica je 9 puta u trajanju od 1:09 minuta cijelom razredu glasno davala upute, a individualno je pomagala 17 puta u trajanju od 6:49 minuta (34, 08%) što je u prosjeku 3, 41% po učeniku (jer smo imali 10 parova).

Na kraju sam sva pitanja i dileme na koje smo nailazili tijekom rada objedinila te sastavila Likertovu skalu procjene koja će nam poslužiti kao kritička refleksija za analizu podataka i provjeru u kojoj mjeri smo ispunili naše polazne odgojne vrijednosti.

	Tvrdnja	Uopće se ne slažem	Ne slažem se	Niti se slažem, niti se ne slažem	Slažem se	U potpunosti se slažem
1.	Samostalno učenje uz pomoć računala omogućava mi lakše usvajanje gradiva			20%	65%	15%
2.	Samostalno učenje uz pomoć računala mi je zanimljivije nego nastava u našem razredu			25%	45%	30%
3.	Računalo mi omogućava da lakše samostalno dođem do zaključaka			10%	70%	20%
4.	Računalo mi pomaže u vizualizaciji matematičkih pojmova			15%	40%	45%
6.	Računalo me tjera da razmišljam više nego kad je nastava u našoj učionici			10%	70%	20%
10.	Napredovanje vlastitim tempom je prednost samostalnog učenja uz pomoć računala			25%	50%	25%
11.	Zbog toga što računalo lako i automatski crta grafove, kasnije imam poteškoća kad moram sam/a nacrtati graf olovkom na papiru	5%	20%	40%	30%	5%
12.	Digitalni materijali dostupni putem weba omogućuju mi da si kod kuće ponovim i razjasnim što na satu nisam dobro shvatio/la			10%	30%	60%

15.	Povratna informacija o točnosti naših odgovora olakšava mi praćenje sadržaja			10%	75%	15%
17.	Radni list na koji olovkom upisujem odgovore pomaže mi da bolje pratim sadržaje na digitalnom materijalu			15%	60%	25%

Tablica 3. Likertova skala procjene (izvadak iz upitnika 3).

Interpretacija akcijskog istraživanja

Kao polazište za ovo istraživanje uzeli smo teoriju konstruktivizma koja naglašava da znanje nastaje aktivnošću učenika te praksu koja pokazuje da to nije lako izvedivo u nastavi organiziranoj na tradicionalni način. U praksi smo uočili da jedan dio učenika ima ozbiljnih problema pri usvajanju matematičkih sadržaja pa se pitamo kako postići aktivnost svih učenika na satu te kako poticati samostalnost u učenju. To su bile polazne vrijednosti od kojih smo krenuli. I kao što Kurnik (2008) predlaže, htjeli smo postići da učenici vlastitim snagama dolaze do vlastitih spoznaja i na taj način stječu nova znanja te smo se odlučili za učenje uz pomoć računala. Međutim, ne misli se na korištenje računala u frontalnoj nastavi uz LCD projektor ili pametnu ploču (Slika 1.) nego da svatko radi za svojim računalom (Slika 2.). Ako tomu dodamo Pólyin heuristički pristup učenju (Pólya, 2004) tada je samostalno učenje otkrivanjem jedno od mogućih rješenja početnog problema.

S obzirom da je u istraživanju prikupljeno podosta kvalitativnog i kvantitativnog materijala, njihovo usklađivanje će poslužiti kao postupak triangulacije čime ćemo nastojati izbjeći jednostranost kvalitativne analize podataka te doći do vrijednih zaključaka.

Iz slika 2., 3. i 4. vidljivo je da smo postigli aktivnost svih učenika na satu. To potvrđuju i rezultati pisanih provjera koji su znatno bolje nego inače. U kratkoj provjeri (grafikon 3) trećina učenika ocijenjena je odličnim, dok je u trećoj pisanoj provjeri (grafikon 4) 80% učenika ocijenjeno pozitivnom ocijenom. Učenici kao prednost ističu napredovanje vlastitim tempom (75% učenika, tablica 3, tvrdnja 10) te činjenicu da više puta mogu ponoviti isti postupak dok si ne razjasne problem. 90% učenika (tablica 3, tvrdnja 6) smatra da ih računalo tjera na razmišljanje više nego tradicionalno organizirana nastava. Za pretpostaviti je da sve to utječe na veću aktivnost učenika na satu. Nadalje, primjećuje se da učenici i kod kuće samoinicijativno posjećuju web stranice s nastavnim materijalima (upitnik 2, 61% učenika) te da im dostupnost putem weba omogućuje da si kod kuće razjasne što nisu shvatili u školi (90% učenika, tablica 3, tvrdnja 12).

Analiza video snimki (grafikon 5) ukazuje na povećanje samostalnosti pri čemu razlikujemo samostalnost u cjelokupnom procesu učenja i samostalnost u dolaženju do zaključaka. U prvom ciklusu (grafikoni 1. i 2.) može se primijetiti da su učenici samostalniji u dolaženju do zaključaka (28%) nego u učenju uz

pomoć računala ukupno gledajući (6%). To objašnjavamo time što su učenici isprva imali poteškoća s praćenjem uputa te su češće pozivali nastavnika u pomoć ili se konzultirali s drugim učenicima da bi nakon shvaćanja upute lako došli do zaključka uz pomoć računala. Samostalno dolaženje do zaključaka učenici smatraju poticajnim (tablica 2), a to je ono čemu težimo.

Kvalitativni podaci (fotografije, video snimke, izjave kritičkih prijatelja, grupni intervju) ukazuju na vrijednost računalom potpomognutog učenja. 75% učenika smatra da im je ovakav način učenja zanimljiviji od tradicionalne nastave (tablica 3, tvrdnja 2). Omogućava im lakše usvajanje gradiva (80% učenika, tablica 3, tvrdnja 1) jer računalo u kratkom vremenu može generirati više različitih primjera, a pomaže i u vizualizaciji matematičkih objekata (85% učenika, tablica 3, tvrdnja 4). Računalo daje brzu povratnu informaciju što učenicima olakšava praćenje sadržaja digitalnih materijala (90% učenika, tablica 3, tvrdnja 15), a vježbe sa sakupljanjem bodova ih posebno motiviraju.

Međutim, uočeni su i nedostaci ovakvog načina rada. Učenici uče uz pomoć računala, a provjerava ih se pisanim provjerama znanja na papiru. Jedan od načina kako ublažiti ovu kontradikciju jest korištenje radnog lista kojeg će učenici ispunjavati paralelno s proučavanjem sadržaja digitalnog materijala. 85% učenika (tablica 3, tvrdnja 17) se izjasnilo da im radni list pomaže da lakše prate sadržaj. Još bolje rješenje bilo bi osmisliti provjeru znanja također na računalu. Nadalje, primjećuje se da neki učenici imaju poteškoća s crtanjem grafova na papiru jer računalo automatski nacrtat graf (tablica 3, tvrdnja 11, komentari Željka Kralja). Uočen je i problem razumijevanja uputa, a uglavnom se pojavljuje kod onih učenika koji i inače imaju poteškoća s razumijevanjem zadatka. Rješenje ovog problema može se potražiti u heurističkom pristupu učenju i umijeću otkrivanja (Pólya, 2004) te bi se trebalo rješavati kontinuirano bez obzira na način organizacije nastave.

Promjene koje sam ovim akcijskim istraživanjem uspjela uvesti u svom nastavnom procesu rezultat su raznih činitelja: spremnosti i mene i učenika za djelovanje na temelju postavljenih vrijednosti, stalna suradnja i uvažavanje svačijeg mišljenja, odgovornost učenika za vlastiti napredak, pomoć i savjeti kritičkih prijatelja te samokritičko uvažavanje njihovih primjedbi. Za razliku od prethodnih generacija učenika, ovi učenici su puno ozbiljnije pristupili učenju uz pomoć računala čemu je uvelike doprinio proces akcijskog istraživanja, a posebice refleksija nakon svakog ciklusa.

Kritički prijatelj Željko Kralj nakon analize prikupljenih podataka izveo je opći zaključak: *“Učenjem uz pomoć računala učenik iz pasivnog promatrača prelazi u aktivnog sudionika, a time i njegova motivacija za učenje postaje veća.”* Međutim, to ne znači da je naše akcijsko istraživanje završeno, sada se otvaraju nove perspektive, uočavaju se problemi i eventualna rješenja, mogućnosti za poboljšanje, a pitanja kojima ćemo se ubuduće baviti su:

- ugradnja Pólyinog heurističkog učenja u dinamičko okruženje za učenje,
- provjera znanja korištenjem računala i interaktivnih digitalnih materijala,
- izrada novih i poboljšanje postojećih digitalnih materijala (ovo je 2. cilj koji je ostvaren u okviru akcijskog istraživanja, a za daljnje provođenje ovakvog

načina poučavanja potrebno je osigurati nove kvalitetne digitalne nastavne materijale).

Zaključak

Učenje matematike otkrivanjem uz pomoć računala i programa dinamičke geometrije Geogebra te uz metodički osmišljene i didaktički oblikovane interaktivne digitalne obrazovne materijale osigurava aktivnost svih učenika u razredu, povećava motivaciju za učenje matematike, potiče samostalno dolaženje do zaključaka, ali i suradnju među učenicima. Pritom je važno da učenik sam ili u paru uči na svom računalu, da sam pokuša otkriti nove spoznaje. Tako stečeno znanje dulje će ostati u pamćenju, a učenik će naučiti misliti, svaki novi problem puno lakše riješiti te ovladati umijećem otkrivanja.

Međutim, je li ovo izvedivo u nastavnoj praksi, uvelike ovisi o školskim uvjetima. Pokazalo se kako su učenici spremni na suradnju pri uvođenju promjena u nastavi, ali ako nastavnik matematike nije u mogućnosti koristiti informatičku učionicu za potrebe svoje nastave, tada neće moći provesti ideje opisane u ovom radu. Ipak, nadam se da će čitanje rada potaknuti i druge nastavnike da pokušaju uvesti promjene u svojoj pedagoškoj praksi u skladu sa školskim uvjetima u kojima rade.

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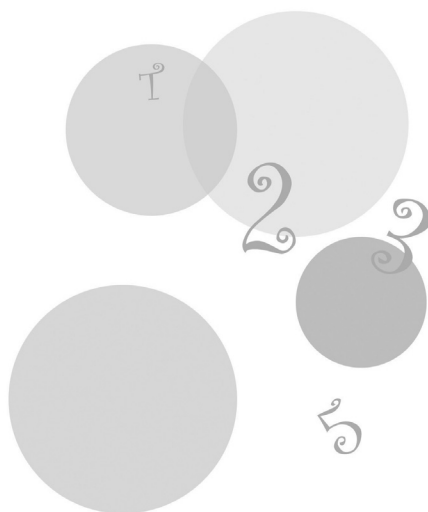
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5.

Što može promijeniti učitelj matematike



U petom poglavlju autori u svojim člancima daju pregled uzroka slabog uspjeha u nastavi matematike te sagledavaju posljedice takvoga stanja. Komentiraju neka opća pitanja o kojima ovisi prihvaćanje matematike kao neophodnog znanstvenoga područja od strane šire društvene zajednice. Istraživanja pokazuju da stav djeteta prema matematici značajno ovisi o stavovima njegovih roditelja, ali i stavu njegova učitelja. Također se analiziraju pitanja vezana za udžbenike, a komentira se i nastavnička perspektiva u Hrvatskoj.

Slab uspjeh u nastavi matematike: uzroci i posljedice

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Sažetak. Slab uspjeh kao i nizak nivo znanja iz matematike nije pusta pretpostavka, već je činjenica koja je potkrijepljena konstatacijama većine izvještaja, kako iz osnovnih tako i iz srednjih škola. Većina anketiranih učenika matematiku karakterizira kao tešku, nerazumljivu, nezanimljivu, prezahtjevnju.

Nesumnjivo “krivaca” za loš uspjeh na polju matematike na svim nivoima ima mnogo, te praviti klasifikaciju i hijerarhisku ljestvicu bilo bi nezahvalno i neodgovorno. Naime, nema određenog i jasno definiranog stava da bi se “krivac” mogao identificirati te kao takvog ili eliminirati ili smanjiti na najmanju moguću mjeru. Ono, što bi se u većini stručnih izvještaja moglo izdvojiti, kao najčešće spominjani razlozi su neadekvatni i prebukirani nastavni planovi i programi, prekobrojna odjeljenja, loša stručna osposobljenost nastavnika, neopremljenost škola, loši udžbenici, zastarjeli i prevaziđeni metodičko – didaktički oblici rada, loši porodični uslovi uvjetovani niskim standardom življenja i tome slično.

Od navedenog, kao i od drugih navedenih uzroka, nezahvalno bi bilo izdvojiti neke konkretne faktore, jer neuspjeh u nastavi matematike je funkcija svih tih faktora.

Većina nabrojanog uglavnom uporište traži u vanškolskim uzročnicima, zanemarujući činjenicu da glavne potencijalne krivce treba prvenstveno tražiti u školi, nastavi, načinu izvođenja nastave, odnosno potrebno je prvo temeljno odgovoriti na pitanja ko?, na koji način? i šta? predaje. Nesumnjivo da bi jedan kvalitetan odgovor na postavljena pitanja barem djelomično rasvijetlio pojam neuspjeha u nastavi matematike.

Na navedenu konstataciju ukazuju mnoge dosada objavljene analize, bazirane na nizu obavljenih testiranja, anketiranja, na pedagoškoj dokumentaciji, kako na našim prostorima tako i u svijetu. Također smo svjedoci dispariteta u ocjenama i načinu ocjenjivanja kod različitih nastavnika u okvirima jednog nastavnog predmeta, a da ne ukazujemo na različite predmete. Činjenica da svršeni osnovnoškolci u srednju školu dolaze najvećim dijelom sa visokim ocjenama iz matematike, baš kao što svršeni srednjoškolci odlaze na fakultete također sa vrlo visokim ocjenama.

U toku školovanja u srednjoj školi, što je pogotovo evidentno na prvoj polugodišnjoj klasifikaciji, rezultati su mnogo slabiji od rezultata donesenih iz osnovne škole, da bi se na kraju školske godine situacija malo poboljšala, ali opet s uočljivom razlikom između zaključnih ocjena osmog razreda osnovne i prvog razreda srednje škole. Disparitet u ocjenama iz nastavnog predmeta matematike ne pojavljuje se samo kao proizvod različitih kriterija pri ocjenjivanju učenika, već na njega imaju utjecaj i drugi faktori koje smo imali namjeru ako ne u potpunosti onda barem dijelom razotkriti. Rezultati istraživanja bi trebali omogućiti praktičnu akciju na području nastave matematike čime bi se barem dio uzroka lošeg uspjeha u nastavi matematike; ako ne u potpunosti onda barem dijelom, otklonio.

Ključne riječi: Škola, matematika, uspjeh, nastava, planovi, metode

Da li je stepen razvijenosti društva neposredna posljedica vaspitanja, a time i obrazovanja, ili je pak stepen vaspitanja i obrazovanja neposredno uslovljen razvojem društva, pitanje je vrlo diskutabilno i ostaje uvijek otvoreno. U okvirima toga i škola, kao važan faktor u sistemu, direktno ili indirektno utiče na postojanje, razvoj i promjene društva u najširem smislu.

Globalno nezadovoljstvo kao posljedica manjkavosti većine društvenih sistema, a što je opet posljedica, između ostalog, nivoa i načina lošeg vaspitno-obrazovnog djelovanja, nije pošteđelo ni naše prostore, te, objektivno gledajući, u okruženju gdje kvantitet masovno potiskuje kvalitet na svim nivoima, mlađim pokoljenjima ne garantuje nimalo svijetlu budućnost.

Vaspitno-obrazovne ustanove postale su kvantitativno i kvalitativno, po načinu rukovođenja i uopšte po samom funkcionisanju, ustanove sa statusom vrlo visoke produktivnosti nekvalitetnog "proizvoda", koji opet direktno ili indirektno djeluje na postojeće društvene odnose. Korekcije ili popravke, ili pak uništavanje kompletne serije materijalnih proizvoda sa greškom u privredi je relativno lako nadoknaditi, no generacija loše odgojenih ili obrazovanih učenika predstavlja nenadoknadiv i vrlo opasan promašaj, za čiju korekciju treba mnogo dodatnog napora, materijalnih sredstava i vremena.

Vaspitno-obrazovni proces je kontinuiran proces koji se obavlja u svako vrijeme i na svim mjestima u raznim oblicima i vidovima, a samo prisustvo komunikacijskih kao i interakcijskih veza nužan je i neizostavan oblik i način sa kojim učesnici prenošenja informacija obavljaju i uspostavljaju kontakte, razmjenjuju informacije, jačaju veze, te se i na taj način intelektualno i kulturno uzdižu.

Nastava kao kontinuirani proces neodrživa je bez interakcijskih veza između učenika i učenika, učenika i nastavnika, učenika i nastavnih sadržaja, pa se samo po sebi nameće kao zaključak da takve veze treba njegovati, obogaćivati i sadržajno unapređivati. Naime, nedostatak interakcijskih odnosa između učenika i učenika, učenika i nastavnika može rezultirati negativnim ishodima kako za vrijeme tako i poslije redovnog školovanja. Interakcijska saradnja utiče na razvoj misaonih aktivnosti, jačanju samopouzdanja, sigurnosti kao i pronalaženju najboljih odgovora i rješenja. Podizanje nivoa matematičke kulture javlja se kao neizbježna pojava

uzrokovana razvojem matematike, koja opet, sa druge strane, svoj razvoj duguje sve većim i većim zahtjevima društva kao i trendovima drugih nauka. Nastali raskol između matematičke kulture koja se njeguje kroz nastavu matematike i naučne matematike, matematike koja će imati konkretnu primjenu ne samo u materijalnom već i u duhovnom dobru, možemo potražiti prvenstveno u metodama i sadržajima koji dominiraju školskom učenju matematike.

Egzaktnost matematike nije ugrožena sadržajem nastavnih planova i programa, već njihovom bespotrebnom preopširnošću što skupa sa reduciranjem broja časova doprinosi da se matematika posmatra kao teško savladiva nastavna disciplina. Nema sumnje da je matematika kao naučna disciplina napravila dubok i širok prodor, prvenstveno u prirodne nauke, a zatim i u brojne društvene, ali svu nauku ne možemo prenijeti u nastavne planove i programe za djecu.

Jezik, simboli, operativni principi, strogost kao i metode rada nisu više prepoznatljivi samo kod naučnika prirodnih nauka, već se funkcionalnim i strukturalnim razmatranjima koriste i istraživači na području društvenih nauka (ekonomisti, sociolozi, pravnici, psiholozi i drugi). U prilog navedenom najslikovitije govori činjenica da se razvoj neke nauke može posmatrati kroz stepen uspješne primjene matematike u njoj.

Masovniji pristup školovanju je trend koji se pojavio kao neposredna posljedica visokoindustrijalizovanog društva koje se svakim danom sve više i više razvija, sa zahtjevima za još školovanim stručnjacima na svim poljima društvenog razvoja. Sa druge strane, pojedinačna znanja, elementi teorije, problemski aspekti, komparativno sticana znanja, zanemarivanje socijalnih aspekata, ukazuju na, ako ne korjenite, onda barem djelimične promjene u vaspitno-obrazovnom procesu.

Vaspitanje i obrazovanje kao fundamentalni elementi svake društvene djelatnosti ni u kom slučaju ne može biti pošteđeno napretka nauke, tehnike i tehnologije, pogotovo uzimajući u obzir sveopšte nezadovoljstvo efikasnošću školskih sistema. Za realizaciju bržeg i ekonomičnijeg načina školovanja nije dovoljno znati samo šta je nužno učiti, već i kako učiti. Još davne 1613. godine predloženo je od strane Ratkea, da se u nastavni proces ugrade i vještine poučavanja. Pored niza ponuđenih rješenja, nije pronađen unikatan model učenja, čime se nametnuo zaključak da jedinstvenog modela nema, već je potrebno koristiti razne, međusobno isprepletane metode, kako bi i rezultati bili bolji.

Kada i koju metodu primijeniti u nastavnom procesu prvenstveno je stvar gradiva koje je nužno kvalitetno obraditi sa učenicima, a zatim i nastavnika u kome se ogleda veličina dobrog metodičara, didaktičara, pedagoga i učitelja. Sa druge strane, ono što u suštini predstavlja noćnu moru, prvenstveno pedagoga, jeste činjenica da će životni kontekst budućih generacija biti drugačiji od današnjeg, a o tome kakva će ta budućnost biti, može se samo nagađati, jer ona ovisi od mnogih faktora.

Nastavnik je u suštini nezamjenjiv akter nastavnog procesa, ali se nameće pitanje u kojoj mjeri je njegova prisutnost neophodna. Zašto sputavati učenike da samostalno uče i grade svoj svijet? Zašto predavačko-receptivni oblik edukacije ne zamijeniti jednostavnijim dijalogom?

Iz godine u godinu ponavljaju se poznati problemi vezani za različite nivoe predznanja koja djeca donose sa sobom prilikom upisa u prvi razred, što je naročito izraženo u oblasti matematike. Zašto je to tako? Ranija praksa nije u velikoj mjeri poznavala predškolsko obrazovanje, te se prihvatilo da: “položaj matematike u razrednoj nastavi, kao i u osnovnoj školi općenito, određen je njenom odgojno-obrazovnom funkcijom. Naime, usvajanjem matematičkih sadržaja učenici se ne osposobljavaju za neko zanimanje, već se prvenstveno odgajaju i obrazuju, pa se s pravom može reći da početna nastava matematike ima isključivo odgojnu i općeobrazovnu funkciju. No osim ove, matematika u razrednoj nastavi ima i značajnu propedeutičku funkciju, a to je pripremiti učenike za matematičko odgajanje u višim razredima osnovne škole (bez znanja matematičkih sadržaja razredne nastave ne mogu se usvajati sadržaji predmetne nastave matematike), a zajedno s tom nastavom i za matematičko obrazovanje u srednjoj školi” (Markovac, 1992, str. 17).

Da predškolska djeca mogu spontano razmišljati i računati, te da kognitivna sposobnost učenja jezika, sposobnost računanja i pisanja broja može biti urođena i univerzalna, nije nikakva novina. Matematika, kao i druge naučne discipline koje su oslonjene na matematički aparat, pružaju velike mogućnosti razvoja logičkog mišljenja i razmišljanja. Matematička egzaktnost zahtijeva prvenstveno misaonu aktivnost utemeljenu na analizi, apstrakciji, generalizaciji i na kraju zaključku. “Danas je potpuno jasno da smo ušli u učeću civilizaciju XXI vijeka i da će biti sretni oni ljudi koji s lakoćom uče, koji se ne boje nepoznatog i novog, koji su sposobni da rješavaju probleme i da prate najnovija saznanja nauke i tehnike. To nam govori da djecu moramo od ranih početaka života osposobljavati za učenje, omogućiti im da zavole učenje. Ovim predškolska pedagogija dobija posebnu vrijednost i izuzetan značaj” (Suzić, 2006, str. 10).

Ova dva navedena stava puno govore, iako ih ne dijeli veliko vremensko razdoblje, ipak se može primijetiti da metodika devedesetih godina nije naglašavala učenje matematike u predškolskom vaspitanju, dok su zahtjevi XXI vijeka na mnogo višem nivou. Predškolsko vaspitanje nije u ranijim godinama zvanično prejudicirano, ali ono je postojalo kao formalno, neformalno i na kraju tehničko. O postojanju predškolskog vaspitanja i u ranijim stadijumima ljudskog razvitka najrječitije kazuje i definicija predškolskog vaspitanja u kojoj se kaže da je to: “svjestan, svrsishodan i manje ili više organizovan proces razvoja individualnosti i društvenosti predškolskog djeteta, društveno determinisan i individualno obilježen sistem aktivnosti, djelatnosti i procesa koji utiču na sveukupno formiranje djeteta od rođenja do polaska u školu” (ibidem, str. 10).

Da bismo govorili o značaju ranog starta u nastavi matematike, prvo treba da razjasnimo principe predškolskog vaspitanja. “Polazeći od najhumanijih zahtjeva, od slobode i sreće kao vrhovnog principa, principe predškolskog vaspitanja možemo predstaviti u sljedećoj nomenklaturi:

- princip optimalizacije razvoja djetetovih potencijala,
- princip igre i igrovnih aktivnosti,
- princip nenasilne komunikacije,
- princip spontanosti i organizovanosti,
- princip primjernosti (Suzić, 2006, str. 11).

Navedeni principi su sasvim jasni i konkretni, iako odgovori na pitanja aspekta humanosti, te pedagoške opravdanosti pomjeranja granice tehničkog odgoja nisu u potpunosti dati. Naime, odgovori su dosta teški i složeni, iako dosadašnja iskustva sa predškolskim vaspitanjem u Francuskoj nepobitno ukazuje da preovladava mišljenje da će dominantni uticaj u naučnoj i tehničkoj superiornosti imati zemlje čiji vaspitno-obrazovni sistem bude začet što ranije.

Ako je vrhovni princip predškolskog vaspitanja sloboda i sreća, te “ako će organizovan sistem predškolskog vaspitanja donijeti više slobode i sreće djeci, tada je opravdano koristiti civilizacijska saznanja u te svrhe” (Suzić, 2006, str. 12). Sa aspekta matematike, rani start bi trebao da odgaja i obrazuje djecu u pravcu razvoja psihičkih, posebno intelektualnih sposobnosti učenika sa posebnim osvrtom na razvoj logičkog mišljenja, pažnje, pamćenja, uočavanja, zaključivanja, uspostavljanja veza među elementima, relacijama i tako dalje. Ovim putem dolazi do izražaja funkcionalna komponenta koja se ogleda u razvoju psihičkih i intelektualnih sposobnosti djece, kao i odgojna komponenta koja jača sa formiranjem i nadgradnjom pozitivnih karakteristika učenikove ličnosti.

Iako svaki od navedenih principa direktno ili indirektno ima uticaj i na pojmovna shvaćanja matematičkih vještina, ipak nećemo ovdje dublje ulaziti u sveobuhvatnost predškolskog vaspitanja kroz navedene principe, već ćemo se osvrnuti na kognitivni proces razvoja matematičkog mišljenja.

Mišljenje kao proces sa svim svojim nejasnoćama i tajnama možemo prihvatiti kao posljedicu unutrašnjeg instinktivnog nagona, poznatog nam pod nazivom znatiželja, nagon za upoznavanjem okolnog svijeta u kojem živimo. Identifikovanjem, upoređivanjem, razlikovanjem itd. dijete rano počinje da razvija svoj razum, svoje logičko mišljenje. Sa otkrivanjem kategorijalnih odnosa (uzrok-posljedica, stvar-osobina, dio-cjelina itd.) i sa upoznavanjem svrhe, dječje mišljenje počinje naglo da se razvija i napreduje (Slatina, 2005, str. 179).

U zavisnosti prvenstveno od starosne dobi mogu se grubo odrediti nivoi kao i vrste mišljenja. Tako možemo naići na podjelu mišljenja u najširem logičkom smislu. To mišljenje se javlja kao posljedica pitanja koja su najčešće u formi: zašto? čime? koliko? pri čemu odgovori na pitanja ne zadovoljavaju dječju radoznalost jer nisu ni postavljena sa nekom konkretnom važnošću. Nasuprot logičkom mišljenju u najširem smislu, imamo mišljenje u najužem smislu, koje se javlja kao posljedica jednostranih ideja, a koje su proizvod ranije usvojenih stavova. Treći vid mišljenja je onaj koji se javlja u analitičkom i sintetičkim svjetlu rasuđivanja, a ta rasuđivanja su posljedica nastupa problema, zagonetnih momenata, prepreka i slično.

Nesporna je činjenica da za razvoj dječjeg mozga nisu dovoljni samo nasljedni, odnosno genetski faktori, već su djetetu potrebni vanjski podražaji, izraženi putem dodira, govora, slika, što dovodi do zaključka da bliža i daljnja okolina oblikuje mozak, odnosno da vanjski podražaji jače ili slabije međusobno povezuju moždane stanice i neurone.

Sva istraživanja donose nove činjenice, koje ukazuju na dosadašnje greške u odgoju djece. Tako Gordon Drajden obrazlaže zašto bebe trebaju gledati oštre

kontraste? “U prvim mjesecima mozak beba uspostavlja svoje vidne putove. Djetetov korteks ima šest slojeva stanica koje prenose različite signale od retine u oku uzduž vidnog živca u mozak. Na primjer, jedan sloj tih stanica prenosi signale za vodoravne crte, a drugi za okomite. Drugi slojevi ili stupci stanica bave se krugovima, kvadratima i trokutima. Kada bi dijete vidjelo samo vodoravne crte, na primjer, tada bi se ono prilikom puzanja ili hodanja neprekidno sudaralo s nogama stolova i stolica jer njegovi vidni putovi ne bi mogli procesuirati okomite crte” (Dryden, 2001, str. 242).

Ronald Koutlack navodi da čak i u slučaju da je mozak neke osobe savršen, ta osoba neće moći vidjeti ukoliko ne procesuir vizualne podražaje do druge godine, i neće nikada naučiti govoriti, ukoliko do desete godine nije čula govor otuda i sugestija da se bebe od rođenja izlažu oblicima jakih crno-bijelih kontrasta, a ne da im zidove oblažemo jednočličnim pastelnim tapetama (Koutlack, 1996).

Prema dosadašnjim mnogobrojnim istraživanjima, naročito Birkenbihla (1989), razlikujemo, grubo rečeno, dva mozga, “stariji” i “noviji”. U starom mozgu smješten je naš biološki program, “hardver”, koji je svojom funkcionalnošću zadužen za empiriju te izbjegavanje bola i neugoda, motiviše ili umanjuje nagon za bijegom, osigurava nagon za preživljavanjem, nagon za zadovoljavanjem vitalnih životnih potreba, upravlja i kontroliše naše ponašanje, blokira naše razmišljanje, utiče na zapažanje stvarnosti, kontroliše naše pokrete tijela; on je sjedište naših osjećaja, emocionalnosti, pohranjuje i čuva naša sjećanja, tu se nalazi naše “ID” (Freud) i dječje “JA” (Berne), naše nadsvjesno i naše podsvjesno, kaos i još štošta nepoznato.

Sa druge strane imamo “novi” mozak koji je integralni dio lijeve i desne hemisfere kore velikog mozga, a predstavlja softvere, sa ulogama i zadacima povezivanja, analiziranja, zaključivanja, razmišljanja o sebi, istraživanja sebe, zadovoljavanja tipičnih ljudskih potreba, inteligenciju i kreativnost (Brajša, 1995, str. 53).

Po Pijažeu (Piaget, 1970), razvoj mišljenja se može grubo podijeliti u četiri šire faze, tako da se svaka nova faza naslanja na staru. Na temelju istraživanja koja su se uglavnom bazirala na eksperimentu, Pijaže pravi skalu baziranu na životnoj dobi te izdvaja prvi period ili period senzornomotorne faze, koja traje od rođenja do približno osamnaest mjeseci. Za navedeni period je osnovna karakteristika da beba uspijeva povezati osjećaj sa radnjom, postojanje predmeta je uslovljeno njenim viđenjem, dok u zrelijem stadijumu postaje svjesna postojanja predmeta i bez viđenja, a takođe dolazi i do spoznaje reverzibiliteta.

Drugo razdoblje, koje traje od osamnaest mjeseci pa do sedme godine života, Pijaže naziva preoperacioni stadij, koje dijeli u dva podrazdoblja, 2a i 2b. Oba dijela imaju svoja obilježja koja su karakteristična za dječja ponašanja i viđenje svijeta dječjim očima, pogotovo u periodu 2a, kada probleme rješava intuitivno, dok logika dolazi do izražaja u fazi 2b, kada dolazi do sukoba između dječje percepcije i mogućnosti logičkog rasuđivanja.

Treće razdoblje je period od sedme do dvanaeste godine i to je tzv. konkretno-operacijski period u kojem do izražaja još više dolazi logičko rasuđivanje, iako je navedeno razdoblje još bazirano na neposrednim opažanjima ili ranijim iskustvima.

Dijete shvata pojam komutativnosti, operacije sabiranja i pojam reverzibilnosti, što mu omogućava da zaključi da je npr. $7 - 3 = 4$ zato što je $3 + 4 = 7$, iako ne povezuje suprotnost navedenih operacija. Takođe je interesantno napomenuti da dijete u ovom periodu shvata pojam tranzitivnosti u formi $A = B, B = C \implies A = C$ ili $A > B, B > C \implies A > C$, dok u zadacima tipa Emir je viši od Gorana, Goran je viši od Zlatana: Ko je najviši?, djeca nisu spremna dati valjan odgovor. Pijaže navedenu nemogućnost vidi u činjenici da logika zauzima sve značajniju ulogu u razmišljanju, ali je ona još većim dijelom oslonjena na traženje rješenja utemeljenog na percepciji.

Četvrto razdoblje obilježeno je sposobnostima dokazivanja apstraktnih stavova, deduktivnim zaključivanjima, logičkim rasuđivanjima i tako dalje. Interesantna je činjenica da djeca razmišljaju strogo hipotetički, a da pri tome ne ulaze u mogućnost praktične realizacije problema, npr. u problemu tipa: ako 15 komada jaja košta 2 K.M. koliko se može kupiti jaja za 1 K.M.? Dječji odgovor je 7,5 komada, no da li se dati problem može u praksi realizovati, to djecu uopšte ne interesuje.

Nedoumice bitnosti se daju pokazati na sljedećem primjeru:

Upitajmo sami sebe šta je bitnije za dijete, da shvati da je $7 \cdot 2$ isto što i $2 \cdot 7$, ili da zapamti da je $2 \cdot 7 = 14$, ili, kraće rečeno, da li je bitnije da dijete zna računati ili da zna shvatiti? Upravo ovdje u naizgled bizarnim stvarima, stvara se nedoumica. "Kada poučavamo vještinu računanja, traže od nas razumijevanje, kada poučavamo vještinu razumijevanja, traže od nas računanje" (Liebeck, 1984, str. 7).

Ne osvrćući se na zahtjeve galopirajućeg, zahtjevnog u konkurenciji bezobzirnog fenomena nazvanog globalizacija društva, ne bi se smjelo zaboraviti da svako dijete u principu bi trebalo da svaki matematički problem, naravno ukoliko je to moguće, doživi vizuelno (slaganje kockica, grupisanje štapića, edukativna igra tangram, abakus i slično), te da na temelju vizuelnog sâmo dočara sliku koju treba skicirati (forma kvadratića, trouglova, kružića i slično), a zatim skiciranim elementima pridruži simbole, odnosno brojeve. Žan Pijaže, opisujući rad Eblija u sferi operativnih metoda, akcenat stavlja na uslove vaspitanja, koji mogu imati ubrzavajuće ili usporavajuće djelovanje, te da je preporučljivo da se simboličkom prikazu da onaj značaj koji konkretan prikaz posjeduje (Aebli, 1982 – vidi kod: Piage, 1983).

Nezaobilazna je činjenica "da se djeca pametna rađaju" te da i prije polaska u školu veći broj djece poznaje elementarne računske operacije sabiranja, oduzimanja, množenja i dijeljenja, naravno na opšte zadovoljstvo roditelja koji su dobrim dijelom zaslužni za razvoj kako fizičkog tako i intelektualnog nivoa svoje djece. Ne pitajući se kako dijete prihvata apstraktne pojmove, kao npr. šta je broj ili neka operacija, dešava se da veći broj učenika u srednjoj školi, zbog same apstraktnosti poimanja operacija, izraz $2a + 2b$ prihvata kao $4ab$, ili pak na neki drugi način.

Istraživanja bazirana na Piažeovim tezama, da se misaone aktivnosti temelje na prenosu i kopiranju realnih aktivnosti koje se vrše na materijalnim predmetima, nastavljaju Galjperin, Lompšer, koji, zajedno sa Brunerom, dolaze do konstatacije da se mentalne radnje dešavaju na nekoliko nivoa koji su međusobno povezani.

1. Nivo praktično-predmetnih radnji koji omogućava primjenjivost eventualnog objekta spoznaje.
2. Nivo neposrednog uočavanja. Valja naglasiti da ovdje nije izražajno samo puko uočavanje objekta već i misaono kompletiranje objekta spoznaje sa većim dijelom njihovih karakteristika.
3. Nivo posrednog uočavanja. Na navedenom nivou dolazi do aktiviranja ranije memorisanog i komparativnog pristupa objektu.
4. Nivo jezičko-pojmovne spoznaje, gdje se mentalna aktivnost odvija prije svega u okviru pojmovnog, tj. u okviru apstraktnih, a ne direktnih struktura.

Ono što bi ovdje bilo posebno interesantno, a vezano je za nivo neposrednog uočavanja, jeste činjenica da učenici misaonim putem mogu jednostavnije utvrditi broj ivica paralelopipeda, nego u praktičnom slučaju. Drugim riječima apstraktno-pojmovna obrada je bila izražajnija nego vizuelno-predmetna. Razrađujući teoriju o razvoju dječijeg mišljenja i operativnim principima, Pijaže ističe dio istraživanja Lompšera, te navodi kako postoje četiri spoznajna nivoa koji predstavljaju, sa jedne strane, genetički redoslijed, svaki viši se razvija iz nižeg kao svoje osnove. S druge strane, niži nivoi ne prestaju da postoje, nego sve ravni koje su jednom nastale nastavljaju da postoje, a nove ravni utiču na prethodne mijenjajući njihov karakter. Između ove četiri ravni postoje uzajamni odnosi, koji se u toku razvoja neprestano mijenjaju (Lompšer, 1972 – vidi kod: Piaget, 1983).

Dosadašnja istraživanja, kao i praksa, pokazala su daleko veću efikasnost pri savlađivanju neke operacije kada se ista operacija kombinuje sa drugim operacijama, tako, na primjer, pri operaciji množenja preporučuje se paralelno obavljanje i sabiranja i oduzimanja, kako pokazuje sljedeći primjer:

$$6 \cdot 9 = 9 \cdot 6 = 9 \cdot 5 + 9 \cdot 1 = 10 \cdot 6 - 1 \cdot 6$$

itd.

Zadaci prezentovani na ovaj ili sličan način kod učenika razvijaju osjećaj fleksibilnosti, kombinatorike, slobode, što jednostavno doprinosi da se tablica množenja mnogo brže i jednostavnije savlada, jer učeniku pruža mogućnost izbora raznih puteva do krajnjeg cilja.

Sa druge strane, receptivni oblik učenja, kod kojeg je učeniku prepušteno da samostalno istražuje i iznalazi najoptimalnije puteve do cilja, pokazuje se vrlo djelotvornim. Ovdje je takođe bitno naglasiti da usvajanje novih matematičkih saznanja, vještina, operacija, novih pojmova kao i adekvatna primjena istih na rješavanje problemskih zadataka mora biti utemeljeno na prethodnim predznanjima. Naime, upravo u ovakvim i sličnim momentima se demonstrira kumulativnost matematičkih vještina i funkcionalna primjena.

U vezi sa navedenim važno je reći da se rijetko upražnjava jedan tip učenja, već se različiti tipovi isprepliću i međusobno upotpunjuju, te u matematici susrećemo uglavnom sa sljedeće tipove učenja:

1. Asocijativno učenje (učenje na osnovu signala, draži, koje prelaze u automatizam kao npr. tablica množenja, operacije sa razlomcima, rješavanje jednačina i slično).

2. Diskriminatorno učenje (svodi se na njegovanje sposobnosti da razgraničavamo različite, ali i slične stvari kao pojmove, kao na primjer pravougaonik, kvadrat i tako dalje).
3. Učenje matematičkih pojmova (aksiomatski zasnovati neke pojmove te na temelju njih razraditi složenije).
4. Učenje matematičkih pravila (definicija, lema – pomoćna teorema, teorema).
5. Učenje heurističkih pravila (učiti sa razumijevanjem, vršiti razdiobu receptivnog i kognitivnog, razlučiti šta je dato, šta se traži? Da li je put dobar?).
6. Rješavanje matematičkih problema (učenje putem rješavanja matematičkih odnosa, raditi na tipskim i konstruktivnim zadacima).
7. Učenje opažanjem (iako liči na relaciju pokazanog, imitirano-pokazanog, ipak ima svoje prednosti (Bandura, 1977, str. 31).

O alternativnom početku planskog i svjesnog učenja kod djece postoje mnoge nedoumice. Tu problematiku je analizirao i Gordon Draiden koji opisuje rad bivšeg harvardskog profesora Barton L. Wajta, te navodi kako u povijesti zapadnjačkog obrazovanja nijedno društvo nije prepoznalo važnost obrazovanja u najranijoj dobi. Takođe nije bilo nikakvih priprema kao ni pomoći porodicama u cilju pokretanja akcije u cilju ranog razvoja djece. Profesor Wajt dalje navodi kako je razdoblje od kada dijete prohoda, pa do dvije godine života najvažnije, te da je taj period temeljan i ključan za razvoj govora, znatiželje, inteligencije i društvenosti. (White, 1991 – vidi kod Draiden, 2001, str. 253).

Optimalizacija razvoja djetetovih potencijala podrazumijeva stvaranje uslova i organizovanje aktivnosti kojima ćemo pomoći djetetu u razvoju njegovih sposobnosti, kompetencija i karakternih crta potrebnih za slobodan i srećan život u društvenom i prirodnom okruženju sredine u kojoj će provesti život. Teško je ili gotovo nemoguće predvidjeti uslove i sredinu budućeg života čovjeka, zato je potrebno već od ranog djetinjstva osposobljavati čovjeka za učenje i razvijanje vlastitih kompetencija, za samoosposobljavanje. Ovo postaje značajnije time što je izvjesno da će civilizacije u XXI vijeku imati obilježja učećeg društva. Stalno razvijanje vlastitih kompetencija, cjeloživotno ili doživotno učenje, postaju uslov slobode savremenog čovjeka.

Rezultate ovakvih istraživanja te ovakva razmišljanja bilo bi nehumano završiti, a ne postaviti, nebrojeno puta nametnuto pitanje: da li previše rano poučavanje krade djeci njihovo djetinjstvo? Odgovor bi se sveo na sljedeće: Dokle god se dijete učeći izvrsno zabavlja, opasnosti po njegovo mentalno zdravlje nema, ali ako dijete u fazi učenja ne nalazi zadovoljstvo, treba sa učenjem prekinuti.

Slab uspjeh kao i nizak nivo znanja iz matematike nije pusta pretpostavka, već je činjenica koja je potkrijepljena konstatacijama većine izvještaja, kako iz osnovnih tako i iz srednjih škola. Većina anketiranih učenika matematiku karakteriše kao tešku, nerazumljivu, nezanimljivu, prezahtjevnju.

Nesumnjivo “krivaca” za loš uspjeh na polju matematike na svim nivoima ima mnogo, te praviti klasifikaciju i hijerarhisku ljestvicu bilo bi nezahvalno i neodgovorno. Naime, nema određenog i jasno definisanog stava da bi se “krivac” mogao

identifikovati te kao takvog ili eliminisati ili smanjiti na najmanju moguću mjeru. Ono, što bi se u većini stručnih izvještaja moglo izdvojiti, kao najčešće spominjani razlozi su neadekvatni i prebukirani nastavni planovi i programi, prekobrojna odjeljenja, loša stručna osposobljenost nastavnika, neopremljenost škola, loši udžbenici, zastarjeli i prevazđeni metodičko – didaktički oblici rada, loši porodični uslovi uslovljenim niskim standardom življenja i tome slično.

Od navedenog, kao i od drugih navedenih uzroka nezahvalno bi bilo izdvojiti neke konkretne faktore, jer neuspjeh u nastavi matematike je funkcija svih tih faktora.

Većina nabrojanog uglavnom uporište traži u vanškolskim uzročnicima, zane-marujući činjenicu da glavne potencijalne krivce treba prvenstveno tražiti u školi, nastavi, načinu izvođenja nastave odnosno potrebno je prvo temeljno odgovoriti na pitanja tko?, na koji način? i šta? predaje. Nesumnjivo da bi jedan kvalitetan odgovor na postavljena pitanja barem djelimično rasvijetlio pojam neuspjeha u nastavi matematike

Na datu konstataciju ukazuju mnoge dosada objavljene analize, bazirane na nizu obavljenih testiranja, anketiranja, na pedagoškoj dokumentaciji, kako na našim prostorima tako i u svijetu. Takođe smo svjedoci dispariteta u ocjenama i načinu ocjenjivanja kod različitih nastavnika u okvirima jednog nastavnog predmeta, a da ne ukazujemo na različiti predmete. Činjenica da svršeni osnovnoškolci u srednjoj školi dolaze najvećim dijelom sa visokim ocjenama iz matematike, baš kao što svršeni srednjoškolci odlaze na fakultete takođe sa vrlo visokim ocjenama na fakultete.

U toku školovanja u srednjoj školi, što je pogotovo evidentno na prvoj polugodišnjoj klasifikaciji, rezultati su mnogo slabiji od rezultata donesenih iz osnovne škole, da bi se na kraju školske godine situacija malo poboljšala, ali opet sa uočljivom razlikom između zaključnih ocjena osmog razreda osnovne i prvog razreda srednje škole. Disparitet u ocjenama iz nastavnog predmeta matematike ne pojavljuje se samo kao proizvod različitih kriterija pri ocjenjivanju učenika, već na njega imaju uticaj i drugi faktori koje smo imali namjeru ako ne u potpunosti onda barem dijelom razotkriti. Ako je neuspjeh u nastavi matematike problem, a jeste, onda za njegovo rješavanje interaktivna nastava ima dio rješenja za koji Ante Vukasović navodi: “Rješavanje problema u nastavi pojavljuje se kao oblik intelektualne, a često i praktične aktivnosti pri čemu učenici intenzivno misle, istražuju, samostalno djeluju, pronalaze rješenja, rješavaju i uče” (Vukasović, 1977, str. 311).

Obilježja neuspjeha u nastavi matematike ovdje su navedena samo dijelom, a kroz interaktivnu nastavu nadamo se doći do konkretnijih rezultata koji bi upućivali na identificiranje, suzbijanje i otklanjanje takvih uzroka. Naučno istraživanje vezano za interaktivno učenje u nastavi matematike bi moralo da nam pruži neke pokazatelje koji bi poslužili za iznalaženje adekvatnih ideja za odgovarajuće didaktičke i metodičke akcije koje bi uslijedile u nastavi matematike. Rezultati istraživanja bi takođe trebali omogućiti praktičnu akciju na području nastave matematike čime bi se barem dio uzroka lošeg uspjeha u nastavi matematike ako ne u potpunosti onda barem dijelom otklonio. Sasvim je neopravdano posmatrati interaktivno učenje kao način učenja čijom bi se primjenom otklonili svi problemi u

vaspito-obrazovnom procesu a time i u nastavi matematike, ali primjenom interaktivne nastave u kombinaciji sa drugim oblicima nastave došlo bi do poboljšanja obrazovnog procesa na svim nivoima.

Neuspjeh u nastavi matematike, kao i neuspjesi u svim drugim sferama života, imaju svoje karakteristike po kojima se lako prepoznaju. Pasivno prisustvovanje nastavnom procesu, bez obzira koja se metoda na času koristila, kao i nastavni sadržaj koji se obrađuje, najslikovitije su karakteristike neuspjeha kod učenika i ove karakteristike ne odnose se samo na matematiku, već i na druge nastavne predmete. Navedo ne smijemo prihvatiti isključivo kao proizvod nastave matematike, jer na psihi mogu uticati mnogi faktori, od kojih se većina manifestuje raznim ignorantskim odnosom prema nastavnom procesu i školi uopšte, a ako se takvo ponašanje javlja relativno rijetko, onda ga možemo odbaciti. Neuspjeh u nastavi matematike se javlja kao posljedica manjkavosti matematičkih znanja ili kao posljedica nedovoljne razvijenosti intelektualnih sposobnosti učenika. Prvu navedenu karakteristiku obilježavaju kvantitativna svojstva, a drugu kvalitativna. Bez obzira o kojim karakteristikama je riječ, prepoznatljivost je veoma uočljiva, naročito u sferi obrazovne djelatnosti, dok se kod vaspitne djelatnosti može teže primijetiti. Posebno negativnu konotaciju sadrži činjenica da kvantitativna svojstva nose osobinu kumulativnosti, tj. postepeno povećanje neznanja matematičkih znanja, što je uglavnom posljedica prirode matematike kao nauke.

Naime, između ostalih uzroka za njenu težinu uglavnom su zaslužni matematički pojmovi, veličine, parametri, operacije i simboli koji su neraskidivo vezani i kompatibilni, što za posljedicu ima da je svaka nova oblast u matematici uslovljena prethodnim usvajanjem kroz definicije, leme (pomoćne teoreme), teoreme, operacije, činjenice, postavke iz drugih naučnih oblasti. Drugim riječima, matematičko gradivo je čvrsto povezano u neraskidivu cjelinu, što mu daje posebnu težinu.

Sa druge strane, nevolja pri učenju matematike je u tome da se simptomi neuspjeha ne javljaju iznenada, već je njihovo prisustvo dosta prikriveno i u funkciji je vremena kao parametra. Racionalizacija i drugi podsvjesni mehanizmi takođe čine svoje, što sve skupa daje konačan ishod, a to je loš uspjeh učenika u nastavi matematike.

Kako navodi poljski pedagog Konopnicki, postoje četiri karakteristične etape u nastavnom procesu nastajanja neuspjeha u nastavi.

Prvu etapu karakteristiše neprimjetno javljanje manjih praznina u znanju učenika, a njih uslovljavaju reakcije učenika koje se manifestuju u formiranju odbojnih stavova prema školi i nastavi. Kako se praznine povećavaju, što je neminovnost, jer prva etapa uglavnom promiče, to nastupa etapa u kojoj učenik gubi motivisanost, a sa njom i volju za učenjem.

Druga etapa se odlikava u sve višem stepenu nemogućnost praćenja nastavnog procesa, kao i pojačanim problemima u samostalnom rješavanju nastavnih zadataka. Bez obzira što su simptomi dosta uočljivi, nastavnici, a i roditelji, ne pridaju veću pažnju tome.

Treću etapu karakteristiše ignorantski odnos prema nastavi, pa i prema školi, tako da učenici zanemaruju sve obaveze, što rezultira povećanim brojem slabih

ocjena, što je prvi ozbiljniji pokazatelj da se sa učenikom nešto negativno dešava. Povećan broj negativnih ocjena rezultira povlačenjem iz aktivnog života razreda. Učenik se teško hvata u koštac sa nagomilanim problemima, te bez pomoći okruženja ne uspijeva da se vrati u nekakvo normalno stanje.

U *četvrtoj etapi* kod učenika se javlja osjećaj nesamostalnosti, nemoći, odbačenosti, što on kompenzuje kroz druge oblike koji nisu karakteristični za obrazovno djelovanje. U zavisnosti od broja slabih ocjena učenik se upućuje na ponavljanje razreda, što se u izuzetnim situacijama i ne mora desiti, pod uslovom da se učenik samostalno prisili na nadoknađivanje propuštenog, uz pomoć nastavnika, ili kroz vid instruktivne nastave (Konopnicki, 1969, br. 4 str. 452).

Navedene pojave dosta je teško u potpunosti spriječiti, ali treba sve učiniti da se one što više smanje. U tome mogu učestvovati svi nastavnici, pedagoške i socijalne službe a naročito roditelji. Ako ne totalno sprečavanje, onda u velikoj mjeri smanjivanje navedenih pojava bilo bi potpomognuto permanentnim praćenjem učenikovog rada i napretka u vaspitno-obrazovnom procesu. Naime, blagovremeno uočavanje problema bilo koje vrste stvara realne mogućnosti za preduzimanje adekvatnih mjera za sprečavanje neuspjeha.

U najkraćim crtama neuspjeh u nastavi matematike manifestuje se u:

- slabom matematičkom predznanju osnovnih pojmova, pravila i zakona,
- slaboj usvojenosti matematičkog jezika i njegove primjene,
- slaboj automatizaciji matematičkih operacija,
- slabom ili nikakvom pamćenju formula.

Interaktivnom nastavom obezbjeđuje se aktivnost svakog učenika, čime dolazi do razmjene stavova, ideja a time i do otklanjanja nejasnih pojmova. Takav rad utiče na razvoj sposobnosti i tehnike slušanja kao i sposobnost izlaganja ideja čime dolazi do ustrojavanja elementarnih pravila i zakonitosti uvažavanja drugog. Poučavanje se svodi na minimum, jer do izražaja dolazi učenje u kojem je učenik subjekat a ne pasivni posmatrač. Pamćenje se odvija spontano jer do izražaja dolazi kreativno mišljenje. Naglašena je kooperativnost, što predstavlja dobar preduslov za otklanjanje mnogih faktora koji uslovljavaju slab uspjeh u nastavi matematike.

Matematika kao ključni predmet ovog projekta doprinosi ostvarivanju niza pedagoških efekata koji zajedno formiraju svojstva ličnosti osposobljene za život u XXI vijeku, o čemu Suzić kaže: “Ako generacije prvog razreda osnovne škole budu učile interaktivnim metodama tokom svog školovanja, nakon izlaska iz srednje škole to će biti građani Evrope koji znaju da komuniciraju, da rade u grupi, da kontrolišu svoje emocije, da prezentiraju ostvarenja, da brzo nađu i koriste informacije. Osim toga, to će biti ljudi širokih pogleda, tolerantni i pomirljivi, spremni na saradnju i podršku ljudima oko sebe. Teško se može zamisliti bolji doprinos pomirenju i trajnom miru kao želji svih ljudi na ovim prostorima” (Suzić, 2001, str. 87).

Uzroci slabog uspjeha u nastavi matematike

Svu ili samo dio težine matematike kao nauke možemo potražiti ili u njenoj egzaktnosti ili u njenoj apstraktnosti. Prvi susret sa pojmovima, kao što je pojam ravno, okruglo, valjkasto, za djecu predstavlja apstrakciju, dok formula $E = mc^2$ predstavlja i za većinu odraslih osoba takođe apstraktan pojam. Pojam broja dva takođe je apstrakcija sve do momenta kada istoj oznaci ne pridružimo skup od dva elementa: dva učenika, dva novčića, dvije knjige i tome slično. Sve matematičke operacije, pa i one elementarne, kao sabiranje ili oduzimanje brojeva, predstavljaju apstrakciju na još višem nivou u odnosu na broj kao operacionalni pojam. Težina korespondencije između konkretnih elemenata i numeričkih oznaka kao krajnji cilj prve faze apstrahiranja nezaobilazni je faza razumijevanja matematike, te možemo i taj detalj prihvatiti kao jedan od prvih uzroka slabog uspjeha u nastavi matematike.

Sve škole bi na pitanje “kako vrše vaspitno-obrazovno proces”, dale odgovor da se on provodi po principima logike, psihologije i filozofije. Ma koliko je to istina, takođe je i istina da rijetko koja škola poučava kreativnom mišljenju, tj. sposobnosti da se otvorenog uma (neopterećenog drugim idejama) traže nove ideje, da se gleda drugačijim očima i u novim pravcima. Javljaju se ideje da su i postojeći termini logičkog i kreativnog mišljenja suviše ograničeni. Ljudski um je osposobljen i za konceptualna, analitička, spekulativna, kritička, luckasta, konvergentna, divergentna, bizarna, reflektivna, vizuelna, simbolička, brojana, metaforička, mitska, poetska, verbalna, neverbalna, praktična, dvosmislena, konstruktivna, realistička, nadrealistička, fokusirana, kreativna, konkretna, objektivna, subjektivna i druga rješenja.

Rješenje bilo kojeg nastalog problema mozak “traži” u empirijskom pretraživanju “datoteke” da bi našao istu ili sličnu situaciju koju je već jednom rješavao. Nenamjerno ograničavanje mozga vrlo je često uslovljeno predrasudama, predubjedjenjima ili tabuima, a to je uglavnom u uskoj vezi sa emocionalnim, nacionalnim, vjerskim, psihološkim, seksualnim ili drugim osjećanjima. Uzgred zapitajmo sami sebe: Da nismo možda ono što su drugi od nas očekivali da budemo? Roditelji, nastavnici i sredina u kojoj prebivamo svakodnevno odašilju očekivanja putem riječi, stavova, ozračja i jezika tijela tako da ta očekivanja postaju učenikova ograničenja.

Škole svih nivoa učinile su ili čine “svoje”, tako da smo već unaprijed predodređeni na direktne odgovore, ne ostavljajući prostor za traženje novih načina ili rješenja. Nema sumnje da će svaka osoba sa završenom srednjom školom ili fakultetom reći da je škola koju je baš ona završila najbolja ili u najgorem slučaju jedna od najboljih škola koju je završila. Navedeni stav treba prihvatiti kao sasvim normalan, jer rijetko koja osoba o sebi ima stopostotnu objektivnu sliku, kao što nijedan vaspitno-obrazovni, religijski ili pak zdravstveni sistem ne odgovara svima na isti način. U svima nama, što je sasvim normalno, postoje strahovi i pristrasnosti, što nije strano ni učenicima. U cilju njihovog prevazilaženja potrebno je u rad sa učenicima unijeti što više šale, zabave i humora. Zabava stvara atmosferu, a ona je uslov kreativnosti koje toliko nedostaje u našim školama. Sve to ne zaobilazi ni matematiku, za koju se uglavnom tvrdi da je prepuna apstrakcija, što čini da je teška, nesavladiva te kao takva odbojna barem većem dijelu učenika.

Jezik kao sredstvo kontinuiranog komuniciranja u osnovi takođe predstavlja apstrakciju, ali za razliku od matematike, u jeziku nije toliko nužna apstrakcija za objašnjenje nekih pojmova, te kao takav ne zahtijeva hijerarhiju pojmova. U matematici je apstrakcija dominantna i uslovljena je prethodnim savladavanjem jednostavnih apstraktnih pojmova, da bi se uspješno savladali složeniji pojmovi. Uspješno savladavanje matematike leži najvećim dijelom u nastavnikovoj sposobnosti da učenike uspješno provede kroz hijerarhiju matematičkih pojmova i operacija uz povezivanje činjenica sa vanjskim svijetom.

Neosporna je činjenica da učenici dolaze u osnovnu školu sa velikom dozom spontanog interesovanja za brojeve i matematiku. Sa godinama taj se spontanitet i zadovoljstvo u radu opada kod najvećeg broja učenika, tako da oni izlaze iz osnovnih škola sa odbojnošću prema matematici i stavom da je matematika nešto što je suviše dosadno i teško za shvatanje. Nastavno-naučne predmete a time i matematiku učenici vide kao visokosložene aktivnosti, koje su tu radi sebe same, a ne vide ih kao sadržaj namijenjen razumijevanju svijeta u kojem žive. Posljedice ovakvih razmišljanja su dosta ozbiljne, jer to kazuje da su životi velikog broja učenika ograničeni, te da je ukupan broj nacionalnih talenata daleko manji nego što bi mogao da bude. Prema izvještaju SCANS-a (*The Secretary's Commission on Achieving Necessary Skills* – Komisija Ministarstva za postizanje nužnih vještina), pod nazivom: "Šta posao zahtijeva od škola" (*What Work Requires of Schools*), više od pedeset posto američke mladeži napušta školu bez znanja i temelja potrebnih za pronalaženje i zadržavanje dobrog radnog mjesta. O posljedicama za budućnost tako profilirane omladine suviše je dalje govoriti. U izvještaju se dalje navodi: "Ti će mladi ljudi platiti vrlo visoku cijenu. Oni se suočavaju sa tužnom perspektivom besperspektivnog rada koji će biti prekidano samo razdobljima nezaposlenosti" (Dryden, 2001, str. 271). Prema procjenama, manje od polovice svih mlađih odraslih osoba postiglo je (propisane) minimume u čitanju i pisanju. Još je manje onih koji zadovoljavaju standarde za matematiku, a današnje škole samo se indirektno bave sposobnostima slušanja i govorenja. Radna snaga je nedovoljno obrazovana, nedovoljno uvježbana i nedovoljno klasifikovana (Ibidem).

Situacija na ovim našim prostorima nije nimalo bolja. Naprotiv, gora je. U ovom momentu bilo bi najfunkcionalnije tvrditi da su škole krive za sve, ne obazirući se na sistem društvenog uređenja koje je proizvelo iste te škole i sam predodređen da proizvodi neuspjehe. Takođe bi bilo najgore tvrditi kako i izvrsne nove tehnike učenja u školama mogu u potpunosti kompenzirati nove uticaje društva koje je i samo takođe predodređeno tako da je većina njegovih članova unaprijed osuđena na neuspjeh. Škole kakve danas jesu, nisu u stanju da promijene situaciju kakva je, no one u svakom slučaju predstavljaju esencijalnu nadu za promjenama.

Evidentno je da nastavljanjem vaspitno-obrazovnog procesa u srednjoj školi dolazi do naglog negativnog dispariteta u ocjenama, što je posebno naglašeno u nastavnom predmetu matematika, a što se ne bi smjelo dešavati. Naime, veći dio matematike u prvom razredu srednje škole u suštini predstavlja osnovnoškolsko gradivo sa temeljitijim pristupom analiziranju zadataka, kao i neznatno širem teoretskom dijelu. Kao i u osnovnoškolskoj matematici, tako i ovdje do izražaja dolazi činjenica da su dobra i kvalitetna dostignuća uslovljena kvalitetnim i cjelovitim znanjima prethodno usvojenim.

Matematika po samoj svojoj prirodi spada u red nauka koje zahtijevaju dosta velika intelektualna naprezanja te je svrstavamo u red "teških" nastavnih disciplina. Deviza da djeca sve znaju, ali malo toga razumiju, pojačava tezu da težina matematike leži i u nedovoljnoj intelektualnoj razvijenosti učenika, što nije tačno. Realizacija matematičkih sadržaja kroz metodičke pristupe koji su uslovljeni karakteristikama sadržaja takođe se može navesti kao jedan od uticajnih elemenata na težinu matematike.

Socijalna sredina, okruženje, mjesto prebivanja učenika, uopšte kultura življenja, takođe se navode kao mogući faktori slabog uspjeha u nastavi matematike. Kao jedan od važnih uzroka slabog znanja matematike jeste i fond nastavnih časova navedenog predmeta, koji je manji na svim nivoima škola u odnosu na evropske standarde, kao i brojnost učenika u većini odjeljenja, čime je kvalitet rada vrlo upitan. Sada se nameće otvoreno pitanje na koga ili u šta uprijeti prstom kao na potencijalnog krivca. Tražiti krivca u jednom činiocu i to van institucionalnih ustanova bilo bi najjednostavnije rješenje, no sva dosadašnja istraživanja pokazuju da je uzrok slabog uspjeha u nastavi matematike multifaktorske prirode, te da se razlozi najvećim dijelom nalaze unutar škole kao institucije.

(Ne)uspjeh u nastavi matematike ovisi o mentalnoj zrelosti učenika, organizaciji i realizaciji nastave matematike, metodici nastave matematike, učeničkoj okolini, stručnoj matematičkoj i pedagoškoj osposobljenosti nastavnika, nadarenosti učenika za matematiku i slično (Frommberger, 1955, str. 168). Šta staviti na prvo mjesto u hijerarhijskoj ljestvici faktora koji utiču na loš uspjeh u nastavi matematike? Svaki od navedenih činilaca ima svoj uticaj. Kao prevenciju neuspjeha u nastavi matematike na neki način trebalo bi prioritetno razvijati trend pedagogije u sljedećim aktivnostima: nastava matematike mora se približiti učeniku, motivisati učenike, razvijati njegovu latentnu energiju, učiti ih da uče, da razvijaju sposobnosti apstraktnog mišljenja.

Jedan broj srednjoškolskih nastavnika pokazuje u radu s učenicima aroganciju i prepotentnost i s visine posmatraju učenike proturajući tezu da srednja škola nije obavezna i tako kod učenika stvaraju odbojan stav, strah, frustracije i druge negativne pojave. Metodika matematike, htjeli to priznati ili ne, otvoreno ukazuje na naš automatizam. Ono što se svjesno ili nesvjesno zanemaruje jeste da uspješan prvi korak u nekom zadatku predstavlja izazov za sljedeći korak, a ovaj opet za naredni. Ovaj lanac uspjeha treba da se odvija uz uspostavljanje dublje veze sa "kognitivnom strukturom" djetetovog CNS-a.

Uvođenjem interaktivne nastave kao inovacije koja bi predstavljala novi model vaspitanja i obrazovanja u školama, nesumnjivo bi se desili mnogi pozitivni pomaci. "Inovacijsku školu sagledavamo kao trajan proces. Ona će se stalno dograđivati. Ne samo da prihvata ono što je kvalitetno, racionalno, pedagoški pozitivno naslijeđeno, već će i sama iznalaziti novo, kvalitetnije i to: u ciljevima i sadržajima vaspitno-obrazovnog, u nastavnom i vannastavnom radu; u izgradnji škole, u metodama, oblicima, medijima, tehničko-tehnološkim rješenjima; racionalnim i efikasnijim načinima nastavnog rada, u modelima i postupcima" (Milijević, 2003, str. 99).

Iz rečenog proizilazi, da od momenta kada nastavnici u svom radu izaberu značajne, pristupačne i uzbudljive teme, kako iz matematike tako i iz ostalih

naučnih disciplina, kada budu favorizovali grupni rad u svim formama, isticali pozitivni takmičarski duh među učenicima, kada težište bace na istraživački stil rada sa razumijevanjem onoga što se izučava, a ne na učenje termina i formula napamet i kada prevladaju ubjeđenja da njihovi učenici znaju da se od njih očekuje da istražuju i da uče i da su rezultati njihovog rada priznati – tada će najveći broj učenika i učiti.

Matematika sa svojim jezikom, operacijama, simbolima, teoremama, dokazima, logičkim pristupima i cjelokupnom aparaturom za svoje uspješno funkcionisanje zahtijeva kontinuirani lanac nadovezivanja cjelina sa cjelinama. “Nagomilavanje” nastavnog gradiva vrlo često dovodi do paničnih reakcija učenika zasnovanih na uvjerenju da se propušteno ne može stići, te kao posljedicu imamo fazu “inertnosti” prema svemu onome što je vezano za matematiku, kao i odbojan stav prema nastavi.

Pasivan stav, koji se vrlo često ne manifestuje samo u nastavi matematike, već ima odraz i u drugim nastavnim predmetima, za posljedicu ima uticaj na kompletan učenikov mentalni sklop, jer je i on u linearnoj zavisnosti od matematičkih saznanja. Smanjenjem usvajanja matematičkih sadržaja dolazi do smanjenja mentalnih sposobnosti, a o posljedicama je suvišno govoriti.

Zbog nedovoljne aktivnosti nastavnika, pedagoga, roditelja, vrlo često se “sindrom” neuspjeha u nastavi matematike prikriveno provlači do faze iz koje se teško vraća u fazu samopouzdanja i povjerenja učenika u samog sebe i svoje mogućnosti.

Na početni stadij nezainteresovanosti za nastavu matematike, kao i druge nastavne predmete, koji nisu nužno proizvod nagomilanih propusta u praćenju redovne nastave, već mogu biti i proizvod drugih faktora, neminovno se kao posljedice javljaju problemi kod učenika koje možemo grubo svrstati u kategorije:

- pedagoške prirode,
- psihološke prirode,
- socijalne prirode,
- ekonomske prirode.

Kao što smo već napomenuli, jedan od najčešćih uzročnika ignorantskog odnosa prema nastavi matematike jeste nagomilavanje nastavnog gradiva, što se u svjetlu pedagoških posljedica javlja u kvantitativnom i kvalitativnom obliku. Kvantitativni oblik prepoznajemo u odsustvovanju radnih navika, osjećaja za obaveze, uglavnom u neusvajanju elementarnih matematičkih činjenica itd., dok se kvalitativni oblik javlja u pomanjkanju logičkih rasuđivanja, reverzibilnosti matematičkih operacija i drugo.

Pomanjkanje kvantitativnih i kvalitativnih sadržaja matematike, odnosno matematički znanja, kod učenika dovodi do pasivnog pristupa matematici, a time do neshvatanja matematičkih sadržaja čime matematika gubi svoju prevashodnu funkciju u vaspitno-obrazovnom procesu.

Učenici se vrlo često ili zatvaraju u svoj svijet ili se u razredu trude da što češće skreću pažnju na sebe, no kako posjeduju oskudno znanje to nisu u mogućnosti učiniti na polju znanja, ostaje im izbor na drugom, da budu predmet ismijavanja, nešto poput klovna. Vremenom postaju sve manje interesantni okolini, što za

posljedicu ima još dublje povlačenje u same sebe, gubitak povjerenja u vlastite mogućnosti, pojavu kompleksa manje vrijednosti, apatičnost i gubitak ambicija.

Socijalne posljedice su vrlo očigledne i uglavnom se manifestuju u asocijalnom ponašanju koje se lako prepoznaje u samovolji, izbjegavanju redovne nastave, odbojnom stavu prema nastavniku, kolegama, agresivnom ponašanju.

Kada je riječ o ekonomskim posljedicama, jedna takva je upućivanje učenika na ponavljanje razreda, a to ima za posljedicu velike finansijske izdatke. Nesumnjivo, matematika se pojavljuje kao jedan od nastavnih predmeta koji zauzima neslavno vodeće mjesto na listi uzroka zbog kojih učenici gube godinu, pa se nameće zaključak da bi poboljšanjem uspjeha u nastavi matematike i ekonomski izdaci bili umanjeni.

Eliminisanje neuspjeha u nastavi matematike

Početni korak ka eliminisanju ili umanjenju neuspjeha u nastavi matematike kao i u razvijanju interesovanja i voljne aktivnosti učenika, pripada predmetnom nastavniku. Njegov zadatak je da se maksimalno angažuje i istraje u tome. Primarna aktivnost pri tome bila bi identifikacija uzroka neuspjeha. Kao glavne uzroke neuspjeha možemo prepoznati:

- djelimično ili nepotpuno znanje prethodnih pojmova, pravila i zakona,
- slabo ili nikakvo poznavanje matematičkog jezika,
- slabo ili nikakvo poznavanje matematičkih operacija,
- pasivno prisustvovanje nastavi,
- slabe ili nikakve radne navike,
- pomanjkanje osjećaja odgovornosti.

U suštini svi uzroci mogu se svrstati u manjkavost intelektualnog, moralnog, estetskog ili fizičkog vaspitanja. Od navedenih uzroka (kojih može biti mnogo više) kod pojedinih učenika može se pojaviti samo jedan uzrok, a najčešće se pojavljuje više njih zajedno. Neki od uzroka se pojavljuju prikriveni, ali sa porastom negativnih posljedica i oni postaju vidljiviji. Blagovremenim identifikovanjem uzročnika problem je uz pomoć nastavnika vrlo jednostavno rješiv. Nakon identifikacije uzroka slijedi strateški pristup problemu a zatim preduzimanje adekvatnih mjera za njihovo suzbijanje i otklanjanje.

Otklanjanje uzroka i potpuno angažovanje učenika u nastavni proces može se obaviti na više načina:

- – dopunskom nastavom,
- – produžnom nastavom,
- – ponavljanjem razreda.

Dopunska nastava ne odvaj se bitno od redovne nastave, osim u načinu realizacije i broju učenika koji joj prisustvuju. Ona je interventna, tj. organizuje se u toku školske godine, kada se za to ukaže potreba, a namijenjena je učenicima koji teže napreduju u savladavanju nastavnog gradiva kroz redovnu nastavu. Dopunska nastava se organizuje paralelno sa redovnom nastavom, prvenstveno sa ciljem

da se popune praznine koje se pokažu u redovnoj nastavi i koje onemogućavaju pojedine učenike da je prate. Iskustva su pokazala da je kompenzacijski oblik redovne u obliku dopunske nastave vrlo produktivan te da učenici vrlo brzo sustižu i nadoknađuju propušteno ili neshvaćeno nastavno gradivo. Efekat dopunske nastave bio bi i veći da se jedan broj nastavnika usavršava i stručno pouči, jer dosadašnja praksa pokazuje da se način rada u okvirima dopunske nastave ne razlikuje bitno od redovne nastave kod većine nastavnika iako bi ona trebala biti prilagođena svakom učeniku, a time i specifična.

Produžna nastava, organizuje se nakon okončanja redovne nastave u toku školske godine, sa ciljem da učenici koji nisu u toku redovne nastavne godine savladali nastavno gradivo dodatnim fondom sati pokušaju da uz pomoć nastavnika nadoknade propušteno. U principu, produžnu nastavu izvode isti nastavnici koji su izvodili i redovnu nastavu, jer su oni najbolje upućeni u probleme učenika koji se upućuju na pohađanje produžne nastave. Dosadašnja iskustva nedvojbeno ukazuju na relativno skromne rezultate rada produžnom nastavom. Razlozi za navedeno su uglavnom psihofizičke prirode. Naime, produžna nastava zahtijeva veliki angažman nastavnika, kao i učenika, koji se ogleda u zahtjevu da u relativno kratkom vremenskom roku učenik nadoknadi najveći dio gradiva koje je propustio u toku redovne nastave. Za ostvarenje jednog takvog zahvata od nastavnika se traži primjena savremenih nastavnih metoda kao i kvalitetno didaktičko-metodičkih oblikovanje nastave.

Ponavlja se razred kada su se sve navedene pedagoške ili nenavedene mjere pokazale bez efekta. Ova krajnja mjera se primjenjuje kako bi učenici ponavljanjem razreda nadoknadili ili uklonili nedostatke u matematičkim znanjima, sposobnostima kao i radnim navikama koje su im potrebne za daljnje napredovanje u nastavi matematike. Upućivanje učenika na ponavljanje razreda, ma koliko to nepopularna pedagoška mjera bila, pokazuje se dosta efektom na generacije učenika, čime respekt prema školi dobija na značaju. Sa druge strane, ova mjera se primjenjuje uz pretpostavku da se propušteni nedostaci mogu u potpunosti nadoknaditi, kao i ojačati učeničke navike i umijeća. "Teorijska osnovica ponavljanja razreda kao načina, mjere i sredstva za suzbijanje neuspjeha u nastavi matematike u osnovnoj i srednjoj školi je spoznaja o funkciji i ulozi matematičkog predznanja u procesu učenja (savladavanja) matematičkih sadržaja po kojoj (spoznaji) uenicima, koji nisu savladali (usvojili) matematičke sadržaje prethodnog razreda, nedostaju nužna predznanja (preduslovi) za uspješno usvajanje matematičkih sadržaja sljedećeg razreda" (Markovac, 1978, str. 98).

Mjera upućivanja učenika na ponavljanje razreda uglavnom se preporučuje, naravno, kao posljednja pedagoška mjera kod učenika koji zaostaju u psihofizičkom razvoju ili kod učenika koji zbog bolesti ili drugih razloga nisu bili prisutni na dijelu redovne nastave, što ima za posljedicu zaostajanje u matematičkim znanjima koja su potrebna za daljnje praćenje redovne nastave.

Dosadašnja istraživanja, na uzorcima učenika koji su bili upućeni na ponavljanje razreda, pokazuju da se ponavljanjem razreda ne ostvaruju očekivani efekti, odnosno da učenici ne dopunjavaju praznine u matematičkim sposobnostima, niti se u značajnijoj mjeri nadograđuju na intelektualnom nivou.

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Stav učitelja razredne nastave prema matematici

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Sažetak. U radu ukazujemo na implikacije učiteljskog odnosa prema matematici na kvalitetu njihove nastave i rezultate učenika. Zanimalo nas je istražiti kakav stav prema matematici imaju učitelji razredne nastave u Hrvatskoj kako bi iz dobivenih rezultata mogli pokušati otkriti neke od uzroka često nezadovoljavajućih rezultata učenika u matematici. Odnos učitelja prema matematici ispitali smo anonimnim anketiranjem na uzorku od četristotinjak učitelja iz raznih krajeva Hrvatske i Bosne i Hercegovine. Rezultati su pokazali da usprkos pozitivnim stavovima koje učitelji eksplicitno deklariraju, oni u svom radu imaju brojne predrasude, kako prema matematici, tako i prema svojim učenicima. Osvještavanjem tih predrasuda moguće je vremenom utjecati na njihovo mijenjanje, a time indirektno i podizati kvalitetu matematičkog obrazovanja u periodu razredne nastave.

Ključne riječi: učitelj, nastava matematike

Uvod

Kvalitetna nastava od početka školovanja traži kvalitetnog učitelja(cu)¹ ili nastavnika(icu), koji dobro poznaje suvremene metodičke spoznaje i zna kako će ih primijeniti u vlastitoj nastavnoj praksi. Bez stalnog, cjeloživotnog usavršavanja, učenja, čitanja i praćenja suvremenih spoznaja o nastavi matematike, teško je udovoljiti potrebama djece u brzom, kompleksnom i promjenjivom svijetu u kojem živimo. Učitelji razredne nastave, kao prve stručne osobe koje sustavno izgrađuju matematička znanja, umijeća i stavove o matematici kod učenika na početku školovanja, imaju pred sobom još važniji i složeniji zadatak. Oni su, naime, interdisciplinarni stručnjaci za više predmetnih područja, a matematika je samo jedan segment njihova profesionalnog djelovanja. Usprkos tome, oni moraju imati široko i duboko, konceptualno razumijevanje i poznavanje matematike kako

¹ U radu će se koristiti termini učitelj ili nastavnik kao opće imenice koje se podjednako odnose i na žene i na muškarce.

bi kod učenika u osjetljivim godinama razvili kvalitetan temelj na kojem će se kasnije nadograđivati matematička znanja i umijeća u predmetnoj nastavi. Kako bi u toj ulozi bili uspješni, od učitelja razredne nastave očekuju se brojne matematičke, pedagoške i psihološke kompetencije.

Mnogi istraživači smatraju da su učiteljevo razumijevanje i primjena suvremenih shvaćanja razvojne psihologije (koja u centar obrazovnog sustava postavljaju dijete) bazičan uvjet za uspješno poučavanje i učenje matematike (MacNab, 2000). Drugi model kojeg su predložili Fennema i Franke (prema Leou, 1998) na sličan način ukazuje na interaktivnu i dinamičnu prirodu učiteljeva matematičkog znanja, pedagoškog znanja i znanja o učenikovom kognitivnom razvoju, ali naglašavaju i važnost i utjecaj učiteljevih uvjerenja o matematici. Leou (1998) predlaže model koji postavlja četiri kategorije koje određuju nastavnikove kompetencije za poučavanje matematike, a to su:

- (a) umijeća poučavanja,
- (b) materijalna organizacija i prezentacija,
- (c) okolina (atmosfera) za učenje stvorena između nastavnika i učenika i
- (d) nastavnički stavovi ili uvjerenja o matematici.

Bez obzira na način klasificiranja učiteljskih kompetencija za poučavanje matematike, treba naglasiti da samo posjedovanje navedenih kompetencija još nije garancija uspješnosti učitelja.

Govoreći o stavovima učitelja prema matematici, podrazumijevamo nekoliko aspekata tog stava. U stav učitelja ulazi njihova procjena važnosti matematike, opći odnos prema matematici, njihova procjena učeničkih mogućnosti unutar matematike, kao i odnos prema nekim čestim predrasudama koje prate nastavu matematike. Tako ćemo smatrati da bolji stav prema matematici ima učitelj koji je smatra važnom, koji je zainteresiran za matematiku, koji smatra da je svaki učenik može savladati i koji nema predrasuda.

“Nužno je, vezano za neuspjeh u nastavi matematike općenito, ukazati na ustaljena mišljenja, stavove i predrasude većeg broja roditelja, učenika pa i mnogih nastavnika, prema matematici i učenju njenih sadržaja, a koji smatraju da je sasvim prirodno i opravdano da neki učenici imaju poteškoća u razumijevanju i usvajanju matematičkih sadržaja, zbog izrazite apstraktnosti tih sadržaja. U tom krivom poimanju ide se čak do te razine da se sasvim ustalilo mišljenje da je matematička znanost (znanje) dostupno samo odabranim, posebice nadarenim, a da su neki učenici, naprosto, nesposobni za učenje matematike, odnosno da im je matematika teško shvatljiva i nedostupna” (Pejić, 2003, 4). Umjesto da se od djece očekuje mnogo, da se svima daje prilika za uspjeh, da se cijeni i potiče njihova upornost, interes i trud, učitelji i šira društvena okolina prihvaćaju neuspjeh u matematici kao nešto razumljivo, prihvatljivo, pa čak i neizbježno. Takvi su stavovi izuzetno opasni, jer su istraživanja pokazala “da se društveni položaj matematike treba prepoznati i da se individualna percepcija matematike ne može razdvojiti od socijalnog okvira u kojem je stvarana i razumijevana od drugih” (Brown i dr., 1999, 319). Stoga treba što prije mijenjati negativnu stigmu koja često prati (školsku) matematiku i koncentrirati se na njezinu važnost, ljepotu i korisnost kako u svakodnevnom životu, tako i u školi.

“Danas je postalo jasno da je učenje cjelovit proces i ne svodi se samo na pamćenje i mišljenje. To je i emocionalni doživljaj, i psihomotorna aktivnost, i socijalni odnos, i proces samoaktualizacije” (Bognar, 1998, 349). “Temelji djetetova matematičkog razvoja postavljeni su u najranijim godinama njegova života, a kasnije ih je potrebno samo nadograđivati i razvijati. Matematičko se učenje treba graditi na znatiželji i oduševljenju djeteta i prirodno se razvijati kroz njegovo iskustvo. Matematika je u tim godinama, ukoliko je primjereno vezana uz djetetov svijet, mnogo više nego spremnost za školu ili uvođenje u elementarno računanje. Primjerena matematička iskustva potiču manju djecu da istražuju zamisli povezane s oblicima, brojevima i prostorom na sve složenije načine” (NCTM, 2000, 73). U početnoj nastavi matematike formiraju se djetetovi stavovi prema matematici koji mogu značajno utjecati na njegov budući matematički razvoj. “Istraživanja su pokazala da se do djetetove jedanaeste godine oblikuje njegov odnos prema matematici. Ako je taj odnos negativan, djeca ne vole, izbjegavaju i zaziru od matematike” (Pavleković, 1997, 302). Ta uvjerenja značajno utječu na njihov daljnji matematički napredak i uspješnost.

Kako bi potaknuo interes i entuzijazam za učenjem matematike kod svojih učenika, učitelj treba ulagati trud i entuzijazam u svoje poučavanje. Naime, “briga i trud koji učitelji ulože u pripremu može pozitivno djelovati na učenike jer po tome mogu zaključiti da je učiteljima stalo da učenici nešto nauče i da su nastavne aktivnosti vrijedne truda” (Kyriacou, 1997, 48). Na početku školovanja, ali i za vrijeme cijelog perioda razredne nastave, većina učenika pokazuje veliki interes za matematiku. Oni matematiku nalaze zanimljivom i korisnom i vjeruju da je učenje matematike važno. Taj je interes i pozitivno uvjerenje potrebno njegovati i održavati, a tu je uloga učitelja izuzetno važna. On mora osmisliti i realizirati nastavu koja će učenicima biti intelektualno izazovna, zanimljiva, korisna, upotrebljiva u svakodnevnom životu i u kojoj će svaki učenik naći svoje mjesto. “Međutim, ako se učenje matematike svodi na proces oponašanja učiteljeva rada i memoriranje činjenica, učenici će brzo izgubiti interes i volju za učenjem” (NCTM, 2000, 143).

Metodologija istraživanja

Brojna su istraživanja pokazala povezanost i pozitivnu korelaciju između učiteljevih kompetencija, načina njegova rada i stavova koje ima o predmetu kojeg poučava sa kompetencijama, rezultatima i stavovima učenika u istom predmetnom području (vidi Wayne i Youngs, 2003; Solomon, 1998; Peterson i dr., 1989; MacNab, 2000; Linares i dr., 2005; Sun Lee i Ginsburg, 2007). Načinom poučavanja, aktivnostima koje provodi, ali i svojim općim odnosom prema matematici, učitelj ostvaruje veliki utjecaj na ukupna postignuća učenika u periodu razredne nastave. “Uvjerenja, stavovi i emotivni doživljaj igraju važne uloge u razvijanju kritičkog i kreativnog smisla za matematiku” (Blum, 2002, 161). Upravo zato je važno i vrijedno ispitati stavove učitelja razredne nastave prema matematici, što je i **cilj** ovog rada.

Da bi ispitili stavove učitelja prema matematici, napravljen je anketni listić kojeg su učitelji anonimno popunjavali². Kod ispitivanja subjektivnih komponenata, kao što je nečije mišljenje ili stav, nikada ne možemo biti sigurni da je odgovor koji smo dobili stvarno odraz mišljenja i ponašanja ispitivane osobe. Deklarirani stavovi učitelja o matematici, kompetencijama ili načinu provođenja nastave matematike, često ne odgovaraju njihovim stvarnim ponašanjima u nastavnim situacijama. Možemo pretpostaviti da će u iskazivanju mišljenja i stavova učitelji biti skloniji birati odgovore za koje pretpostavljaju da ih ispitivač želi čuti ili dobiti (Mužić, 1999). Iako smo kod provođenja ankete pokušali naglasiti činjenicu da u anketi nema točnih i netočnih odgovora, pretpostavljamo da dobivene rezultate trebamo uzimati s velikom rezervom.

Uzorak je činilo 400 učitelja razredne nastave iz Republike Hrvatske i Bosne i Hercegovine. Opći podaci koji su traženi od ispitanika odnosili su se na *državu u kojoj rade, stručnu spremu, spol te duljinu radnog staža*. Broj ispitanika u pojedinim kategorijama prikazan je u Tablici 1.

		N			ukupno	bf ukupno
		M	Ž	0	N	%
država	Bosna i Hercegovina	21	167	1	192	47
	Republika Hrvatska	12	199		208	53
	Total	33	366	1	400	100
stručna sprema	srednja stručna sprema	3	13		16	4
	viša stručna sprema	18	200		218	55
	visoka stručna sprema	12	148	1	161	40
	neodređeno		5		5	1
	Total	33	366	1	400	100
radni staž	do 5	4	75		79	20
	od 6 do 10	4	62		66	16
	od 11 do 20	9	131	1	141	35
	od 20 do 30	8	51		59	15
	preko 30	8	47		55	14
	Total	33	366	1	400	100

Tablica 1. Ispitanici prema državi, stručnoj spreml, godinama staža i spolu.

Obzirom na državu iz koje ispitanici dolaze, njih 208 ili 53% djelovalo je u trenutku ispitivanja u Republici Hrvatskoj, dok je 192 ili 47% ispitanika bilo zaposleno u Bosni i Hercegovini. Obzirom na veličinu uzorka ($N = 400$), smatramo da je uzorak reprezentativan za učiteljske populacije u obje države. Prema *stručnoj spreml* učitelja, vidimo da najveći broj ispitanika ima višu stručnu spremu (55%), zatim visoku stručnu spremu (40%), a najmanji broj ispitanika (svega 4%) ima srednju stručnu spremu. Zbog malog broja ispitanika u kategoriji srednje stručne

² Anketiranje je provedeno u okviru šireg istraživanja standarda matematičkih kompetencija u početnoj nastavi matematike, a za potrebe ovog rada izdvojen je samo segment koji se odnosio na stavove učitelja.

spreme u odnosu na druge dvije kategorije, njihovi su nam rezultati bili nekomparabilni. Stoga smo u daljnjoj obradi podataka morali tu kategoriju zanemariti, odnosno, zadržati se na samo dvije kategorije ispitanika prema stručnoj spremi, viša i visoka stručna sprema. Ustanovili smo da je uzorak izrazito neujednačen prema faktoru *spol* ($M = 33$, $\check{Z} = 366$, jedan ispitanik se nije izjasnio), stoga smo odustali od analiziranja rezultata prema toj kategoriji. Takav odnos muških i ženski učitelja nas nije iznenadio i odraz je stanja u razrednoj nastavi i školstvu općenito u gotovo svim zemljama (vidi Polić, 2003).

Ispitanike smo, prema godinama *staža*, razvrstali u pet kategorija, definiranih kao vremenske intervale od 0 do 5 godina, od 6 do 10 godina, od 11 do 20 godina, od 21 do 30 godina i preko 30 godina staža. Iako kategorije nisu vremenski ujednačeni intervali, smatramo da takva razdioba omogućava zaključivanje o radnom iskustvu ispitanika iz svake od kategorija. Razdioba na dobar način izdvaja učitelje početnike, čime smatramo učitelje s radnim stažem do 5 godina, te učitelje pred mirovinom, s preko 30 godina radnog staža.

Rezultati i rasprava

Stavove učitelja o matematici utvrđivali smo kroz deset tvrdnji o kojima su ispitanici izražavali svoje slaganje na Likertovoj ljestvici od 1 do 5 (od 1-potpuno neslaganje, do 5-potpuno slaganje). Tvrdnje su vidljive u Tablici 2. Tvrdnje smo izabrali na način da smo kombinirali pet pozitivnih tvrdnji o matematici, te pet uobičajenih predrasuda o njoj. Smatramo da je stav pojedinog ispitanika pozitivniji ukoliko se više slaže s pozitivnim tvrdnjama o matematici, odnosno ukoliko se manje slaže s uvriježenim predrasudama. Drugim riječima, smatramo da se stav o matematici iskazuje iznošenjem pozitivnog mišljenja o njoj, kao i nesklonosti negativnim predrasudama o matematici.

Kako bi testirali tu svoju teorijsku pretpostavku o dva različita izražavanja stava o matematici, proveli smo postupak faktorske analize. Prije samog postupka, testirali smo prikladnost podataka za provođenje faktorske analize. U tu svrhu izračunali smo Bartlettov test sfericiteta, koji se pokazao veoma značajnim ($p < 0,01$). Proveli smo i Kaiser-Meyer-Olkinovu (KMO) mjeru prikladnog uzorkovanja čiju smo vrijednost od 0,8 ocijenili kao vrlo zadovoljavajuću, pa smo zaključili kako je opravdano provesti postupak faktorske analize.

Postupkom faktorske analize primijenjenom na deset navedenih tvrdnji, izlučena su dva glavna faktora, koje najbolje uočavamo u rotiranoj faktorskoj matrici u Tablici 2.

Tvrdnja iz ankete	Factor	
	1	2
Učenje matematike je zadovoljstvo.	0,710	
Matematiku ne može svatko uspješno svladati.		0,518
Matematika je usko povezana sa stvarnim životom.	0,518	

Inteligentna su djeca uspješna u matematici.		0,388
Osnovna je svrha matematike naučiti računati.		0,487
Matematika je zabavna i kreativna.	0,772	
Ili jesi, ili nisi za matematiku.		0,651
Matematika je svakome u životu jako korisna.	0,498	
Matematika je jako težak predmet.	0,409	−0,573
Ja volim matematiku.	0,591	

Tablica 2. Rotirana faktorska matrica stavova o matematici.

Kao što se vidi iz Tablice 2, gotovo sve tvrdnje su jasno razdijeljene u jedan od dva izlučena faktora. Jedino tvrdnja “Matematika je jako težak predmet” nije jasno opredjeljena, odnosno zasićuje djelomično i jedan i drugi faktor. Stoga smo navedenu tvrdnju isključili iz daljnje obrade i nismo je uzimali u izračunavanju općeg stava ispitanika.

Prvi faktor povezuje pet tvrdnji koje smo i u izradi ankete osmislili kao pozitivne tvrdnje, pa ga možemo nazvati “pozitivno mišljenje o matematici”. Riječ je o tvrdnjama “Učenje matematike je zadovoljstvo”, “Matematika je usko povezana sa stvarnim životom”, “Matematika je zabavna i kreativna”, “Matematika je svakome u životu jako korisna” i “Ja volim matematiku”. Veći brojevi na Likertovoj ljestvici pridruženi tim tvrdnjama, pokazuju pozitivniji ispitanikov stav o matematici. Drugi faktor zasićen je sa četiri tvrdnje koje možemo nazvati “predrasude o matematici”. To su tvrdnje “Matematiku ne može svatko uspješno svladati”, “Inteligentna su djeca uspješna u matematici”, “Osnovna je svrha matematike naučiti računati” i “Ili jesi, ili nisi za matematiku”. Visoko slaganje s tim tvrdnjama na Likertovoj ljestvici pokazuje negativniji stav o matematici, budući da predrasude pokazuju negativna, uvriježena, ali pogrešna mišljenja o matematici. Stoga smo vrijednosti pridodate tim četirima tvrdnjama simetrično okrenuli, na način da smo uzlazne vrijednosti Likertove ljestvice od 1 do 5, pretvorili u silazni niz od 5 do 1 (tko je zaokružio 5, dobio je 1 bod; tko je zaokružio 4, dobio je 2 boda i tako dalje). Tako je veći broj bodova pridružen ispitaniku koji pokazuje manje slaganje s određenom predrasudom.

Iako je faktorska analiza pokazala da naš mjerni instrument mjeri dvije različite stvari koje smo nazvali “pozitivno mišljenje o matematici” i “predrasude o matematici”, smatramo da jedan i drugi faktor čine dio općeg učiteljeva stava o matematici. Naime, veće slaganje s tvrdnjama koje zasićuju prvi faktor pokazuje pozitivniji stav o matematici, a veće slaganje s tvrdnjama koje zasićuju drugi faktor pokazuje negativniji stav. Kako bi dobili jedinstveni stav svakog ispitanika, zbrojili smo pripadne vrijednosti u svih devet tvrdnji (uz napomenu da su vrijednosti koje su ispitanici pridodali predrasudama simetrično zamjenjene) i dobiveni zbroj podijelili s brojem tvrdnji. Na taj smo način kvantitativno izkazali stav prema matematici svakog ispitanika na intervalu od 1 do 5. Prema našoj interpretaciji, izrazito negativnim stavom smatramo stav ispitanika ispod 2,0, dok negativnim stavom o matematici smatramo stav od 2,0 do 3,0. Zadovoljavajućim stavom smatramo stav od 3,0 do 4,0, stav iznad 4,0 smatramo pozitivnim, a iznad 4,5 izuzetno pozitivnim stavom o matematici.

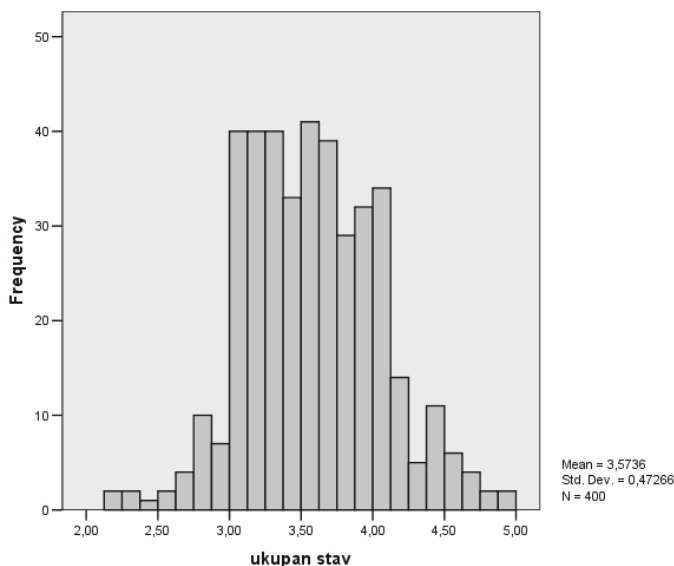
U Tablici 3 prikazane su srednje vrijednosti rezultata svih ispitanika u svakoj pojedinoj tvrdnji, kao i srednja vrijednost ukupnog stava ispitanika o matematici koja iznosi 3,6. Iako je srednja vrijednost stava u gornjoj polovici ljestvice, s takvom vrijednošću općeg stava prema matematici ipak ne možemo biti potpuno zadovoljni, budući je riječ o učiteljima razredne nastave koji veliki dio svog nastavnog rada poučavaju matematiku. Promatrajući dobivene rezultate, uočavamo da su rezultati u pojedinim predrasudama o matematici relativno niski (u donjoj polovici ljestvice), što ukazuje na relativno visoko slaganje ispitanika s tim predrasudama. Rezultati u tvrdnjama koje iskazuju pozitivno mišljenje o matematici u svim su pripadnim tvrdnjama veći od 4,0. Možemo pretpostaviti da su, usprkos velikom slaganju ispitanika s pozitivnim tvrdnjama, njihove predrasude bitno snizile ocjenu ukupnog stava. Ovaj nas zaključak ujedno upućuje na potrebu osviješćivanja predrasuda o matematici kod učitelja razredne nastave, čime bi se njihov stav o matematici mogao podići na viši nivo.

	srednja vrijednost	standardna devijacija
Učenje matematike je zadovoljstvo.	4,20	0,735
Matematiku ne može svatko uspješno svladati.	2,34	1,030
Matematika je usko povezana sa stvarnim životom.	4,38	0,726
Inteligentna su djeca uspješna u matematici.	2,32	1,053
Osnovna je svrha matematike naučiti računati.	3,04	1,212
Matematika je zabavna i kreativna.	4,34	0,745
Ili jesi, ili nisi za matematiku.	2,87	1,180
Matematika je svakome u životu jako korisna.	4,34	0,782
Ja volim matematiku.	4,43	0,804
ukupan stav	3,5736	0,47266

Tablica 3. Stavovi ispitanika o matematici.

Predrasuda s kojom se ispitanici najviše slažu je tvrdnja koja kaže *“Inteligentna su djeca uspješna u matematici”*. Takvo uvjerenje, koje često susrećemo kod učitelja i nastavnika, doprinosi doživljavanju i shvaćanju matematike kao selekcijskog predmeta, što je često razlog zaziranja od matematike kod učenika. Jasno je da inteligencija olakšava postizanje uspjeha u bilo kojem predmetu, pa i u matematici, ali postoje brojna inteligentna djeca koja u matematici ipak ne postižu zadovoljavajuće rezultate. Prema Sharmi (2001), čak oko “25% ljudi s prosječnim ili iznadprosječnim općim stupnjem kognitivne inteligencije ima nedovoljno razvijeno matematičko mišljenje” (Sharma, 2001, 10). Slaganje učitelja s tom predrasudom može biti destruktivno u podizanju uspješnosti učenika u matematici. Naime, ukoliko učitelji smatraju da samo pametna djeca mogu biti uspješna u matematici, onda se ona koja to po njima nisu, mogu odmah prestati truditi, jer oni nikada neće doseći uspjeh (Brown, 1998, 88). Pozitivna tvrdnja u kojoj su ispitanici iskazali najveće slaganje i najpozitivniji stav, jest tvrdnja *“Ja volim matematiku”*. Jasno je, međutim, da je ta tvrdnja potpuno subjektivna i sama po sebi se ne može uzimati kao isključivi dokaz nečije sklonosti prema matematici.

Ukupan stav ispitanika o matematici kretao se u intervalu od 2,2 do 5,0, što znači da nijedan ispitanik nije iskazao potpuno negativan stav o matematici. Promotrimo li razdiobu učiteljskih stavova na Grafikonu 1, vidimo da je najviše ispitanika sa stavom u intervalu između 3,0 i 4,1 što smatramo zadovoljavajućim stavom o matematici.



Grafikon 1. Razdioba stavova učitelja o matematici.

Svega je 28 ispitanika iskazalo negativan stav o matematici, a samo njih 14 iskazalo je izuzetno pozitivan stav. Možemo zaključiti da većina ispitanika izražava pozitivan stav o matematici, odnosno da je vrijednost njihova stava najčešće u gornjoj polovici ljestvice. Ipak, obzirom da se radi o populaciji učitelja koji poučavaju mlade generacije matematici u veoma osjetljivom razdoblju njihova života, trebalo bi težiti podizanju stava iznad ocjene 4,0. Mali broj ispitanika s izuzetno pozitivnim stavom o matematici (3,5%) također je indikativan i ukazuje na potrebu dubljeg istraživanja ove problematike.

Stav ispitanika prema matematici promatrali smo i u odnosu prema općim varijablama (državi, stručnoj spremi ispitanika, radnom stažu), a rezultati su prikazani u Tablici 4.

	država		Stručna sprema			Godine staža				
	BiH	Hr	srednja	viša	visoka	od 0 do 5	od 6 do 10	od 11 do 20	od 21 do 30	više od 30
srednja vrijednost stava	3,49	3,65	3,26	3,52	3,68	3,63	3,55	3,61	3,60	3,41

Tablica 4. Srednja vrijednost ukupnog stava ispitanika o matematici u odnosu na opće kategorije.

Promatrajući stav ispitanika u dvije države, uočavamo da ispitanici u Hrvatskoj pokazuju nešto pozitivniji stav o matematici ($M = 3,65$) od ispitanika u BiH

($M = 3,49$). Kako bi provjerili statističku značajnost te razlike, proveli smo t-test značajnosti razlika srednjih vrijednosti rezultata na ljestvici stavova o matematici između ispitanika iz BiH i Hrvatske koji je pokazao da je ta razlika statistički značajna ($t = -3,478$; $p < 0,01$).

Ovaj bi rezultat mogao biti posljedica nedavnog reformiranja školstva u Hrvatskoj. Naime, u Republici Hrvatskoj je od 2006. godine primijenjen HNOS, a u okviru njegove implementacije održavani su brojni seminari, predavanja i radionice u kojima su se učitelji pripremali za osuvremenjivanje pristupa u nastavi matematike. Reforme sustava obično povlače pojačano financiranje sustava, bolju materijalnu opremljenost, uvođenje modernijih pedagoških standarda, ali usmjerenje pažnje šire društvene javnosti na prosvjetni sektor. Možemo pretpostaviti da reformiranje sustava povećava interes učitelja za osuvremenjivanje vlastite nastave, budi entuzijazam u radu i općenito poboljšava njihov stav.

Pomotrimo li stav ispitanika u odnosu na njihovu stručnu spremu, primjećujemo da ispitanici s višom stručnom spremom iskazuju negativniji stav ($M = 3,52$) od ispitanika s visokom stručnom spremom ($M = 3,68$). Značajnost razlika srednjih vrijednosti rezultata na ljestvici stavova između ove dvije skupine ispitanika ispitati smo t-testom i pokazali da je uočena razlika statistički značajna ($t = -3,197$; $p < 0,01$). Izračun Pearsonova koeficijenta korelacije između ukupnog stava ispitanika i njihove stručne spreme također pokazuje postojanje male, ali značajne povezanosti ($r = 0,206$; $p < 0,01$). Možemo stoga zaključiti da veća stručna sprema učitelja uglavnom ukazuje i na njihov pozitivniji stav prema matematici. Ta činjenica može biti posljedica bolje educiranosti visokoobrazovanih učitelja, a bolja educiranost o nekoj pojavi ima za posljedicu i pozitivniji stav o njoj. U svakom slučaju ovaj rezultat potvrđuje važnost i nužnost produljavanja obrazovanja učitelja razredne nastave na 5 godina, kao i uvođenja obaveze cjeloživotnog obrazovanja kroz sustav licenca koje uvode i Hrvatska i BiH.

Usporedimo li srednje vrijednosti stavova ispitanika o matematici u odnosu na duljinu njihova radnog staža, iz Tablice 4 vidimo da među njima neke uočljive pravilnosti. Najveća je vrijednost na ljestvici stavova dobivena u skupini učitelja početnika (do 5 godina staža) ($M = 3,63$), najmanja kod učitelja s preko 30 godina radnog staža ($M = 3,41$), ali u daljnjoj razdiobi ne vidimo neku očitu pravilnost. Kako bi ipak utvrdili značajnost tih razlika između pet podskupina ispitanika s različitim duljinom staža, proveli smo analizu varijance koja je potvrdila da te razlike ne možemo smatrati statistički značajnima ($F = 2,365$; $p > 0,05$).

Zaključci

Uloga matematike u uvjetima suvremenog življenja postaje sve značajnija, kako za pojedinca, tako i za društvo u cjelini. "Potreba za razumijevanjem matematike i njezinim korištenjem u svakodnevnom privatnom i profesionalnom životu nikada nije bila veća, a u budućnosti će se ta potreba nastaviti i još više rasti" (NCTM, 2000, 4). Upravo zato matematičko obrazovanje na svim nivoima mora učenicima osigurati kvalitetna i korisna matematička znanja i kompetencije s kojima će se moći uključiti u dinamičan i nepredvidiv svijet sutrašnjice.

Loši rezultati učenika na svim nivoima obrazovanja navode nas na preispitivanje svih parametara koji imaju utjecaj na rezultate matematičkog obrazovanja, a jedan od najznačajnijih je upravo učitelj ili nastavnik koji poučava matematiku. Područje našeg interesa je prvenstveno početna nastava matematike koja se realizira u razrednoj nastavi, pod vodstvom učitelja razredne nastave. "Naime, uloga učitelja je nezamjenjiva u nastavi matematike, posebno u početnoj nastavi, a samim tim je nezamjenjiv i njegov utjecaj i prinos uspjehu odnosno neuspjehu nastave" (Pejić, 2003, 9). Učitelj planira i vodi nastavni proces, odabire probleme na kojima će učenici raditi, usmjerava komunikaciju i vodi učenike prema konceptualnom razumijevanju matematike. Učitelj utječe na učenike na mnogo eksplicitnih i implicitnih načina. "Uvjerenja, znanje, prosudbe, razmišljanje i odluke učitelja imaju dubok učinak i na način na koji poučavaju i na proces učeničkog učenja. Uz to, učiteljska uvjerenja, razmišljanja, prosudbe, znanja i odluke utječu i na način na koji oni sami doživljavaju i razmišljaju o poučavanju novih kurikulumskih sadržaja koje dobivaju kao i o širini implementacije u vježbanju i poučavanju" (Peterson i dr., 1989, 2).

Odnos učitelja prema matematici u velikoj mjeri utječe na uspješnost učenika i njihov stav o matematici. Učitelj razredne nastave radi s djecom u veoma osjetljivim godinama i to u kontinuitetu od 4 ili 5 godina. U radu smo istražili odnos učitelja razredne nastave prema matematici, te zaključili da dobivene rezultate ne možemo smatrati potpuno zadovoljavajućim. Naime, pokazalo se da učitelji imaju brojne predrasude prema matematici, a te predrasude zasigurno utječu i na njihov stav prema učenicima, njihovim rezultatima i cjelokupnom nastavnom procesu. Stoga trebamo težiti da te predrasude učitelja osvijestimo i mijenjamo, što će dovesti do podizanja njihova općeg stava prema matematici. Utjecaj na njihov stav moguće je ostvariti kroz obrazovanje budućih učitelja na učiteljskim fakultetima, ali i kroz njihov cjeloživotni, profesionalni razvoj i unaprjeđivanje.

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1	2	3	4	5
uopće se ne slažem	uglavnom se ne slažem	niti se slažem, niti ne slažem	uglavnom se slažem	u potpunosti se slažem

Učenje matematike je zadovoljstvo.	1	2	3	4	5
Matematiku ne može svatko uspješno svladati.	1	2	3	4	5
Matematika je usko povezana sa stvarnim životom.	1	2	3	4	5
Inteligentna su djeca uspješna u matematici.	1	2	3	4	5
Osnovna je svrha matematike naučiti računati.	1	2	3	4	5
Matematika je zabavna i kreativna.	1	2	3	4	5
Ili jesi, ili nisi za matematiku.	1	2	3	4	5
Matematika je svakome u životu jako korisna.	1	2	3	4	5
Matematika je jako težak predmet.	1	2	3	4	5
Ja volim matematiku.	1	2	3	4	5

Učitelj i udžbenik u nastavi geometrije

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Sažetak. Ovaj rad se bavi pitanjima vezanim uz ulogu nastavnika, kurikuluma i udžbenika u nastavi geometrije. U nastavi matematike događa se susret “dviju matematika” – matematike kao znanstvene discipline, te svakodnevne matematike bliske učenicima. Stoga je posebno značajna uloga nastavnika kao stručnjaka-posrednika između ovih dviju kultura (Prediger, 2004). Nastava geometrije je pritom naročito zanimljiva za analizu, jer geometrijski problemi objedinjuju kako praktične tako i apstraktne ideje.

Stoga se postavlja pitanje na koji način se taj susret matematike kao znanosti i svakodnevne matematike događa u nastavi geometrije u Hrvatskoj. Istraživanje matematičkih udžbenika može djelomično odgovoriti na to pitanje. U ovom članku prikazani su rezultati istraživanja geometrijskih sadržaja u matematičkim udžbenicima viših razreda osnovne škole.

Rezultati pokazuju da u zadacima iz udžbeničkih kompleta prevladavaju zahtjevi operiranja, a da su u manjini zadaci koji traže argumentiranje i refleksiju o geometrijskim konceptima. Problem povezivanja svakodnevne matematike s gradivom je potenciran činjenicom nedovoljnog broja zadataka s realističnim i autentičnim kontekstom. Stoga se na temelju rezultata diskutira o ulozi udžbenika, nastavnika i kurikuluma u interkulturalnoj perspektivi nastave geometrije.

Ključne riječi: analiza, nastava geometrije, učitelj, udžbenik, interkulturalna perspektiva

Uvod

O širokoj upotrebi udžbenika u nastavi matematike svjedoče mnoga istraživanja (Pepin i Haggarty, 2001.). Istraživanje od Glasnović Gracin i Domović (2009) pokazuje veliku primjenu matematičkih udžbenika u višim razredima osnovne škole u Republici Hrvatskoj. Ispitani nastavnici koriste udžbenički komplet za pripremu nastave, dok ga učenici najviše koriste za vježbanje zadataka i za domaću zadaću. Nastavnik pritom u nastavi ima ulogu medijatora između udžbeničkog teksta i učenika. Ovi nalazi se odnose na cjelokupnu nastavu matematike, pa tako i na nastavu geometrije.

Nastava geometrije je od davnina bila sastavni dio poučavanja matematike. Najstariji poznati matematički papirusi odnose se većinom na geometrijske probleme koji su nastali iz svakodnevnih potreba. Uostalom, sam naziv “geometrija” otkriva potrebu za mjerenjem zemljišta, tj. primjenu matematike u svakodnevici. S druge strane, starogrčka matematika utemeljuje jedan drugačiji pogled na geometriju – apstraktan, deduktivan i filozofski orijentiran pristup geometrijskim temama koji je daleko od direktne primjene za svakodnevni život. Taj pogled se najbolje očituje u aksiomatskom pristupu prikazanom u Euklidovim Elementima. Euklidovi Elementi su bili najvažniji izvor za učenje geometrije u Europi u Srednjem vijeku, ali i kasnije u nekim europskim školama i učilištima (Dadić, 1982.; Jembrih, 2008.).

Možemo stoga primijetiti dvojaku snagu geometrije u nastavi – geometrija koja izvire iz potreba svakodnevnog života (svakodnevna geometrija), te geometrija koja teži u strogu apstrakciju (geometrija kao dio matematike kao znanstvene discipline). Oba aspekta trebaju biti prisutna u nastavi geometrije, prožimati se i povezivati se gdje god je to moguće. Udžbenik i učitelj matematike pritom imaju važnu ulogu. Povezivanje svakodnevne geometrije s geometrijom kao strogom znanstvenom disciplinom odnosi se na tzv. interkulturalnu perspektivu u nastavi matematike (Prediger, 2004).

Interkulturalna nastava matematike

U svojoj knjizi “Mathematiklernen in interkultureller Perspektive” Susanne Prediger (2004) govori o odnosu matematike kao znanosti, nastave matematike i svakodnevne kulture. Autorica nastavu matematike vidi kao interkulturalni proces između svakodnevne kulture i matematike kao znanstvene kulture. Primjena matematike u realnim i autentičnim situacijama na nastavi doprinosi njihovom povezivanju. Heymannov koncept “*matematike kao inventara svijeta u kojem živimo*” (“Mathematik als Inventar der Lebenswelt”, Heymann, 1996.) se podudara s ovom idejom. Dodajmo da se između svakodnevne i matematike kao znanosti stvorila još jedna “kultura”, a to je kultura nastave matematike koja ima dugu tradiciju i koja također ima svojih osobitosti, svoj specifičan jezik i sl.

Između ostaloga, primjena matematike u svakodnevici je jedna od zadaća svih današnjih matematičkih kurikuluma. Tako i Nastavni plan i program iz 2006. naglašava matematiku kao predmet primjenjiv u svakodnevici:

“Nastavni program naglašava primjenu matematike na koju treba gledati i kao na praktični, koristan predmet koji učenici moraju razumjeti i mogu znati primijeniti na razne probleme u svojem okružju. (MZOS, 2006, str. 238.)

Također, Nacionalni okvirni kurikulum (2010.) spominje cilj osposobljavanja učenika za primjenu matematike u različitim kontekstima.

No, budući da matematička kultura nema svoje geografsko mjesto, uloga učitelja i udžbenika je izuzetno važna u povezivanju matematičkog znanja koje učenik donosi od kuće i onog koje treba naučiti. To se posebice odražava u

nastavi geometrije gdje postoji jak imperativ za primjenom u svakodnevici s jedne strane, i jaka tradicija poučavanja apstraktnih geometrijskih sadržaja s druge strane. Možemo reći da su u tom procesu udžbenik i učitelj izuzetno važni faktori koji utječu na “uspješnost susreta” svakodnevne i matematičke kulture. Stoga je zanimljivo istražiti na koji način matematički udžbenici pristupaju temama iz svakodnevne matematike, te koja je pritom uloga nastavnika matematike. Sljedeća dva poglavlja uključuju rezultate i diskusiju o ulozi udžbenika i učitelja u interkulturalnoj nastavi geometrije.

Udžbenik u nastavi geometrije

Nastavni plan i program (MZOS, 2006., str. 238.) spominje važnost poimanja prostornih odnosa i objekata u nastavi matematike, važnost primjene matematike u svakodnevici te korištenja modernih tehnologija u nastavi. Iste ciljeve naglašava i Nacionalni okvirni kurikulum (2010.). Stoga je zanimljivo istražiti koje zahtjeve pred učenike stavljaju zadaci u matematičkim udžbenicima u Hrvatskoj.¹ Analizirani su svi zadaci i primjeri iz udžbeničkih kompleta triju najzastupljenijih udžbenika za 6. razred u školskoj godini 2005./06., triju najzastupljenijih udžbenika za 7. razred u 2006./07. godini, te triju najzastupljenijih udžbenika za 8. razred u školskoj godini 2007./08. Više od 80% ukupne učeničke populacije u svakoj spomenutoj generaciji koristilo je jedan od tih udžbenika². Svaki zadatak i primjer iz udžbenika analizirao se iz više aspekata. Prvi cilj je bio saznati koje kompetencije se traže od učenika u određenom zadatku: traži li se prikazivanje (crtanje, modeliranje), traži li se računanje (ili neko drugo operiranje po danom pravilu), traži li se interpretacija dobivenog matematičkog prikaza, ili pak argumentiranje. Također, zanima nas traži li određeni zadatak od učenika direktnu primjenu nekog pravila, formule ili definicije; ili se u zadatku možda traži znanje povezivanja različitih pojmova i aktivnosti; ili se pak traži refleksivno znanje koje nije direktno vidljivo iz nekog pravila i sl. Kompetencija refleksije obuhvaća promišljanje o matematičkim odnosima koji nisu direktno vidljivi iz zadatka, promišljanje o matematičkim modelima, interpretacijama, argumentacijama i dokazima u određenom kontekstu. Ova sistematizacija o kompetencijama preuzeta je iz Austrijskih obrazovnih standarda koji su se pokazali kao dobra podloga za istraživanje (IDM, 2007.). Analiza je također obuhvaćala vrstu pitanja iz udžbenika, tj. radi li se o pitanju zatvorenog ili otvorenog tipa. Interkulturalni zahtjevi analizirali su se promatranjem konteksta zadatka: radi li se o čistom simboličkom (unutarmatematičkom) zadatku bez konteksta, o realističnom ili autentičnom kontekstu. Realistična situacija je u ovom istraživanju definirana kao situacija koja vjerno imitira stvarni život, dok je autentična situacija zaista uzeta iz stvarnih (autentičnih) podataka. Primjer zadatka s *realističnom* situacijom: “Ivan ima monitor na računalu s duljinama stranica 33 cm i 24 cm. Kolika je površina njegovog ekrana?”. Primjer zadatka

¹ Analiza je provedena u sklopu doktorskog istraživanja autorice na Institutu za didaktiku matematike na Aplen-Adria Universität-u u Klagnefurtu.

² Udžbenicima su dodijeljeni kodovi A, B i C, a čitatelji zainteresirani za original mogu se obratiti autorici.

s *autentičnom* situacijom: “Izmjeri duljine stranica svog monitora ili televizora. Izračunaj površinu ekrana.”

Osnovnoškolska nastava geometrije od šestog do osmog razreda obuhvaća teme: trokut, četverokut, mnogokut, kružnicu i krug, sukladnost, sličnost, izometrije, Pitagorin poučak, odnose u prostoru te geometrijska tijela (MZOS, 2006.). Ovdje će biti prikazani rezultati istraživanja nekih od nabrojanih tema.

Rezultati analize udžbenika o matematičkim zahtjevima za cjelinu Trokut u šestom razredu pokazuju značajnu razliku u broju zadataka i primjera u analiziranim udžbenicima (Tablica 1). Naime, broj u zagradi u prvom retku tablice označava ukupan broj zadataka i primjera koji se stavljaju pred učenika unutar određene cjeline, a postoci prikazuju postotni udio određene aktivnosti u ukupnom broju. Tako u cjelini Trokut u udžbeničkom kompletu A6 ima ukupno 568 zadataka i primjera, zatim njih 677 u B6, te 837 zadataka i primjera u udžbeniku C6 (Tablica 1).

U prva četiri retka tablice nalaze se aktivnosti prikazivanja, računanja, interpretiranja i argumentiranja. Obzirom da jedan zadatak može tražiti od učenika više od jedne aktivnosti (npr. i modeliranje i računanje), napomenimo da zbroj po aktivnostima ne mora davati ukupno 100%. Rezultati analize pokazuju da u cjelini Trokut dominiraju aktivnosti računanja i operiranja u svim istraženim udžbenicima (Tablica 1). Računanje se odnosi na računanje opsega i površine trokuta, dok se operiranje uglavnom odnosi na izvršavanje osnovnih koraka konstrukcija trokuta. Zahtjevi argumentiranja, refleksije i zadaci otvorenog tipa nisu uopće ili nisu značajno zastupljeni niti u jednom analiziranom udžbeničkom kompletu u cjelini Trokut.

Trokut	A6 (568)	B6 (677)	C6 (837)
Prikazivanje, modeliranje	38%	30%	46%
Računanje, operiranje	59%	68%	58%
Interpretiranje	29%	41%	37%
Argumentiranje	0%	3%	5%
Direktna primjena pravila, reproduksijsko znanje	49%	50%	48%
Znanje povezivanja	50%	49%	52%
Refleksija	1%	1%	0%
Pitanja zatvorenog tipa	98%	97%	96%
Unutarmatematički kontekst	98%	97%	99%
Realističan kontekst	2%	3%	1%
Autentičan kontekst	0%	0%	0%

Tablica 1. Zahtjevi u cjelini Trokut (6. razred).

Rezultati također pokazuju da u zadacima istraženih udžbenika dominiraju unutarmatematički zadaci, tj. zadaci koji nisu stavljeni u kontekst. Slični rezultati dobiveni su za cjeline Četverokuti i Mnogokuti. Nedostatak zadataka koji od učenika zahtijevaju refleksiju i argumentiranje o određenoj matematičkoj temi primjećuje

se i u cjelini Kružnica i krug (Tablica 2), kao i dominacija pitanja zatvorenog tipa s unutarmatematičkim kontekstom. Uz to, primjećujemo da računanje i operiranje dominiraju nad prikazivanjem (crtanjem) i interpretiranjem. Ovaj nalaz možemo povezati s temama Opseg i Površina kruga u kojima ipak dominiraju zahtjevi računanja uz primjenu odgovarajućih formula. S ovim rezultatom možemo povezati i dominaciju reprodukcijских задатака, osobito u udžbenicima A7 i C7 (Tablica 2).

Kružnica i krug	A7 (753)	B7 (306)	C7 (381)
Prikazivanje, modeliranje	25%	27%	26%
Računanje, operiranje	62%	71%	59%
Interpretiranje	22%	10%	28%
Argumentiranje	0%	5%	3%
Direktna primjena pravila, reprodukcijско znanje	72%	45%	58%
Znanje povezivanja	28%	55%	40%
Refleksija	0%	0%	2%
Pitanja zatvorenog tipa	100%	94%	93%
Unutarmatematički kontekst	96%	92%	79%
Realističan kontekst	3%	6%	17%
Autentičan kontekst	1%	2%	4%

Tablica 2. Zahtjevi u cjelini Kružnica i krug (7. razred).

Jedno od važnijih geometrijskih područja koje proizlazi iz svakodnevnih potreba je svakako gradivo Pitagorinog poučka. S druge strane, to gradivo učenike usmjerava i prema matematici kao znanstvenoj disciplini zbog dokazivanja Pitagorinog poučka, pravila za Obrat Pitagorinog poučka i slično. U Tablici 3 prikazani su rezultati udžbeničkih zahtjeva u zadacima iz Pitagorinog poučka. U sva tri analizirana udžbenička kompleta primjećujemo dominaciju računskih zadataka. Ti se zadaci odnose na primjenu formule za Pitagorin poučak. Prikazivanje i interpretiranje zastupljeno je u manjoj mjeri, dok argumentiranje, refleksija i pitanja otvorenog tipa gotovo uopće nisu zastupljeni u toj cjelini. S druge strane, u sva tri istražena kompleta dominiraju zadaci u kojima se traži znanje povezivanja. Zanimljivo je primijetiti da naglasak u udžbenicima nije stavljen na zadatke s kontekstom, iako se tema Pitagorinog poučka može lako primijeniti na pravokutne trokute iz svakodnevice.

Pitagorin poučak	A8 (797)	B8 (474)	C8 (723)
Prikazivanje, modeliranje	8%	8%	7%
Računanje, operiranje	92%	89%	95%
Interpretiranje	13%	13%	13%
Argumentiranje	0%	4%	2%
Direktna primjena pravila, reprodukcijско znanje	41%	45%	37%
Znanje povezivanja	59%	55%	63%

Refleksija	0%	0%	0%
Pitanja zatvorenog tipa	100%	96%	98%
Unutarmatematički kontekst	91%	89%	98%
Realističan kontekst	9%	8%	2%
Autentičan kontekst	0%	3%	0%

Tablica 3. Zahtjevi u cjelini Pitagorin poučak (8. razred).

Osim planimetrijskih tema, svakako unutar nastave geometrije treba spomenuti i nastavu stereometrije. Na početku prvog razreda uče se raspoznavati osnovna geometrijska tijela, zatim se u 3. razredu obrađuje mjerenje obujma tekućine (s naglaskom na mjerenje), pa u četvrtom razredu dolazi gradivo vezano uz osnovne elemente kvadra i kocke (MZOS, 2006). U višim razredima osnovne škole geometrija prostora dolazi tek na kraju osmog razreda – prvo se učenici upoznaju s pravcima, točkama i ravninama u prostoru i njihovim međusobnim odnosima, a zatim i geometrijskim tijelima te formulama za njihovo oplošje i obujam.

U Tablici 4 prikazani su udžbenički zahtjevi iz cjeline Geometrijska tijela u osmom razredu. Rezultati pokazuju dominaciju aktivnosti računanja u pitanjima zatvorenog tipa i bez konteksta. Oko trećine zadataka u svakom kompletu od učenika je tražilo i aktivnosti interpretiranja. Interpretiranje se uglavnom odnosi u ovom slučaju na tumačenje skice ili crteža sa ciljem rješavanje zadatke. Primjerice, to može značiti traženje relevantnog pravokutnog trokuta na skici piramide kako bi se mogla pronaći nepoznata veličina potrebna za daljnja računanja. Zato je u zadacima s geometrijskim tijelima prisutan prilično velik postotak zadataka koji traže znanje povezivanja. Možemo reći da su zadaci s računanjem oplošja i obujma raznih pravilnih tijela vrlo pogodni za sastavljanje zadataka s kontekstom, bilo onih realističnih ili pak autentičnih. No, ipak takvi zadaci nisu zastupljeni u velikom postotku u našim udžbenicima matematike.

Geometrijska tijela	A8 (1048)	B8 (913)	C8 (726)
Prikazivanje, modeliranje	4%	4%	3%
Računanje, operiranje	86%	84%	92%
Interpretiranje	32%	30%	33%
Argumentiranje	0%	0%	1%
Direktna primjena pravila, reproduksijsko znanje	59%	59%	34%
Znanje povezivanja	41%	39%	65%
Refleksija	0%	2%	1%
Pitanja zatvorenog tipa	100%	96%	97%
Unutarmatematički kontekst	89%	75%	94%
Realističan kontekst	8%	19%	5%
Autentičan kontekst	3%	6%	1%

Tablica 4. Zahtjevi u cjelini Geometrijska tijela (8. razred).

Problem da se geometrijska tijela u razrednoj nastavi rade na nivou prepoznavanja, a zatim tek na kraju osmog razreda s jakim naglaskom na računanje, može utjecati na razvijanje kompetencije prostornog zora kod učeničke populacije u Hrvatskoj. Primjerice, Čizmešija i Milin Šipuš (2009) prikazuju problem s usvajanjem prostornog zora na populaciji studenata nastavnčkih smjerova matematike.

Dobiveni rezultati sugeriraju da gradivom geometrije dominira računanje u čistom matematičkom okruženju bez konteksta. Također, rezultati pokazuju manjak geometrijskih zadataka u kojima se od učenika traže sposobnosti argumentiranja. Mnogi postojeći zadaci mogli bi se dopuniti dodatnim zadatkom otvorenog tipa poput "Objasni" i sl. No, kompetencije refleksije, zadataka otvorenog tipa i argumentiranja o matematičkim temama i nisu propisane zadaćama i ciljevima u tekućem Nastavnom planu i programu, već su u planu naglašeni zahtjevi računanja i operiranja (MZOS, 2006). Analiza pokazuje da udžbenici prate ove zahtjeve Nastavnog plana i programa.

S druge strane, Nastavni plan i program naglašava primjenu geometrijskih sadržaja u svakodnevici, dok istraživanje pokazuje da je većina geometrijskih zadataka u udžbenicima unutarmatematičkog karaktera i bez konteksta. Dakle, uočavamo nesklad između kurikulumskih zadaća i udžbeničkih sadržaja u pogledu povezivanja gradiva s geometrijskim problemima iz realnosti.

Učitelj u nastavi geometrije

Tradicionalni pristup u Nastavnom planu i u udžbenicima odražava se na stavove i aktivnosti učitelja u nastavi. U istraživanju od Glasnović Gracin i Domović (2009.) ispitani učitelji matematike su pokazali da sadržaji iz udžbenika direktno utječu na njihovu nastavnu praksu. Pritom učitelj ima važnu ulogu kao posrednik između udžbenika i učenika. U istraživanju od Baranović i Štibrić (2009.) pokazano je da ispitani osnovnoškolski i srednjoškolski nastavnici u Hrvatskoj u svojoj nastavi naglasak stavljaju na rješavanje zadataka, *"a manje razvijaju potrebu matematike u svakodnevnom životu i razvoj kritičkog promišljanja o matematičkim konceptima i postupcima"* (ibid, str. 202.). Ovaj nalaz se podudara s deficitom interkulturalnih i refleksivnih zahtjeva u udžbenicima.

Zaključujemo da postupci učitelja ovise o kurikulumskim i udžbeničkim zahtjevima, ali uvijek treba imati na umu i snažan utjecaj osobnih stavova i mišljenja učitelja (Leder, 2002.). Važno je prvo istražiti kakav stav nastavnici matematike imaju prema matematici samoj, jer se mišljenje nastavnika prema matematici kao znanosti reflektira i u njihovoj nastavnoj praksi (Cross, 2009.). Ako nastavnik ima stav da je matematika prvenstveno skup procedura, računanja i operacija, i njegova nastavna praksa će slijediti ta uvjerenja. Također treba imati na umu da se stečeni stavovi sporo mijenjaju. Reforme za promjenom nastave neće uroditi plodom ako se ne pokuša utjecati na promjenu tih stavova. To je dug proces s kojim bi svakako trebalo započeti na sveučilišnim studijima.

Konkretno, u nastavi geometrije učitelj bi pridonio interkulturalnosti i refleksivnom znanju aktivnostima poput primjene autentičnih zadataka i projekata, te

poticanjem obrazlaganja i argumentiranja o geometrijskim konceptima. Također, potencijali računala, posebice softvera dinamične geometrije, bi se na ovom mjestu mogli iskoristiti u istraživačkoj nastavi za cjelovito shvaćanje geometrijskih ideja (Schneider, 2002.). Uz to, učitelj koji poznaje svoj razred najbolje može procijeniti i pridonijeti uspješnosti diferencirane nastave u kojoj se opet mogu ogledati kako interkulturalnost tako i naglasak na ideji umjesto rutini i slično.

Zaključak

Uvid u nastavni plan i program (2006.) pokazuje da u geometriji dominiraju zahtjevi računanja i operiranja, što se podudara sa zahtjevima iz istraženih matematičkih udžbenika u šestom, sedmom i osmom razredu. Ovi nalazi se također podudaraju s rezultatima ankete o tradicionalnim karakteristikama matematičke nastavne prakse (Baranović i Štibrić, 2009.). S druge strane, Nastavni plan i program jasno naglašava primjenu matematike u svakodnevnici, ali analiza udžbenika pokazuje da zadaci s kontekstom ipak nisu značajno zastupljeni u geometrijskim poglavljima udžbenika. Stoga možemo zaključiti da postoji nepodudarnost između kurikulumskih i udžbeničkih zahtjeva na interkulturalnom nivou.

Dobiveni nalazi sugeriraju bolje povezivanje svakodnevnog okruženja s apstraktnim geometrijskim sadržajima, poticanje argumentiranja i objašnjavanja, refleksivnog znanja te razumijevanja geometrijskih koncepata u cijeloj obrazovnoj vertikali. U tom pogledu novi Okvirni kurikulum (MZOS, 2010.) donosi pomake u odnosu na postojeći Nastavni plan i program (MZOS, 2006.). Osim kurikulumskog nivoa, interkulturalnu i refleksivnu dimenziju treba poticati i na nivou sveučilišnih programa za buduće nastavnike matematike, u nastavnoj praksi, kao i u području razvoja udžbenika.

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Matematika u ulozi izgradnje dobro obrazovane ličnosti

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Sažetak. U radu se stavlja u fokus uticaj matematike na izgradnju dobro obrazovane i poželjne ličnosti. Provedeno je istraživanje koje je pokazalo kako studenti gledaju na te karakteristike, nakon završetka općeg obrazovanja. Nakon sagledavanja svih aspekata nastave matematike na formiranje dobro poželjnih karakteristika ličnosti, u rezultatima je konstatovano da je nastava matematike “kao” stvorena za glavnog aktera u formiranju dobro obrazovane ličnosti. Kao i kod promjena, i ovdje nema recepata, imaju samo dobra iskustva nastavnika od kojih zavisi, u kojoj će mjeri nastava matematike ostaviti traga na formiranje mlade poželjne ličnosti za 21. stoljeće. Na kraju su navedene preporuke nastavnicima kako da nastavu matematike učine interesantniju i učinkovitiju u misiji izgradnje karakteristika poželjne mlade osobe.

Ključne riječi: dobro obrazovana mlada osoba, metodika nastave matematike, poželjan nastavnik matematike

Uvod

U vrijeme velikih kurikularnih promjena, ne samo u Bosni i Hercegovini, nego i u svijetu, kada se pišu novi standardi i mijenjaju programi stari desetljećima, treba dobro razmisliti kakva bi trebalo biti nastava matematike u današnjem vremenu. Svjetska istraživanja i studije iz područja metodike nastave matematike donose važne zaključke o tom pitanju. Ti zaključci utieću na promjene kurikuluma u zemljama u kojima ti stručnjaci djeluju. U tom smislu su se zemlje Evropske unije usaglasile o tome kakvu oni osobu zamišljaju kao obrazovanu i poželjnu za 21. stoljeće. Usaglašeno je da su slijedeće osobine ili ishodi koje se priželjkiju nakon završetka općeg obrazovanja. Po njima je dobro obrazovana mlada osoba:

- nezavisan učenik i donositelj odluka,
- ima dobre odnose sa vršnjacima i odraslim,
- pismen i uspješan u komunikaciji,
- prilagodljiv,

- avanturista i voljan isprobavati nove stvari,
- kooperira kao dio tima,
- ima osjećaj za odgovornost i disciplinu,
- moralno i duhovno osviješten,
- u mogućnosti je da funkcioniše kao dio tima,
- pripremljen na izazove koje mu društvo nudi,
- Tolerantan je i prevazilazi stereotipe,
- ima osjećaj blagostanja i može voditi siguran i ispunjujući život,
- preduzimljiv je učenik, koji je u mogućnosti da preuzme kontrolu i odgovornost za aspekte sopstvenog učenja.

Ovi zahtjevi, bolje reći želje, su potaknute promjenama u društvu u kojem živimo u kome se mora mijenjati i sistem obrazovanja, gdje nije pošteđena ni nastava matematike. Ona je dugo smatrana za kraljicu među naukama. Međutim, to ne znači da nije podložna promjenama koje zadiru u koncepte, pristupe, naročito one metodičke. Dugo se *“metodika nastave matematike u mnogim zemljama i matematičkim krugovima još ne prihvata kao znanstvena disciplina, ne ulaže se u istraživanja iz ovog važnog područja, ne postoje svugdje postdiplomski znanstveni studiji iz metodike nastave matematike koji bi bili rasadnici novih stručnjaka i garancija zauzimanja za primjerenu satnicu i kvalitetne planove u cijeloj obrazovnoj vertikali”* (Glasanović Gracin, 2010). Tu se stvari moraju mijenjati, jer novi trendovi kazuju da nastava matematike opisuje kao nastava orijentirana prema učenicima što znači da se dosadašnja dominantna uloga nastavnika stavlja u drugi plan, a povećava se učenikova aktivnost u nastavi matematike, kroz razne oblike rada, učenje u parovima, grupnom radu, radu na projektima, učenjem kroz istraživanje i td. Time nastavnik nije više u poziciji glavnog aktera prenosnika znanja, već postaje koordinator i organizator nastavnog procesa ili, kako kaže Delors on proces *“vodi, a ne oblikuje ga”* (Delors, 1998:162). Mora se shvatiti da je obrazovna usluga najsuptilnija i najvažnija u svijetu usluga. Potrebno je raskrstiti sa stereotipima o odnosima između nastavnika i učenika. Kotler (1991) kaže da, *“mi [nastavnici] kupcu [učeniku] ne pružamo zadovoljstvo time što ga uslužujemo, nego on nama čini zadovoljstvo što nam daje priliku da ga uslužimo”* (Kotler 1991:23). Prosvjetarima treba biti jasna njihova misija, koja se ne mijenja stoljećima, a to je: odgajati, obrazovati i jačati učenike-buduće nosioce društvenog razvoja.

U ovom radu je dat osvrt na gornje zahtjeve koji krasi dobro obrazovanu osobu, te ulogu i mogućnosti matematike u njihovom dostizanju. Istovremeno, prikazano je istraživanje među studentima I godine fakulteta Univerziteta u Tuzli, o njihovom stepenu zadovoljstva o dostizanju, gore navedenih, ishoda pri završetku srednjoškolskog obrazovanja. Na osnovu vlastitih saznanja i iskustava ispitnika, te razmatranja raznih teoretičara, izvedene su preporuke sa smjernicama za nastavnike, koje govore o poboljšanju uloge nastave matematike u formiranju poželjne osobe za 21. stoljeće.

Paradigma i metodologija istraživanja

Cilj istraživanja u ovom radu je identifikovati postojeće stanje u učinkovitosti općeg obrazovanja u dostizanju općih poželjnih obrazovnih ciljeva, definisanih u zemljama EU, te dovesti ih u vezu sa primjenom različitih metoda i oblika rada u nastavi matematike.

Zadatak istraživanja u ovom radu je: utvrditi kako su studenti zadovoljni svojim općim obrazovanjem sa aspekta prihvatanja osobina dobro obrazovane ličnosti, te na osnovu tih istraživanja i teoretskih razmatranja raznih autora uvidjeti kakve su mogućnosti nastave matematike u dotizanju tih ciljeva – kreiranja poželjnih karakteristika dobro obrazovane osobe.

Shodno tome, postavlja se slijedeća **hipoteza**: “Nakon završetka općeg obrazovanja postižu se, u dovoljnoj mjeri, dobri rezultati u formiranju kompetencija poželjne ličnosti”.

Nastava matematike bi trebala u ovom postupku dati svoj veliki doprinos. Nadalje će se na osnovu teoretskih i praktičnih iskustava, istraživača i studenata, dati preporuke za učinkovitiju ulogu nastave matematike u kreiranju kompetencija dobro obrazovane ličnosti u srednjoškolskom obrazovanju.

Potrebno je bilo odrediti **“uzorak studenata**, koji će garantirati, na osnovu svojih stajališta, egzistenciju istih stajališta i mišljenja svih učenika poslije završetka srednje škole, “kao osnovnog skupa ili populacije.” (Mužić, 1979:534)

Uzorak učesnika u ovom istraživanju je bio namjerni (purposeful sample), jer su uzeti studenti prve godine nastavničkih fakulteta, njih 292, koji imaju istančaniji odnos prema nastavi i njenim ciljevima. Dakle, izabran je takav uzorak od koga se očekuje da će se “moći najviše saznati”, odnosno dobiti najviše korisnih informacija (Merriam, 1998:61).

U ovom istraživanju je korištena **anketa** (engl. survey) koju Mužić (1999:82) poistovjećuje sa “*postupkom istovremenog i pismenog prikupljanja podataka od većeg broja ljudi (ispitanika) o nečemu što oni znaju, osjećaju ili misle*”. Imajući u vidu da je nefunkcionalno i veoma skupo dobiti odgovore cijelog osnovnog skupa (svih studenata brucša), odlučeno je da se provede anketno istraživanje, kako bi se dobili odgovori na postavljena pitanja izabranog uzorka, kao podskupa osnovnog skupa “*kako bi mogli odrediti određenu distribuciju ponašanja, mišljenja i stavova u osnovnom skupu*” (Ristić, Ž. 2006:350).

Anketiranje je obavljeno sa anketnim upitnicima sa 11 pitanja sa Likertovom skalom i jednim otvorenim pitanjem, bolje reći mogućnosti, da studenti kažu što god žele, vezano za tretiranu problematiku. Dakle, upitnik pokriva 11 “želja” sa spiska zahtjeva za “dobro obrazovanu osobu”.

Ograničenja u istraživanju su povezana sa generalizacijom ili poopštavanjem. Uzorak u provedenom istraživanju je izabran spontano. Kao takav ne može se garantirati da je moguće poopštavanje na širu populaciju, odnosno na prostor Bosne i Hercegovine. To nije ni bila namjera istraživača, jer su prostori BiH različitim intenzitetom tretirani kako u provođenju reformi u obrazovanju, od segmenata

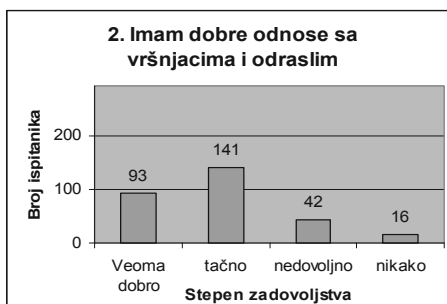
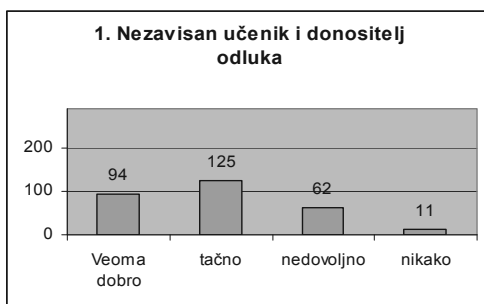
kurikuluma, tako i do osiguranja kvaliteta i profesionalnog i stručnog razvoja i jačanja, kako nastavnika, tako i školskih menadžment timova.

Rezultati istraživanja

Prikupljanje podataka je izvršeno pomoću upitnika. Rezultati za svako pitanje iz upitnika su predstavljeni grafički, gdje su korišteni alati iz Windows okruženja. Na osnovu toga i odgovora na otvoreno 12. pitanje, te na osnovu iskustava različitih autora, na kraju su dati prijedlozi za poboljšanje uloge nastave matematike u kreiranju kompetencija ličnosti, koja je, po mišljenju stručnjaka iz zemalja EU, “dobro obrazovana ličnost”.

Matematika i nezavisan učenik – donositelj odluka

Istraživanje je pokazalo da studenti 1. godine fakulteta, jučerašnji srednjoškolci, njih čak 75%, smatraju da su nakon završetka školovanja, postigli to da su postali nezavisni i sposobni ili veoma sposobni samostalno donijeti odluku. Njih 25% smatra da tu kompetenciju nisu dovoljno ili skoro nikako usvojili. Oni smatraju da se “*ipak, i poslije srednje škole, ne mogu donositi neke odluke samostalno, treba nam savjet od starijeg*” (iz upitnika), jer “*nakon završene srednje škole niko nije toliko sposoban da otpočne samostalno, da živi i preuzme brigu i odgovornost za sebe. Neizbježna je pomoć roditelja u svim segmentima, iako smo dosta ozbiljniji i pouzdaniji.*” (iz upitnika)

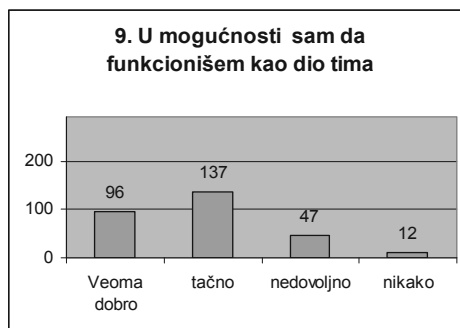
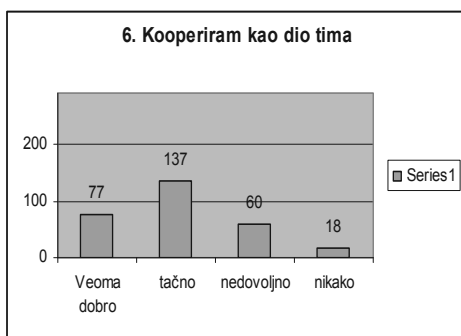


Grafikoni 1. Odgovori na 1. i 2. pitanje.

Rješavanje zadataka iz različitih oblasti matematike su pune mogućnosti samostalnog rada i donošenja odluka. Tradicionalna nastava matematike se uglavnom oslanja (la) na nastavnika, kao predavača, te učenike, koji potom samostalno rješavaju zadatke. Drugim riječima, učenik je imao pasivnu ulogu prilikom sticanja novog znanja. Jačanje učenika u samostalnom radu i istraživanju, rješavanju problema, izboru metoda učenja i prezentacije dostignuća, dovodi do stvaranja kod učenika osjećaja sigurnosti i sposobnosti samostalnog, nezavisnog i argumentovanog iznošenja stavova, što je u konačnici potrebno za donošenje pravih životnih i provodivih odluka, prihvatljivih od svih onih kojih se ona tiče.

Matematika i međuvršnjački odnosi i spremnost za timski rad

Tradicionalna nastava matematike je uglavnom bila bazirana na ovodu i glavnom dijelu, gdje nastavnik dominira, uz saradnju jednih te istih učenika, koji dopunjavaju prazan prostor, te završni dio časa koji se sastoji od kratkog ponavljanja pređenog gradiva, te zadavanja domaće zadaće i upućivanje na literaturu. Jednostavno, grupni rad se upotrebljavao samo u prilikama ogledmo-uglednih časova ili posjeta nastavnim satima od strane direktora ili savjetnika iz pedagoških zavoda. Neke studije su pokazale da je puno učinkovitije učenje vršnjaka od vršnjaka, putem grupnog rada. Svakako osim individualnog grupnog rada, te rada u parovima i frontalni pristup ima svoje mjesto u nastavi, zavisno od prirode materije i posebnih ciljeva nastavnih jedinica. No, ako individualni razvoj ima uticaj na razvoj nezavisnosti učenika, te osjećaja za pravilno i racionalno, ali i rutinsko odlučivanje (Agić i ost., 2006), onda je grupni rad “odgovoran” za razvoj osjećaja za timski rad, ali ne svakakav, nego učinkoviti timski rad, te za uspješnu komunikaciju i dobre odnose, kako sa vršnjacima, tako i sa odraslima. Grupni rad je naročito značajan za utvrđivanje gradiva (homogene grupe), ali i za časove obrade (heterogene grupe). Kao što vrijedi da je svaki specijalista ljekar, a da svaki ljekar nije specijalista, tako vrijedi i za grupe i timove. Svaki tim je grupa, ali svaka grupa nije tim (Belbin, 1996). Čestom upotrebom grupnog rada, sa jasnim ciljem, vještini poticajem od strane nastavnika, zasigurno dovodi do razvoja, gore navedenih, kompetencija, korisnih za život. Konzumiranje grupnog rada nije efektivno, ako se nema za cilj prelaska u timski rad, koji prema (Belbinu, 1996) kartakteriše: mala brojnost grupe, zajednički pristupaju zadatku, komplementarna znanja i vještine, predanost zajedničkoj svrsi, konkretni radni ciljevi, zajednička odgovornost, entuzijazam, zajedničko geslo i identitet, zajednički proživljeni događaji i uzajamna naklonost članova. Engleski znanstvenik **Belbin** (1996) dokazao je da su **timovi sastavljeni od najboljih stručnjaka u nekoj struci daleko manje uspješni nego timovi sastavljeni od vrlo različitih članova**. O tome se mora voditi računa kada se formiraju grupe, nikako dijeliti učenike po grupama, po principu dvije klupe jedna grupa, nego na osnovu kriterija da grupa sadrži različite tipove učenika sa aspekta njihovog poimanja učenja i nivoa njihovih mogućnosti.



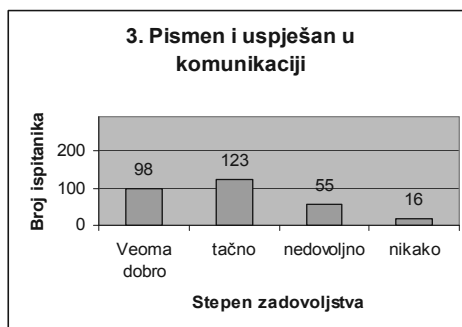
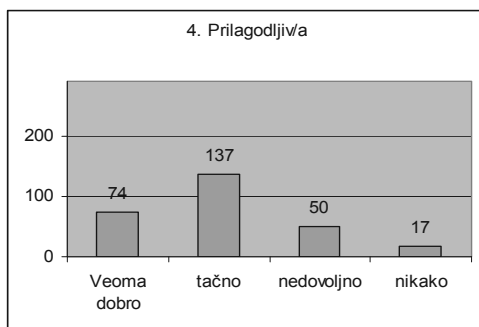
Grafikoni 2. Odgovori na 6. i 9. pitanje.

Istraživanje je pokazalo visok stepen slaganja ispitanika sa tvrdnjom da je “osnovno i srednje orazovanje pomoglo učenicima da **“izgrade dobre odnose sa**

vršnjacima i odraslim". Njih, čak 80% ima visoko i veoma visoko slaganje sa tom tvrdnjom. Ipak svaki 5-ti učenik ne misli tako. Iz upitnika se mogao uočiti stav: *"Kada sam ja u pitanju sa odgovornošću mogu da kažem da sam, nakon završetka srednje škole, postala nezavisan donositelj odluka i da sam razvila svoje odnose kako s vršnjacima tako i sa ljudima svih starosnih dobi i uspjela sam da funkcioniram"* (iz upitnika). Podatak da je preko 79% ispitanika u mogućnosti da funkcionira kao učinkovit član tima, može zavarati, jer je pitanje da li studenti, uopće, poznaju teoriju o timovima i timskom radu, jer u formalnom obrazovanju to nije na programu. Taj podatak upućuje na potrebu uvođenja u nastavni program (bar) nastavnih fakulteta sadržaje iz obrazovnog menadžmenta, tačnije oblasti *Human Resource Managemnt*, gdje spada i timski rad, potrebno je više pažnje posvetiti grupnom i timskom radu. Tu, svakako, treba dati podršku nastavnicima o upotrebi i značaju timova, kako bi mogli motivisati isto među svojim učesnicima prilikom realizacije redovnog kurikuluma. Van nastavne aktivnosti i rad sekcija, svakako pružaju više mogućnosti za organizaciju timskog rada. Ne snije se zaboraviti, sa timovima treba početi od najnižeg nivoa obrazovanja.

Matematika i pismenost te uspješna komunikacija

Uz uspješnu komunikaciju i donošenje dobrih racionalnih odluka, ruku pod ruku, ide pismenost, koja se u matematici itekako može afirmisati. Treba raskrstiti sa stereotipnim mišljenjima i starim definicijama matematičke pismenosti, koja polazi od sintakse i fonetike, a ne od matematičke retorike i razmišljanja, argumentovanog iznošenja stavova, diskusiji o raznim mogućnostima, ali ne, kako to mi radimo, pa i u nižim razredima osnovne škole, da diskutujemo rješenja jednačina ili nejednačina, nego razmatranje po sistemu "šta bi bilo kad bi bilo" i td. Istraživanje je pokazalo da su studenti, u velikoj mjeri, zadovoljni sa stečenom pismenosti u srednjoj školi. Njih više od 75% smatra da su zadovoljni ili veoma zadovoljni sa tom stečenom vještinom. No, ne može se zanemariti činjenica da je ostatak od 25% nije dovoljno (19%) ili nikako (6%) zadovoljno sa stečenom pismenosti nakon završetka srednješkolskog obrazovanja. Komunikacija i pismenost su u veoma uskoj vezi.



Grafikoni 3. Odgovori na 3. i 4. pitanje.

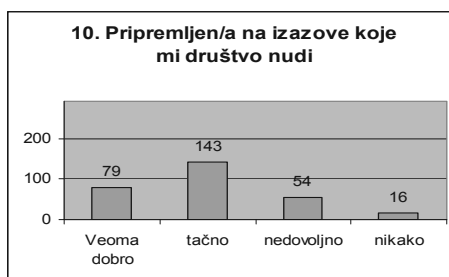
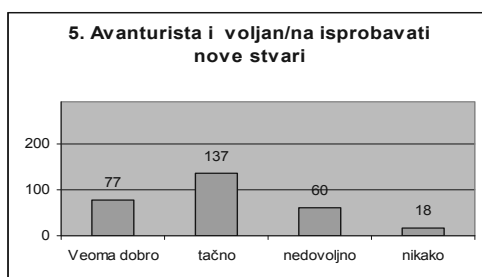
Jedan od ispitanika kaže: *"Ja sam uspješna u komunikaciji, ali mislim da nisam dovoljno pismena"* (iz anketnih upitnika). Kada je u pitanju matematička

pismenost, jasno je da ona prevazilazi značenje jezičke pismenosti i da ona ide dublje u razumijevanje problematike koja je podloga za efektivnije promišljanje, zaključivanje i u konačnici odlučivanje. OECD opisuje matematičku pismenost kao *“sposobnost pojedinca da prepozna i razumije ulogu koju matematika ima u svijetu, da donosi dobro utemeljene odluke i da primjenjuje matematiku na načine koji odgovaraju potrebama života tog pojedinca kao konstruktivnog, zainteresiranog i promišljajućeg građanina”* (Braš Roth i dr., 2008:124). Interesan je podatak dobiven u istraživanju, a koji se tiče prilagodljivosti. Velika većina studenata misli da su nakon završetka srednješkolskog obrazovanja, stekli u velikoj ili dovoljno velikoj mjeri, vještinu prilagodljivosti. Njih, čak 25% misli da su veoma prilagodljivi, dok je skoro 47% studenata zadovoljno sa tom stečenom vještinom. Pitanje je kakva je tu bila uloga matematike. Svakako, ne zanemarljiva. Opet se vraćamo na ulogu nastavnika, koji dosljedno primjenjuje principe primjerenosti, gdje se zna prilagoditi ne samo nivoima razvoja učenika, nego i njihovom trenutnom psihičkom stanju i specifičnim zahtjevima.

Matematika i spremnost na avanturizam i isprobavanje novih stvari

Nastava matematike je kao stvorena da kod dijeta razvije osjećaja za avanturizam i želju za isprobavanje novih stvari. Podobna metoda u ovom slučaju je učenje otkrivanjem, koje se odnosi na mogućnost da “učenici samostalno, kroz eksperimentiranje, dođu do novih spoznaja, ideja i rješenja problema” (Glasanović Gracin, 2010). Treba biti jasno da se u relizaciji nastave po heurističkim metodama zapostavlja naučnost i klasična usmjerenost na definicije i dokaze, ali to ne znači da ta nastava nije sistematski organizirana i da nije od koristi. Naprotiv, nastavnik treba dobro organizirati takav sat, poznavajući prikladne metode i mogućnosti, kako sopstvenih, tako i mogućnosti učenika. (npr. u upotrebi računara).

Dakle, istraživačka metoda i njena primjerena upotreba može kod učenika izazvati osjećaj za put u nepoznato, po sistemu “šta bi bilo kad bi bilo”. To je uslov da se kod mladog čovjeka stvore pretpostavke za spremnost za avanturizam, odnosno snalaženje sa iznenadnim i nepoznatim. Istraživanje je pokazalo da su studenti, nakon završetka škole, poprilično spremni na isprobavanje novih stvari (njih preko 73%) i na nove izazove (76%), što se vidi sa grafikona 4. *“Zadovoljna sam postignutim uspjehom do sada i sa zadovoljstvom želim još mnogo znanja steći i naći se pred mnogim izazovima”*, konstatovale jedna od ispitanika. (iz upitnika)



Grafikoni 4. Odgovori na 5. i 10. pitanje.

Svakako i ovdje je uloga nastavnika presudna. On treba podržavati i poticati učenika u njihovoj kreativnosti, ali i poteze koji mogu imati i nepovoljne ishode, zbog kojih učenici ne bi trebali imati eventualne probleme zbog svojih stavova, sa kojima se nastavnik ponekad i ne slaže. Ne mogu se isprobavati nove stvari, ako od učenika tražimo upotrebu samo uhodanih formi i klišea. Naprotiv, moramo im dati podršku u iznošenju svih mogućih stavova, vezanih za bilo koji problem, a onda učiti ih da analiziraju sve mogućnosti i da metodom eliminacije dođu do željenih rezultata. Nema kreativnosti i avanturizma kod učenika, ako ih budemo kočili i kažnjavali za netačno izrečene stavove. Tako ćemo imati “ziheraše” i “klimoglavce” koji ništa dobro ne mogu donijeti ni sebi ni zajednici.

Nastavnici smatraju da su za njihov uspješan rad sa učenicima nephodne sljedeće karakteristike:

- da poznaje predmet koji predaje,
- da je sposoban da povezuje matematiku sa drugim predmetima,
- da voli djecu,
- da poznaje razvojne probleme djece,
- da zna više od prosječnih ljudi, da je svestran i dovitljiv,
- da ima smisla za humor,
- da ima jaku volju,
- da razvija pozitivne međuljudske odnose, da je suptilan i ljubazan,
- da vjeruje svojim učenicima,
- da cijeni neobične ideje,
- da dozvoljava i potiče samoinicijativu.

Iz ovih osobina proizilazi da nastavnici veoma cijene one osobine svojih kolega, koje su kompatibilne sa zahtjevima sa početka ovog rada. Naročito zadnjih nekoliko osobina koje nastavnici najviše cijene, potiču kod učenika usmjeravanje ka razvoju pozitivnih međuljudskih odnosa, odgovornosti, te kreativnosti i nezavisnosti.

Isto istraživanje govori i o poželjnim karakteristikama nastavnika sa aspekta učenika:

- duhovitost, odmjeranost i staloženost,
- ozbiljnost, prijatnosti i ljubaznost,
- kulturno ponašanje, otvorenost i snalažljivost,
- pažljivost, saosjećajnost i spremnost za praštanje,
- pristupačnost, skromnost i da se lahko ne ljuti,
- upornost, radinost i iskrenost,
- zanimljivost i duhovitost u izlaganju,
- ne prekida učenike kada odgovaraju, **ne čita iz knjiga prilikom predavanja,**
- pravilno ocjenjuje, često ispituje učenike,
- blago ocjenjuje, javno daje ocjenu te dozvoljava učenicima da se koncentriraju.

Očito je da učenici veoma cijene osobine koje promiču otvorenost, odmjerenost, ozbiljnost i prijatnost, što veoma podstiče razvoj odgovornosti i poštivanja ljudi, uopće. Interesantno je da su učenicima važnije te osobine nastavnika, od njegove usmjerenosti na gradivo i permanentno ocjenjivanje učenika. Sagledavajući poželjne osobine nastavnika iz oba ugla, može se zaključiti da posjedovanje i konzumiranje većine njih, garantuje razvoj i sticanje poželjnih osobina učenika, mladog čovjeka. Mogla bi se napraviti i funkcionalna povezanost između stavki ova dva skupa osobina. No, to bi iziskivalo puno više vremena i prostora. Reklo bi se da je ovo razmatranje izvan teme rada, ali to je samo prividno tako. Uz ono što se izučava, ključni su oni koji se podučavaju i oni koji podučavaju ili koordiniraju proces izvođenja nastave. U svakom slučaju, osobine iz zadnjeg podnaslova su u uskoj povezanosti sa osobinama i načinom rada nastavnika, te stila vođenja učenika u procesu podučavanja. Ne može se očekivati da učenici gaje tolerantnost ako je ne doživljavaju od svog nastavnika i tako redom. Ne može se reći da učenici ništa ne uče, kada je to u suprotnosti sa definicijom samog učenika, gdje je u korijenu imena glagol učiti. Tu smo mi da nađemo nove strategije učenja, koje će uspjeti kod učenika izazvati veću mobilnost.

Zaključak i preporuke

Imajući u vidu istraživanje, može se zaključiti da su rezultati pokazali da su studenti zadovoljni ili veoma zadovoljni sa dostignućem skoro svih atributa dobro obrazovane ličnosti. To zadovoljstvo je visoko ili veoma visoko izraženo u opsegu između 70% i 86% po svim postavljenim pitanjima. Stoga je moguće ustvrditi da je postavljena hipoteza potvrđena, tj. da se “nakon završetka općeg obrazovanja, u dovoljnoj mjeri, postižu dobri rezultati u formiranju kompetencija poželjne ličnosti”.

Neke bi to možda i zadovoljilo. Međutim odgovori na otvoreno pitanje sa slobodnim komentarima studenata ukazuju na oprez. Može se postaviti pitanje da li bi neka druga istraživačka metoda npr. intervju ili upitnika sa otvorenim pitanjima, dala iste rezultate? To, svakako, može ostati nedoumica i predmet dodatnog istraživanja. No, bitnije je pitanje o tome kako bi matematika bila od veće koristi u dostizanju gore postavljenih zahtjeva.

Imajući to u vidu se daju preporuke, koje su nastale kao plod proučavanja mnogih teoretičara i praktičara, koje mogu biti od korissti nastavnicima matematike u cilju učinkovitijeg postizanja gornjih zahtjeva.

Preporuke

Može se preporučiti da nastavnik matematike ne mora biti okorjeli znanstvenik i da se u nastavi matematike moraju znanstvene metode staviti u prvi plan. Dakle, prvo trebamo koristiti heurističko (otkrivačko) razmišljanje, pretpostavke i ideje kako bismo izgradili i pripremili teren za generalizaciju ili čak izvođenje pravih dokaza. Usmjeravanje na iskazivanje tvrdnji i njihovo dokazivanje, bez prethodnog ukorjenjavanja toga u učeničku kulturu, nema opravdanje, ako hoćemo da od njih

napravimo samomisleće i korisne članove zajednice. U nastavi matematike, učenici mogu novo gradivo prvo ispitivati sa svih aspekata, iznijeti sve svoje stavove vezane za nastavno gradivo, a zatim analizirajući sve te aspekte, poopštavati stvari i postepeno prelaziti na “strožiji” matematički nivo. Ovakav način rada, učenicima daje mogućnost da rade vlastitim tempom te da se više poštuju **razlike među učenicima i njihovim gledištima**

Nastava je živ proces, koji se ne može u potpunosti isplanirati i kanalisati. Svaki matematički problem je unikatan i ima svoj početak i kraj. U svom rješavanju se nailazi na nešto otkrivačko i stvaralačko. Zato je potrebno da nastavnik u svojim učenicima istinski razvija radoznalost, te sklonost za samostalan umni rad i da im ukazuje na putove do novih otkrića, kako bi oni shvatili da “svi putevi vode u Rim”. Kreativan nastavnik matematike u kreativnoj nastavi ima velike izgleda da kod svojih učenika razvije kreativne osobine, naročito osjećaj za učinkovito nošenje sa **nepoznatim stvarima i izazovima**.

Samo kukavice rade opći slučaj. Pravi učitelji se bave primjerima. (Brueckler, 2006)

Opće poznato je da se učenici uglavnom ugledaju na svoje nastavnike, od toga da se profesionalno usmjeravaju zbog njih, ili oponašanjem usvajaju čak i neke njihove karakteristike. Dakle, za učenike, matematika, kao nauka, bar u većoj mjeri, nije motiv za njeno proučavanje. Isti autor (Brueckler, 2006) kaže da izreći teorem i onda ići dalje je logično, ali to nije podučavanje jer ne doprinosi razumijevanju. Nisu dozvoljene netačnosti, ali dozvoljena su pojednostavljenja. Odnosno, veći je interes upotrebljavati induktivnu nastavu (od jednostavnih problema i primjera ka generalizaciji), nego deduktivni pristup (izricanje tvrdnji, formula, a onda njihova primjena), jer se induktivnim pristupom u prvi plan stavlja učenik i njegova aktivnost i kreativnost, što više ide u prilog u postizanju poželjnih osobina i vještina učenika. U principu je važno učenike pridobiti za interes za matematikom.

Nastavnici su “najveći krivci” za učenikovo interesovanje za matematikom. Oni pokreću učenike da se bave matematikom, svojim ponašanjem znaju preneti na učenike **odgovornost, odnos prema radu, poštivanje, slušanje drugih, prilagodljivost, isprobavanje novih stvari, tolerantnost** i druge veoma važne osobine koje su ključne za davanje pečata u razvoju ljudske ličnosti.

Zato bi nastavnik trebao znati različitim metodama zainteresirati učenike da usvoje (a ne nauče napamet) sve predviđeno gradivo i tako steknu odgovarajući nivo znanja i vještina iz matematike. Ukratko, poželjana nastavnik, u tom slučaju, bi trebao biti:

- uspješan motivator,
- da se sviđa svojim učenicima,
- da mu vjeruju i da ga se ne boje,
- da učenici uoče njegovo zalaganje za njihovo dobro,
- da im ne prijeti negativnim ocjenama,
- da pismeni ispiti ne budu jedino mjerilo za ocjenjivanje njihovog znanja,
- da na učenike prenosi i odgojne vrijednosti,

- da redovno prate što se zbiva u znanosti i pedagoškoj praksi,
- da zna pripremati “teren” za formiranje novih pojmova i da ih analizira sa svojim učenicima, umjesto da se samo bavi definicijama i teoremama.

Ako nastavnik stalno upotrebljava iste strategije podučavanja,
a učenik je stalno neuspješan, ko od njih dvojice, ustvari, sporo uči?

Eric Jensen

Osim toga, udžbenici znaju ograničiti nastavnika u primjeni kreativnih pristupa. Kreativnost nastavnika matematike često trpi zbog pretjeranog oslanjanja na način obrade matematičkih sadržaja u udžbenicima (Kurnik, 2008). Zato je potrebno da se strategije učenja rade zajedno sa učenicima koji znaju koristiti i druge izvore. Ukratko, treba zaboraviti na stereotipove o tradicionalnoj nastavi i izvođenju sata obrade: uvod, ponavljanje potrebnog obrađenog gradiva, glavni dio sa obradom nepoznatih sadržaja, te završni dio sa utvrđivanjem predenog gradiva i zadavanjem domaće zadaće. Umjesto toga, nastavnicima se preporučuje da:

- otvaraju problemsku situaciju u kojoj će se nenametljivo ponoviti potrebno gradivo za novu lekciju,
- pokazuju zgode ilustracije i primjere,
- zadaju motivacijske zadatke,
- problem prikazuju sa očiglednim primjerima,
- daju neki zanimljivi podatak iz povijesti,...

Nastavnik se treba sjetiti svog učeničkog staža. Treba raditi drugim ljudima ono što bi volio da drugi rade njemu ili, tačnije, da radi djeci ono što bi volio da drugi rade njegovoj djeci – učenicima.

Pored navedenih preporuka, može biti od koristi i slijedeće:

- treba dozirati predavačku nastavu (učenik nije “magacin” za punjenje znanjem),
- smišljeno treba koristiti razne nastavne metode (osmišljeni obrazac ponašanja i djelovanja s ciljem olakšavanja učenja i poboljšavanja ishoda učenja),
- veću pažnju davati očiglednosti i primjenjivosti u nastavi,
- treba znati da je nastavni sat kreativna etapa nastavnog procesa,
- da svaki učenik treba biti aktivni sudionik u nastavi,
- da je vrlo djelotvorno saradničko učenje u **grupama ili u parovima**, te u projektima grupe pretvarati u **timove**, sa njegovanjem prihvatljive međuvršnjačke korespodencije, iznošenja **stavova** koristeći snagu argumentata, a ne argumente autoriteta i moći,
- uspješno podučavanje se može opisati kao "organizirani kaos",
- da učenici pokazuju veći interes za učenje kada se upotrebljavaju nove tehnologije, čime se postiže elegancija u rješavanju zadataka gdje se učenici upućuju da to uočavaju i koriste,

- treba učenike ohrabrivati pohvalama i prijateljski se odnositi prema njima,
- ne dopustiti da učenici “zaglibe” s negativnom ocjenom, već im dopustiti da “ispravljaaju” dio po dio gradiva.

Kako je već rečeno, **timski rad** je veoma značajan za uspješan rad i dobru komunikaciju. Jasno je da svaka grupa nije tim, pogotovo onaj uspješni. Uspješne timove karakteriše:

- jasno i transparentno definirani opći i specifični ciljevi
- uspješan voditelj,
- pojedinačna i zajednička odgovornost,
- poštivanje razlika,
- otvorena komunikacija,
- efikasno donošenje odluka,
- međusobno povjerenje,
- konstruktivno rješavanje konflikata (Miljković i Rijevec, 2005),

iz čega je jasno da pravilno korištenje timskog rada u nastavi matematike može kod učenika razviti veoma poželjne karakteristike sa početka ovog teksta.

Mnogi nastavnici kažu da njihove učenike ništa ne interesuje, da su nemirni i td. To je možda samo djelimično tačno. Sama riječ “učenik” je nastala od riječi “učiti”. Prije se treba upitati šta je sa nastavnim programom ili našim metodama podučavanja. U takvim slučajevima se nastavnicima preporučuje da se koriste umjesnim nastavnim metodama i da se profesionalno ponašaju. Dobro isplanirane nastavne aktivnosti, koje će poticati i održavati učeničku pažnju, nastavnikovo zanimanje, skoro zanos (“uživljenost”) za posao koji obavlja, te osjetljivost nastavnika za uzajamno poštovanje i razumijevanje će zasigurno dati određene rezultate u mobilisanju učenika u procesu podučavanja, jer nastavnik koji voli ono što predaje prenosi entuzijazam na učenike (Kovač, 2010).

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Nastava matematike u osnovnim školama u Hrvatskoj – nastavnička perspektiva

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Ključne riječi: mišljenje nastavnika, osnovne škole, nastava matematike, matematička kompetencija, učeničke aktivnosti, nastavna sredstva i pomagala

Prezentacija se zasniva na rezultatima empirijskog istraživanja o nastavničkoj percepciji nastave matematike u osnovnim školama u Hrvatskoj. Istraživanje je provedeno 2010. god. na reprezentativnom uzorku od 625 nastavnika iz cijele Hrvatske primjenom anketnog upitnika. Upitnik se sastojao pretežno od pitanja zatvorenog tipa kojima su nastavničke procjene i uvjerenja o nastavi matematike mjereni Likertovim skalama od pet stupnjeva te od manjeg broja pitanja otvorenog tipa.

Rezultati istraživanja indiciraju na prevalenciju tradicionalnog pristupa nastavi matematike u odnosu na suvremeni (konstruktivistički) u većini ispitivanih dimenzija nastave matematike kao što je razumijevanje matematičke kompetencije učenika, primjena nastavnih metoda i nastavnih sredstava te stav prema nastavi matematike. Kada je riječ o viđenju matematičke kompetencije podaci pokazuju da nastavnici pri procjeni učeničkih postignuća/ishoda koji opisuju matematičku kompetenciju manju važnost pridaju onim postignućima koja su karakteristična za suvremeni koncept matematičke kompetencije (PISA) nego tradicionalnim. Npr. samo je 13% do 18% nastavnika odgovorilo da među pet najvažnijih ishoda spada učeničko argumentiranje vlastitih ideja o matematičkim pojmovima i postupcima, kritičko promišljanje matematičkih pojmova i postupaka te međusobno komuniciranje vlastitih ideja i rezultata, dok je tradicionalnim učeničkim postignućima pridat veći značaj – 32% do 35% nastavnika je među najznačajnije učeničke ishode ubrojilo njihov marljiv i savjestan rad na nastavi, stjecanje sigurnosti u rješavanju osnovnih tipova matematičkih zadataka i savladavanje nastavnog programa.

Prevalencija tradicionalnih obilježja nastave matematike vidljiva je i iz podataka o primjeni nastavnih metoda i nastavnih sredstava u nastavi. Podaci pokazuju da učenici na nastavi matematike najčešće rješavaju zadatke: vježbaju zadatke slične zadacima koji će biti na ispitu ($M = 4.21$, $SD = .73$), rješavaju zadatke postupcima koje im je pokazao njihov nastavnik ($M = 4.19$, $SD = .55$),

na svome mjestu rješavaju standardne zadatke ($M = 4.08$, $SD = .67$), dok najrjeđe samostalno eksperimentiraju i istražuju matematičke pojmove i zakonitosti ($M = 2.43$, $SD = .75$), pismeno obrazlažu vlastite ideje i zaključke ($M = 2.46$, $SD = .92$), samostalno rade svoje učeničke projekte ($M = 2.73$, $SD = .81$). Slično je i s nastavnim sredstvima i pomagalicama – najčešće se koriste tradicionalna sredstva kao što su udžbenici i priručnici, a najrjeđe informacijsko-komunikacijska tehnologija (ICT).

Indikativno je da su stariji nastavnici i nastavnici s višim obrazovanjem skloniji primjeni suvremenih nastavnih metoda i nastavnih sredstava nego oni mlađe dobi (posebice oni u dobi do 29 godina starosti) i s visokim (fakultetskim obrazovanjem). Izuzetak je primjena ICT, koju češće koriste mlađi nastavnici. Ove razlike tumačimo duljim radnim iskustvom i opsežnijim pedagoškim obrazovanjem starijih nastavnika, za razliku od mlađih generacija sa završenim nastavničkim fakultetima u čijem je školovanju akcent bio na matematičkom obrazovanju.

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Zahvala našim uglednim matematičarima (poster)

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Sažetak. U zadnjih nekoliko godina Hrvatska je ostala bez niza uglednih matematičara koji su dali veliki doprinos nastavi matematike u Hrvatskoj. Taj doprinos možemo grubo podijeliti u dvije bitno različite, ali ipak nerazdvojne cjeline.

Dva su matematička velikana – prof. dr. Svetozar Kurepa (1929. – 2010.) i prof. dr. Krešimir Horvatić (1930. – 2008.) na ovim prostorima (Osijeku) pokrenuli, organizirali i koordinirali provedbu i realizaciju dopunskoga studija iz matematike nastavnoga smjera sedamdesetih i osamdesetih godina prošloga stoljeća ispred Matematičkoga odjela Prirodoslovno-matematičkoga fakulteta iz Zagreba. Dopunskim studijem stotinjak je polaznika steklo visoku stručnu spremu i diplomu učitelja matematike. Time je učinjen dalekosežni poticaj stručnom, znanstvenom i gospodarskom zamahu na prostorima Slavonije, Baranje i dijelu Bosne i Hercegovine.

Drugu skupinu matematičara ponajprije ćemo pamtiti po sjajnim knjigama, časopisima, provedbi matematičkih natjecanja, kvizova te drugih aktivnosti s ciljem popularizacije matematike, ali i znanstvenoga doprinosa nastavi matematike. To su profesor Boris Pavković (1931. – 2006.) i akademik Stanko Bilinski (1909. – 1998.), zatim Dominik Palman (1924. – 2006.) i Zdravko Kurnik (1937. – 2010.), redom profesori s Matematičkoga odjela Prirodoslovno-matematičkoga fakulteta u Zagrebu. Posebna zahvala pripada i osebujnoj osobi – matematičaru, književniku i japanologu, koji je radio na Fakultetu strojarstva i brodogradnje prof. dr. sc. Vladimiru Devidu (1925. – 2010.). Zaljubljenik u matematiku i umjetnost uvijek je nalazio vremena pisati za najmlađe.

Organizacijski odbor želi reći hvala velikim učiteljima matematike!

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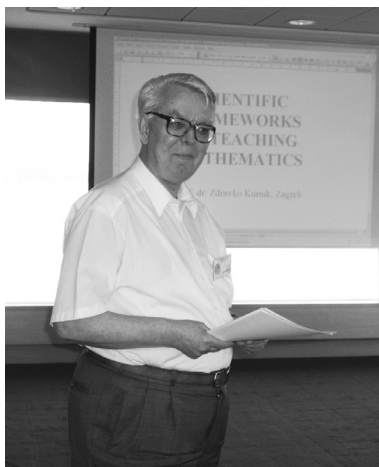
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Zdravko Kurnik **(Travnik, 2. 2. 1937. – Zagreb, 25. 7. 2010.)** **svestrani hrvatski matematičar**

Mirko Polonijo i Sanja Varošaneć

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Prof. dr. sc. Zdravko Kurnik na Prvom međunarodnom skupu
MATEMATIKA I DIJETE u Osijeku, 13. travnja 2007. god.

Prije četiri godine, u Osijeku je 13. travnja 2007. održan Prvi međunarodni znanstveni kolokvij *Matematika i dijete* u organizaciji Učiteljskog fakulteta i Odjela za matematiku Sveučilišta J. J. Strossmayer. Tema skupa je bila *kako učiti i poučavati matematiku*. Očekivano i s velikim zanimanjem svih sudionika skupa, svojim izlaganjem sudjelovao je prof. dr. sc. Zdravko Kurnik, najznačajniji hrvatski metodičar matematike. Naslov njegova izlaganja je bio “Znanstveni okviri nastave matematike”, tema koja ga je dugo zaokupljala i koja je u njegovim promišljanjima i cjelokupnom opusu zauzimala središnje mjesto. Dokazuje to i upravo isti takav naslov knjige prof. Z. Kurnika objavljene 2009. u biblioteci *Metodika nastave matematike* u izdanju zagrebačkog *Elementa*. Trebala je to biti prva u nizu knjiga iz metodike matematike. U njima je prof. Z. Kurnik želio okupiti mnoge svoje objavljene tekstove i buduća promišljanja posvećena metodici nastave, tom važnom a često zanemarenom i u nas nedovoljno priznatom području matematike.

Na veliku žalost cijele naše matematičke zajednice, u ljeto 2010. godine prof. Zdravko Kurnik je iznenada preminuo. Njegov prerani i neočekivani odlazak izazvao je mnogobrojne dojmive usmene i pismene reakcije, prepune poštovanja i zahvalnosti. Znajući i poznavajući koliko je toga napravio za hrvatsku matematičku "obitelj", posebice za poučavanje matematike, jasnije utemeljivanje i definiranje metodike matematike te za popularizaciju matematike na svim razinama i kroz dugi niz godina, svi ostajemo impresionirani postignućima profesora Kurnika.

Zdravko Kurnik je diplomirao 1960. godine i nakon trogodišnjeg rada na gimnaziji dolazi u Geometrijski zavod Prirodoslovno-matematičkog fakulteta Sveučilišta u Zagrebu, na poziv tadašnjeg predstojnika, profesora Stanka Bilinskog.

Magistrirao je 1968. temom *Teorija mjerenja sadržine u neeuklidskoj geometriji* a doktorirao 1989. temom *Granične vrijednosti za krivulje u euklidskim i neeuklidskim prostorima*. Stručno se usavršavao u Bonnu na području Riemannove geometrije pod vodstvom poznatog geometričara profesora W. Klingenberga. Bavio se diferencijalnom geometrijom, te geometrijom euklidskog i neeuklidskih prostora.

U nastavnoj djelatnosti izradio je programe i izvodio niz kolegija: *Konstruktivna geometrija*, *Euklidski prostori*, *Pedagogija matematike*, *Metodika nastave matematike I, II*, *Seminar iz metodike nastave matematike II*, *Nacrtna geometrija*, *Elementarna matematika*, *Metodika početne nastave matematike*. Bio je voditelj Katedre za metodiku i računarstvo od 1994. do umirovljenja 2002.

Interes za metodička pitanja učenja i poučavanja matematike prof. Zdravko Kurnik iskazivao je već od početka svoga nastavničkog rada. Primjerice, bio je vrlo aktivan u organiziranju skupa GIRP-12 godine 1983. kada se sastanak te međunarodne grupe za istraživanja u području pedagogije matematike (GIRP = franc. Groupe international de recherche en pédagogie de la mathématique) održavao u Zadru. Međutim, upravo razdoblje od 1995. do njegove prerane smrti karakterizira intenzivan rad prof. Z. Kurnika u području metodike nastave matematike. Neumorno i ustrajno se tada bavi pitanjima unapređenja i osuvremenjivanja nastavnog procesa, cjeloživotnog obrazovanja nastavnika matematike, rada s naprednijim učenicima i popularizacijom matematike.

Stalni mu je cilj bio usustaviti metodiku, učiniti je jasnom i preglednom. Istodobno, uvijek je naglašavao svoje mišljenje da je od strane naših matematičara nedovoljno priznata važnost metodike matematike. To ga je "gonilo" da još ustrajnije piše i govori o temama matematičke edukacije. Posljedica takvih pogleda i stremljenja prof. Z. Kurnika i njegova neumornog rada veliki je broj publikacija, mnoštvo knjiga i članaka, te originalnih doprinosa u organiziranju i oblikovanju rada s učiteljima i profesorima matematike te njihovim učenicima.

Više nego zaslužen i opravdano profesor Zdravko Kurnik je dobio državnu Nagradu "Ivan Filipović" za 2006. godinu za životno djelo u području znanstvenog i stručnog rada.

A stigao je mnogo i uspješno pisati i izvan područja matematike. Objavio je tri knjige aforizama znakovitih naslova (*Na oštrici riječi*, *Otkucaji srca*, *Budno oko*),

dobio osam nagrada u Japanu za svoje haiku pjesme, a upravo je u 2010. godini “navršio” 60 godina priznatog enigmatskog rada.

Profesor Zdravko Kurnik je u radu i suradnji bio neumoran, sistematičan i precizan, uredan i u svakom pogledu istinoljubiv, savjestan i svestran. Uvijek s matematikom u težištu, usredotočen na one aspekte matematike i matematičkog djelovanja koji se ponekad ne smatraju znanstveno najvažnijima. Istodobno s mnogim interesima izvan matematičkih granica. Bio je smiren ali uporan i temperamentan u odbrani vlastitih stavova i gledanja. Nezaobilazno zvan i pozivan na sve “naše” skupove u bilo kojoj vezi s nastavom matematike.

Pamtit će se metodički radovi profesora Zdravka Kurnika, njegove knjige i mnogobrojni tekstovi koji će i dalje motivirati i koji će se još dugo navoditi, prepisivati, dorađivati, “prepravljati” pa s vremenom, nije isključeno, i potpisivati. Malo tko će imati strpljivosti pokušati tu složenu problematiku metodike matematike sistematizirati i obuhvatiti njegovom strpljivošću i minucioznošću. Malo tko će imati sposobnosti i znanja to učiniti. Malo tko će to moći s njegovom strašću, voljom i marom realizirati.

Lik i djelo profesora Zdravka Kurnika trajati će i dalje kroz sve nas koji smo ga poznavali, voljeli i cijenili, ali i puno više od toga, i puno trajnije, trajat će kroz bezbrojne stranice njegovih raznovrsnih tekstova i uradaka, kroz njihovu sadržajnost, raznolikost i neprikosnovenu kvalitetu.

Detaljnije informacije o životu i radu profesora dr. sc. Z. Kurnika mogu se naći u sveobuhvatnom članku Sanje Varošaneć i Vladimira Voleneca (*Glasnik matematički-Prilozi* 45(65)(2010), u tisku), te nadahnutim i znalačkim tekstovima

Branimira Dakića (*Matematika i škola* br. 56 (2010), str. 2–6; *Matematika i škola* br. 43 (2008), str. 142–143, *Matematika i škola* br. 16 (2002), str. 38–41) i Sandre Gračan, *Matematika i škola* br. 56 (2010), str. 7).

S. Varošaneć je u spomenutom članku priredila popis matematičkih publikacija koje je objavio prof. dr. sc. Zdravko Kurnik i ovdje taj popis prenosimo kako bi bio dostupan širem matematičkom čitalačkom krugu.

Popis matematičkih znanstvenih i stručnih knjiga i članaka prof. dr. sc. Zdravka Kurnika

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Kazalo imena

-
-
- Abikoff, H. , 233, 536
Abrami, P. C. , 98, 367
Abuzaid, R. A. , 149, 417
Adedeji, T. , 156, 424
Agić, H. , 246, 551
Ala-Mutka, K. , 129, 397
Alexander, R. , 28, 294
Allwein, G. , 46, 327
Alsina, C. , 232, 535
Alvir, J. , 232, 536
Ambrus, A. , 70, 339
Amir, R. , 144, 441
Andersson, S. , 139
Andrilović, V. , 231, 525
Antić, S. , 26, 292
Antonijević, R. , 26, 292
Arambašić, L. , 11, 190, 267, 472
Ausubeli, 231, 525
Ayalon, M. , 50
Ayotola, A. , 156, 424
Babić, N. , 23, 279
Bakovljević, M. , 231, 525
Ball, D. L. , 62
Bandura, A. , 150, 222, 418, 515
Baranović, B. , 189, 239, 255, 471, 544, 560
Barth, B. M. , 173
Barton, B. , 27, 293
Barwise, J. , 46, 327
Bass, H. , 50
Benšić, M. , 12
Benson, P. G. , 89, 139, 144, 357, 407, 412
Bernard, R. M. , 98, 367
Bertić, D. , 89, 358
Biembengut, M. S. , 232, 535
Birenbaum, M. , 142, 410
Bishop, A. , 112, 181, 379, 458
Bjelanović Dijanić, Z. , 126, 198, 394, 491
Blum, W. , 232, 528
Bodrovski, K. , 22, 278
Bognar, B. , 196, 199, 210, 232, 481, 492
Bork, A. , 129, 397
Borokhovski, E. , 98, 367
Bouleau, N. , 232, 535
Braš Roth, M. , 248, 553
Brajša, P. , 220, 512
Brajković, S. , 174, 451
Bratanić, M. , 231, 525
Brewer, C. , 126, 394
Brincková, J. , 170, 438
Broadbridge, P. , 126, 395
Broutin, C. , 179, 456
Brown, T. , 232, 527
Brueckler, F. M. , 158, 163, 426, 431
Bruner, J. S. , 180, 221, 440, 457, 513
Bryant, M. J. , 157, 425
Bucat, B. , 159, 427
Büchel, F. , 51
Cai, Y. , 157, 425
Cañadas, M. C. , 50, 56
Cardy, R. L. , 80, 349
Carpenter, T. , 233, 536
Castro, E. , 50, 56
Chandrasekaran, R. M. , 145, 413
Chapman, O. , 27, 293
Chavez, J. , 127, 395
Chen, N.-S. , 149, 416
Chick, H. L. , 28, 294
Christou, C. , 49, 127, 395
Clark, J. M. , 181, 458
Clark, R. E. , 92, 361
Clements, D. , 22, 278
Coffin, S. T. , 170, 438
Cohen, L. , 202, 231, 495, 525
Cole, M. , 108, 181, 376, 458
Coleman, L. , 127, 395
Colker, L. J. , 21, 277
Confrey, J. , 232, 535
Cooper, T. J. , 127, 395
Cramer, K. , 31, 297
Cross, D. I. , 240, 544
Cunningham, S. , 46, 327
Currier, S. , 33, 300
Čižmešija, A. , 90, 195, 239, 359, 477
Čudina-Obradović, M. , 174, 231, 456, 525
Dadić, Ž. , 235, 539, 544
Davies, D. , 173
Davis, G. E. , 32, 298
De Leon, O. , 127, 395
Dejić, M. , 26, 292
Delors, J. , 243, 548
Deschenes, D. , 186, 462
Dietz, L. , 185, 462
Dinka, J. , 179, 456
Dinov, I. D. , 127, 395
Divjak, B. , 92, 150
Domović, V. , 234, 538
Dorothy, E. , 179, 456
Dowson, M. , 151, 419
Drent, M. , 130, 398
Dress, A. W. M. , 116, 384
Dryden, G. , 219, 512
Duncan, T. , 151, 418
Džuriková, I. , 170, 438
Đorđević, J. , 231, 525

- Đuranović, M. , 191, 473
 Đurović, I. , 190, 472
 Đurović, J. , 190, 472
 Ebeling, D. , 186, 462, 465
 Edwards, M. , 233, 536
 Egerić, M. , 26, 282
 Elezović, N. , 81, 196, 350, 489
 Engelbrecht, J. , 127, 395
 Erjavec, Z. , 92, 95, 361, 364
 Ernest, P. , 105, 373
 Etchemendy, J. , 46, 327
 Even, R. , 50
 Falus, I. , 70, 339
 Farkas, G. , 22, 278
 Fast, L. A. , 157, 425
 Felder, R. M. , 141, 409
 Fennema, E. , 233, 527
 Ferla, J. , 157, 425
 Freudenthal, H. , 110, 378
 Galbraith, P. , 232, 535
 Gardiner, A. , 170, 438
 Gardner, M. , 170, 438
 Garner, H. , 185, 462
 George, D. , 196, 489
 Ginsburg, H. , 233, 528
 Giudicci, C. , 173, 449
 Glaser, R. , 51
 Glasnović Gracin, D. , 127, 171,
 211, 234, 395, 439, 503, 538
 Glasser, W. , 189, 471
 Govindarajan, M. , 145, 413
 Graves, B. , 231, 525
 Graves, M. F. , 231, 525
 Gregurović, M. , 254, 558
 Gudjons, H. , 231, 525
 Haggarty, L. , 234, 538
 Hanley, U. , 232, 535
 Hanna, G. , 50
 Harding, A. , 127, 395
 Hart, S. , 185, 462
 Hativa, N. , 142, 410
 Haury, D. L. , 171, 439
 Haviar, M. , 170, 438
 Haymore, J. , 34, 300
 Heath, R. V. , 170, 438
 Hejini, M. , 173
 Henderson, S. , 126, 395
 Henn, H. W. , 232, 535
 Hersh, R. , 108, 376
 Heymann, H. W. , 235, 539
 Ho, O. , 170, 438
 Hodgson, B. R. , 27, 232, 293, 535
 Hoffman, B. , 151, 419
 Hohenwarter, M. , 201, 494
 Howe, A. , 173
 Hoyles, C. , 50
 Hurh Kim, R. , 170, 438
 Huson, D.H. , 119, 387
 Ikeda, T. , 232, 535
 Inhelder, B. , 62, 184, 461
 Irović, S. , 21, 277
 Ivić, I. , 26, 292
 Jagodić, B. , 197, 489
 Jain, S. , 151, 419
 Jarrett, D. , 159, 427
 Jelas, Z. M. , 144, 411
 Jembrih , 235, 539
 Jensen, J. H. , 104, 371
 Jensen, E. , 252, 557
 Jones, L. , 232, 535
 Jursić, A. , 189, 471
 Jurčić, M. , 80, 196, 349, 478
 Kamii, C. , 173
 Karadag, Z. , 200, 493
 King, J. P. , 168, 436
 Kinshuk, Wei, C.-W. , 149, 416
 Kirsten, A. , 175, 452
 Klasnić, I. , 189, 471
 Klauer, K. J. , 50
 Klein, F. , 27, 293
 Klippert, H. , 196, 489
 Kobetić, D. , 463
 Kolaczek, M. , 179, 456
 Kostović Vranješ, V. , 88, 357
 Kotler, P. , 243, 548
 Kovač, A. , 254, 558
 Kozma, R. , 92, 361
 Krampač-Grljušić, A. , 185, 462
 Krowatschek, D. , 196, 489
 Krsnik, R. , 173, 444
 Krstanović I. , 139, 407
 Küchemann, D. , 49
 Kudo, M. , 145, 413
 Kurnik, Z. , 127, 199, 252, 257,
 395, 492, 557, 562, 563
 Kyriacou, C. , 232, 528
 Latyk, O. , 179, 456
 Leder, G. C. , 240, 544
 Leou, S. , 233, 527
 Lepičnik Vodopivec, J. , 21, 277
 Lewis, J. L. , 157, 425
 Liebeck, P. , 24, 170, 173, 179, 221,
 280, 438, 441, 456, 513
 Lim, K. H. , 31, 297
 Linares, O. , 232, 528
 Lingefjärd, T. , 232, 535
 Lisak, N. , 186, 463
 Liu, C.-C. , 149, 416
 Loef, M. , 233, 536
 Long, M. , 29, 295
 Loparić, S. , 159, 427
 Lotus, P. , 127, 395
 Lowerison, G. , 98, 367
 Lučić, Z. , 119, 387
 Ljubetić, M. , 88, 357
 Ma, L. , 28, 294
 MacNab, D. , 233, 527
 Manion, L. , 202, 495
 Marcetić A. , 139, 397,
 Marendić, Z. , 179, 456
 Marinić, I. , 186, 464
 Mariotti, M. A. , 50

- Markovac, J. , 217, 510
 Markočić Dekanić, V. , 254, 558
 Marks, R. , 34, 300
 Markuš, M. , 254, 558
 Massini, B. , 179, 456
 Masuyama, N. , 149, 417
 Matijević, M. , 23, 195, 196, 199, 279, 477, 481, 492
 McClave, J. T. , 84, 139, 144, 353, 400, 412
 McDougall, D. , 200, 493
 McGowen, M. A. , 32, 298
 McKeachie, W. , 151, 418
 McNamara, O. , 232, 535
 McRobbie, C. J. , 127, 395
 Mellissen, M. , 130, 398
 Merriam, B. Sharan, 244, 549
 Metijević, M. , 25, 281
 Mignon, P. , 179, 456
 Mihanović, V. , 186, 463
 Milin Šipuš, Ž. , 90, 239, 359, 544
 Miljković, D. , 254, 558
 Mišurac Zorica, I. , 89, 232, 357, 526
 Mitton, J. , 179, 456
 Mitton, S. , 179, 456
 Molnár, E. , 113, 381
 Monk, D. , 32, 298
 Morrison, K. , 202, 503
 Mrkonjić, I. , 35, 196, 302, 488
 Muller, E. , 232, 535
 Muschla, G. R. , 170, 438
 Muschla, J. A. , 170, 438
 Mužić, V. , 233, 536
 Naimie, Z. , 142, 410
 Nakahara, T. , 181, 458
 Niss, M. , 104, 112, 232
 Norman, E. , 170, 438
 Norton, S. , 127, 395
 Ostroški, M. , 98, 364
 Paivio, A. , 181, 458
 Pajares, F. , 152, 420
 Papageorgiou, E. , 49, 62
 Pastuović, N. , 186, 463
 Pavleković, I. , 185, 462
 Pavleković, M. , 89, 98, 141, 233, 367, 409
 Pešikan, A. , 26, 292
 Pejić, M. , 233, 527
 Pellegrino, J. , 51
 Peller, J. , 70, 339
 Penglase, M. , 29, 32, 295, 298
 Pepin, B. , 234, 538
 Perner, D. , 186, 469
 Peteh, M. , 173
 Peterson, P. , 233, 528, 535
 Phe, G. D. , 50, 58
 Piaget, J. , 62, 181, 220, 440, 458, 512
 Piaw, C. Y. , 149, 417
 Pickover, C. A. , 170, 438
 Piskalo, A. M. , 70, 339
 Polić, M. , 233, 530
 Pólya, G. , 52, 63, 163, 211, 431
 Ponte, J. P. , 27, 293
 Post, T. , 33, 300
 Pound, L. , 173
 Prediger, S. , 235, 538
 Preiner, J. , 201, 494
 Prentović, R. , 173
 Prok, I. , 113, 381
 Pumfrey, L. , 170, 438
 Punieand, Y. , 139, 407
 Rajić, V. , 89, 358
 Ramsden, P. , 98, 367
 Reš, J. , 197, 489
 Rešić, J. , 215, 242, 507, 547
 Redecker, C. , 139, 407
 Reid, D. , 51, 56
 Reis, S. M. , 81, 350
 Renzulli, J. S. , 81, 350
 Rijavec, M. , 254, 559
 Rinaldi, C. , 173, 442
 Ristić, Ž. , 244, 549
 Roberts, H. , 173
 Romstein, K. , 20, 21, 276, 277
 Rosbruch, N. , 233, 536
 Rose, J. , 33, 299
 Saaty, Thomas L. , 127, 395
 Salamon, 196, 488
 Sanchez, J. , 127, 395
 Sandusky, L. , 170, 438
 Sang, G. , 130, 398
 Sarama, J. , 22, 278
 Scavo, T. , 170, 438
 Scharnhorst, U. , 51, 62
 Schmid, R. F. , 98, 367
 Schneider, E. , 240, 545
 Schoenfeld, A. H. , 50, 63
 Selvarajan, T. T. , 80, 349
 Severinac, A. , 11, 18, 195, 267, 274, 477
 Shagholi, R. , 149, 417
 Sharma, M. , 189, 233, 471, 532
 Shimbo, M. , 149, 417
 Shulman, L. S. , 28, 294
 Silverman, L. , 143, 411
 Simon, W. , 170, 438
 Sinnich, T. , 89, 139, 357, 407
 Siraj, S. , 143, 411
 Skovsmose, O. , 109, 376
 Slatina, M. , 219, 231, 511, 525
 Slunjski, E. , 174, 451
 Soloman, B. A. , 141, 149, 409, 416
 Solomon, Y. , 233, 536
 Sotirović, V. , 173
 Southwell, B. , 29, 34, 295, 300
 Sprague, J. , 186, 462, 468
 Spurlin, J. , 144, 411
 Stacey, K. , 50, 63
 Starc, M. , 179, 456
 Steinberg, R. , 28, 34, 294, 300
 Stern, M. , 232, 536
 Stojanović, M. , 119, 387
 Stylianides, G. , 50, 63
 Suljić, S. , 215, 507
 Sun – Joo Shin , 46, 327
 Sun Lee, J. , 233, 528
 Surkes, M. A. , 98, 367

- Suzić, N. , 218, 231, 510, 525
Šiljković, Ž. , 80, 89, 349, 358
Štibrić, M. , 195, 239, 255, 477, 544, 560
Tamim, R. , 98, 367
Teknomo, C. , 145, 413
Teppo, A. , 70, 339
Thompson, A. G. , 31, 297
Thompson, P. W. , 31, 297
Tomović, D. , 26, 292
Tondeur, J. , 130, 140, 398, 407
Toyama, J. , 149, 417
Tucker, K. , 173
Usher, E. L. , 152, 157, 420, 425
Uzelac, Z. , 139, 407
Valcke, M. , 130, 140, 157, 398, 407, 425
Van Braak, J. , 130, 139, 398, 407
Van Hiele, P. , 64, 70, 333, 339
Varošanec, S. , 158, 426, 565
Verdet, J. P. , 179, 456
Verschaffel, L. , 232, 535
Vidaček Hainš, V. , 98, 367
Vigotszkij, L. S. , 184, 458
Vlahović-Štetić, V. , 11, 18, 179, 195,
267, 274, 456, 477
Vos, J. , 231, 525
Vrgoč, D. , 197, 489
Vukasović, A. , 231, 516, 525
Wade, C. A. , 98, 367
Walker, G. , 233, 536
Wang, S. , 232, 435, 535
Wayne, A. , 233, 528, 536
Wedge, T. , 104, 112, 371, 379
Weisstein, Eric W. , 170, 438
Weller, M. , 98, 368
Wertsch, J. , 184, 461
Wilburne, J. M. , 29, 34, 295, 301
Williams, K. , 157, 425
Williams, T. , 157, 425
Williams, L. , 231, 525
Wittmann, E. Ch. , 23, 276
Wolfram, S. , 127, 135, 395, 403
Woodhead, C. , 33, 299
Yackel, E. , 50, 63
Youngs, P. , 233, 528, 536
Zekić-Sušac, M. , 141, 409
Zimmerman, W. , 46, 327
Zvonarević, M. , 231, 525
Žić Ralić, A. , 186, 463, 468
Živčić-Bećirević I. , 157, 425

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